

# Criticality Avoidance

A new paradigm for congestion control based on science of phase transition

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**Abstract**—Network QoS control is generally difficult due to the complexity, dynamism, and limited measurability of networks. As an alternative, we seek a network phenomenon that is simple, universal and consequential to control. The result is a framework for proactive dynamic network congestion control that is based on the science of continuous phase transition. Key beneficial properties of continuous phase transition are its early onset warning signs and universality. The former allows the detection of proximity to congestion *before* its occurrence; while the latter implies that any criticality-based network control would likely be insensitive to network details and, in particular, not require any a-priori knowledge of the values of critical loads. Preliminary experimental results demonstrating these promises are presented.

**Keywords**—congestion; QoS; quality of service; phase transition; criticality; onset detection; control.

## I. INTRODUCTION

Quality-of-Service (QoS) control for communications networks is generally difficult due to the complexity, dynamism, and limited measurability of the network. These attributes arise because of the existence of a large number of interacting functional and architectural components that operate on different time scale, are opaque to each other, and yet can affect the same QoS metrics. For example, rate allocation can involve end-to-end congestion control, local scheduling, per-hop resource allocation, routing, and even human network operators [1], [8]. As a consequence, QoS controls tend to be ad hoc and incomplete.

The most well-known effort to reorganize and simplify network architecture evolves from the application of optimization decomposition to reverse engineer TCP [10], [11], [12], into using it as a layering principle [1], with numerous applications, such as traffic management at carriers [8] and a new version of TCP for high-speed long-latency networks [13]. In our view, a key benefit of this approach is that it allows systematic derivations of QoS controls that are grounded in principle and are less likely to suffer from the usual shortcomings of ad hoc, case-by case QoS controls.

Against this backdrop, the goal of this article is to suggest, for the purpose of controlling network congestion, another opportunity to simplify network control and reap similar (and additional, see below) benefits by viewing network congestion from the perspective of the well established science of continuous phase transition (CPT) or continuous criticality. We will offer evidence that network congestion is indeed a

CPT and that the properties of CPT afford criticality-based congestion control the aforementioned benefits.

The study of network congestion as a phase-transition phenomenon exists in the physics community. In [15] and [16], simulation results on wired networks show that the congestive transition is of the continuous variety. However, their scope was to understand congestive criticality, rather than to leveraging its properties for network control, as we desire to. The knowledge that congestive phase transition is continuous is emphasized here, because the behaviors of warning signs for continuous criticalities are well understood, while those for discontinuous criticalities are not. Incidentally, they are also called second-order and first-order phase transitions, respectively.

Certain shared properties of CPTs are particularly valuable due to their implications for network control. First, a system with CPT exhibits measurable, advanced warning signs, as it approaches the critical, or transition, point [18], [19], making it possible to proactively (vs. reactively) avoid the CPT. Such warning signs in complex dynamical systems including ecological, climatic, and financial are discussed in [17].

Second, CPTs are universal [18], [19] in the sense that all systems progressively “behave the same” (upon scaling) as a transition point is approached. (See Figure 1. and 2.) In particular, this implies that no a priori knowledge of the value of transition point is required for the control of a CPT. An issue congestion-control designers might be concerned with is how much network details matter. The answer is very minimal, at least for the explicit control of congestive criticality; this is a major simplification.

While network congestion is well studied, its warning signs and the value of its critical load via network information theory remain poorly understood. Employing the science of CPT allows for a systematic understanding of the former and renders the latter less important for network-congestion control.

The main contributions of this article are to draw attention to the significant potential of the CPT-view of network congestion and to report initial simulation results demonstrating the predictability and measurability of network congestion warning signs.

This work is solely supported internally by Telcordia Technologies, Inc. A US patent application is filed based on this work.

## II. CRITICALITY ONSET WARNING SIGNS

Formulating a phase transition requires identifying the pertinent degree of freedom (DoF), whose statistics constitute the phase indicating order parameter and the onset warning signs. For network congestion, the growth rate of queue occupancy, or length, is an appropriate DoF, as its mean value is zero when there is no congestion, but becomes positive in the congested phase, and hence is good indicator of congestion. For this reason, queue occupancy is popularly used in congestion control designs. Our approach is differentiated by our exploitations of the knowledge that the criticality associated with queue occupancy is a CPT [15], [16], in terms of early detection of congestion onset and the insensitivity of such detection to network details due to universality of CPT. Advanced nature of these detections will enable proactive, rather than reactive, network control, such as, routing around crowded regions and reducing traffic admissions, all before congestions actually set in. Universality implies high potential for such detection methods to succeed in many networking scenarios. To accentuate the universality of CPT, a figure in [15] showing the collapse of near-critical order-parameter plots from topologically distinct networks onto the same curve is reproduced as Figure 1, and a figure in [14] demonstrating the collapse of coexistence curves of different fluids is reproduced as Figure 2. Each shows that for apparently unrelated systems, a common equation can describe the plotted quantities near criticality. In the network case, the equation describing the curve is  $\eta = \frac{p/p_c - 1}{p/p_c}$ , where  $\eta$  is the average growth rate of queue-occupancy, the order parameter for congestive criticality, and  $p/p_c$  is the common source rate normalized by its critical value. The fluid coexistence curve is given by  $\rho \triangleq \rho_{\text{liquid}} - \rho_{\text{gas}} \propto (T_c - T)^\beta$ , where  $\rho_{\text{liquid}}$  and  $\rho_{\text{gas}}$  are the densities of the fluid on the right and left branches of the existence curves, respectively, both at temperature  $T$ , and  $\beta$ , which is 1/3 in the plot, is called a critical exponent that is a good fit for all the fluids shown.

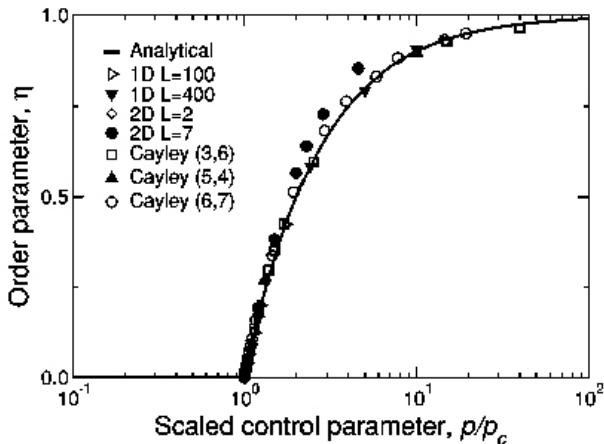


Figure 1. Reproduced from Fig. 5 in [15]. Packet accumulation rate normalized by source rate (proportional to the average queue occupancy normalized by source rate) vs. scaled source rates. Plots for one-dimensional, two-dimensional and Cayley networks all collapse onto the same curve.

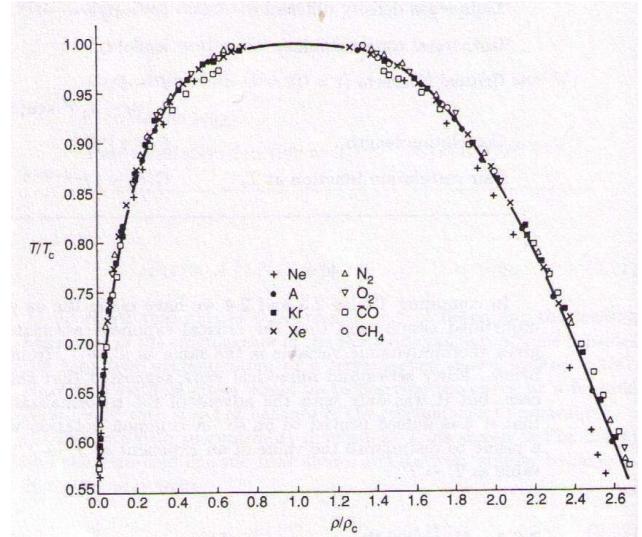


Figure 2. Reproduced from [14]. The coexistence curve of different fluids plotted in scaled temperature vs. scaled density difference.

Onset detection utilizes three quantities that are considered universal advanced warning signs of *any* CPT, and are known to undergo predictable and significant changes when a system approaches criticality [18], [19], [17]. They are spatial-correlation (function), temporal- or auto-correlation (function), and fluctuation in aggregate DoF (sum of all DoFs). The first two are the correlation coefficients of any two DoFs, separated in space and time, respectively. They are functions, as they depend on spatial and temporal separations. Correlation indicates the extent to which the two DoFs involved have similar values; fluctuation indicates the extent to which the aggregate DoF is far from its mean value.

Their behaviors as the network approaches, but *not* cross, the congestive criticality (from either the uncongested or the congested side) are as follows [18], [19]. During the approach, both correlations, at given spatial and temporal separations, respectively, become stronger, and fluctuation in aggregate DoF becomes larger, all in completely predictable manners. That is, different parts of the network become progressively more correlated with each other and with time-lagged versions of themselves, and DoFs fluctuate progressively further away from their common mean value for the current phase (which is also the order parameter, and remains unchanged as long as the transition is not crossed).

Specifically, spatial correlation as a function of spatial separation,  $r$ , for *all* CPTs is given, asymptotically in  $r$ , by

$$\Gamma(r) \sim r^{-a} \exp(-r/\xi) \quad \text{and} \quad \sim 1/r^b, \quad (1)$$

where  $a, b > 0$ ,  $b$  depends linearly on spatial dimension and  $\xi$  is the length scale indicating the “range” of spatial correlation, when the system is far from, and close to a CPT, respectively. Both are decaying functions; but negative exponential in the first expression does so much faster than power law in the second expression. Faster decay means correlations between DoFs tend to be weaker. Behaviors of auto-correlation are analogous but with different scaling exponents.

The growths of correlations and fluctuations are consequences of the nucleation, growth and coalescence of islands of the opposite phase upon approaching the transition point. See Figure 3. If the network is approaching the transition point from the uncongested phase, then the islands of the opposite phase are the congested islands forming inside the sea of uncongested network. Growing islands increase not only spatial correlations directly, but also temporal correlations, as it is more difficult, and hence takes longer, for the DoFs on a larger correlated island to fluctuate together as a block. As values of DoFs on an island deviate from their mean value in the current phase as blocks, large fluctuations in aggregate DoF result. These islands can be seen in plots in Figure 3, reproduced from [15]. Based on their simulations, the networks in both panels are in the globally uncongested phase, but the one in the right panel is much closer to criticality (because of having higher source traffic rates) and have more obvious congested islands.

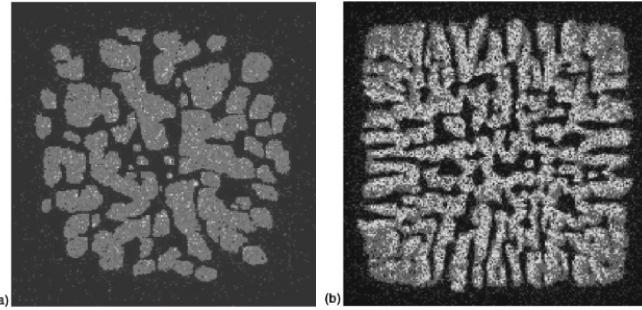


Figure 3. Reproduced from Fig. 8 in [15], illustrating the formation of congested islands in simulations of globally uncongested two-dimensional networks. Bright and dark dots represent congested and uncongested queues, respectively.

For a spatially region  $D$ , the aggregate DoF is defined as

$$X \triangleq \sum_{\vec{r} \in D} x(\vec{r}), \quad (2)$$

where  $x(\vec{r})$  is the individual DoF at location  $\vec{r}$ . It can be shown [18] that the fluctuation in  $X$ ,  $\sigma_X$ , is related to spatial-correlation like

$$\sigma_X^2 \triangleq \langle (X - \langle X \rangle)^2 \rangle = \sum_{\vec{r}_1, \vec{r}_2 \in D} \Gamma(\vec{r}_1, \vec{r}_2), \quad (3)$$

where  $\langle \cdot \rangle$  denotes ensemble average and  $\Gamma(\vec{r}_1, \vec{r}_2)$  is the spatial-correlation. When a homogeneous (in traffic-source statistics) region of the system is approaching a CPT,  $\Gamma(\vec{r}_1, \vec{r}_2)$  becomes slower decaying in the separation  $|\vec{r}_2 - \vec{r}_1|$  (see (**Error! Bookmark not defined.**)), causing the double sum in (3) to diverge like  $\|D\|^c$ , where  $\|D\| = \sum_{\vec{r} \in D}$  is the number of DoFs in region  $D$ . For a CPT, the constant  $c$  satisfies  $c > 1$  due to constraints on critical exponents [18].  $\|D\|$  is normally a large number in practice and is often approximated by infinity in theory. It is in this sense that fluctuation is said to be growing as a system approaches a CPT.

The ensemble averaging in the definition of  $\sigma_X$  implies that it measures the fluctuation between independent realizations of the system, which are implementable in simulations using independent pseudo-random number sequences. Yet, there is only a single realization in a real network. If all the DoFs in a system are strongly correlated with one another, then  $\sigma_X$  can be

measured only by averaging over multiple realizations. However, if the system contains non-unique regions that are weakly correlated with each other, such as the islands emerging during an approach to criticality, then they can be “stand-ins” for multiple realizations, and  $\sigma_X$  is approximately proportional to the sample standard deviation of individual DoFs in a single realization:

$$\sigma_X \propto \sqrt{\frac{1}{N} \sum_{i=1}^N (x(\vec{r}_i) - m_X)^2}, \quad (4)$$

$$\text{where, } m_X \triangleq \frac{1}{N} \sum_{i=1}^N x(\vec{r}_i) \quad (5)$$

is the sample mean of DoFs and  $N$  is the number of DoFs being simulated.

Equation (2) indicates that fluctuation does not contain more information than spatial-correlation; however, it is simpler to measure, if (4) and (5) are used. To do so correctly, measurements from the latter equations need to be interpreted carefully. As the system approaches criticality, although fluctuation increases, the regions that are uncorrelated with one another become fewer, until eventually, the entire system becomes correlated at the transition point. Hence, fluctuation as measured by (4) and (5) first increases due to the occurrence of uncorrelated regions, during the approach to criticality, but then paradoxically decreases due to their coalescence, when extremely close to criticality. To the extent that (4) and (5) include enough effectively independent realizations in uncorrelated regions, they form a good estimate of fluctuation. The effects of this complication will be shown in the experimental results. Another potential complication is that the proportionality constant in (4) depends on closeness to criticality and could hence affect the fidelity of how sample standard deviation represents  $\sigma_X$ . Fortunately, our results appear to allay this concern.

### III. ONSET DETECTION AND CONGESTION CONTROL

Congestion-onset detection methods will exploit the advanced nature of onset warning signs (to be proactive rather than reactive) and the universality property of CPT. The latter states that the behavior of a systems near a CPT does not depend on the details of interactions between the fundamental DoFs [18], [19]. Behavior close to CPT depend only on spatial dimensionality, the symmetry of the degrees of freedom, and whether interactions between DoFs are short- or long-ranged, and not on any other details of the system. For networks of common spatial dimensionality, this means that the manners in which correlation-length and correlation-time grow when approaching CPT are nearly identical, *irrespective* of any other details of the network, as show in Figure 1. and Figure 2. In fact they, probably fortuitously, show stronger universalities than CPT theory warrants. Figure 1. shows a case in which universality beyond networks of common spatial dimensions, and Figure 2. shows a case in which universality extends far from the critical point (at  $T = T_c$  and  $\rho = \rho_c$  ).

Universality implies, for instance, if we interpret a set of threshold values for spatial- and auto-correlation and fluctuation as a certain degree of closeness to criticality for one two-dimensional network, the interpretation of this set of threshold values for another two-dimensional network is likely

to be very similar. Another important implication of the universality of CPT is that the a-priori knowledge of critical load (network capacity) is not required for onset detection or control.

However, similarly strong statements cannot be made for fluctuation as measured by (4) and (5) due to the complications mentioned in Section II and the lack of a natural range for the value of fluctuation. Nevertheless, experimental results shown below suggest that (4) and (5) remain a useful fluctuation measure for the purpose of onset detection.

The simplest idea for onset detection consists of setting thresholds for spatial-correlation, auto-correlations, and fluctuation for signaling that a local region is near congestion criticality. Instead of depending on network capacity, the exact values of these thresholds should be determined by the tradeoffs amongst control-response time, network utilization and risk of false detections, and hence criticality crossing.

All three onset warning signs are local quantities measured over spatial and temporal neighborhoods having homogenous traffic conditions. This means that onset detection and criticality avoidance can be applied locally in different homogenous patches of the network, as well as in a multi-resolution fashion to larger parts of the network (by measuring the onset indicators over those larger parts). As an uncongested region can have congested sub-regions and vice versa, control decisions at different resolutions need not coincide.

Local signals are most suitable for local control, for delay avoidance reason. Timely advanced warnings can help enable local controls such as dynamic re-routing, traffic balancing and dropping to not only reduce local congestion after it occurs, but avoid it altogether. Keeping both monitoring and control local should also minimize communication overheads.

#### IV. SIMULATION EXPERIMENTS

The presented data are results of ns-3 [22] simulation experiments. Each one consists of a 10-node by 10-node wired grid with 100 randomly chosen source-destination (src-dst) pairs with fixed shortest-path routes, and each source has the same statistical traffic process. Queue buffer sizes are effectively infinite. In these homogeneous scenarios, traffic source type and intensity are varied to produce the following figures. Each simulation produces a queue-length fluctuation (see below), a rate of queue-length growth and an average delay, which are plotted in their individual figure panels. Different color symbols at the same network load (source rate) on the same plot correspond to otherwise identical network scenarios simulated with different random-number sequences. Different figures correspond to different traffic types.

Each figure shows normalized queue-occupancy fluctuation (top), rate of queue-occupancy growth normalized by the per-node offered load (the order parameter; middle), and average delay (bottom), all versus per-node offered load. Queue-occupancy fluctuation is measured by the time average of ratio of right-hand side of (4) to (5). It is normalized by the maximum of this ratio,  $\sqrt{N-1}$ , where  $N$  is the number of queues in the network. The maximum is estimated by assuming that only one queue among all has non-zero occupancy. Although the top panels show the fluctuation in queue length

rather than that in its time rate of change; the two are equivalent because the average growth rate presented in both (4) and (5) ( $x$  is growth rate of queue occupancy) are canceled by taking their ratio.

Since the rate of queue-occupancy growth indicates which phase the network is in, it is the order parameter of the phase transition. Denote the per-node offered load as  $y$ . Below the critical load,  $y_c$ , queue occupancy fluctuates close to zero but does not have any long-term growth; while above the critical load, normalized queue occupancy grows at the positive normalized rate of  $(y - y_c)/y$ , by simple packet accounting. The fact that the order parameter as a function of  $y$  is continuous is the reason why the congestive phase transition is a continuous one [18], [19], blessed with desirable properties. Note also that this is the same expression describing the order-parameter plot in Figure 1.

As the offered load increases, the order parameter plots (normalized rates of queue-length growth) show where criticalities start. However, the normalized queue-occupancy fluctuation has been acting as advanced warning for the criticality. Its value grows as the offered load approaches the critical load and reaches its maximum at the transition point. As an illustration of the advanced nature of this warning sign (top panels), we can see that its value becomes significant well before another popular performance metric, average delay (bottom panels), does. We expect the behaviors of spatial- and auto-correlations to be similarly. The drop and noisiness in normalized queue-occupancy fluctuation around the critical load are interpreted as evidence of lack of uncorrelated regions near criticality for (4) to remain accurate; this matches the expectations discussed near the end of Section II.

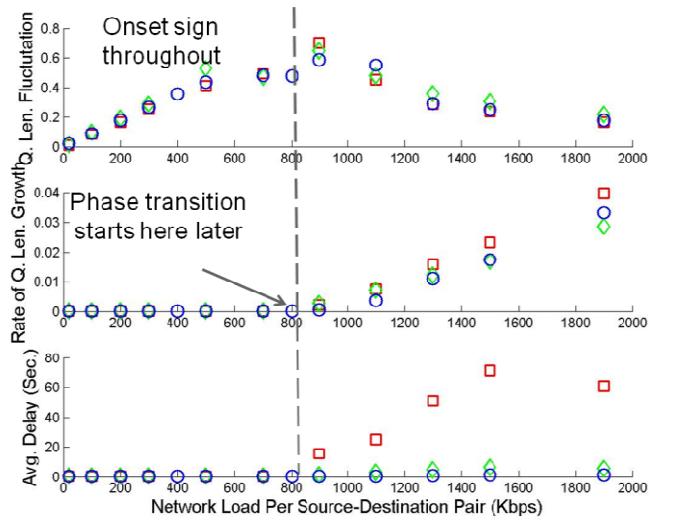


Figure 4. Criticality warning sign for constant-bit-rate traffic sources.

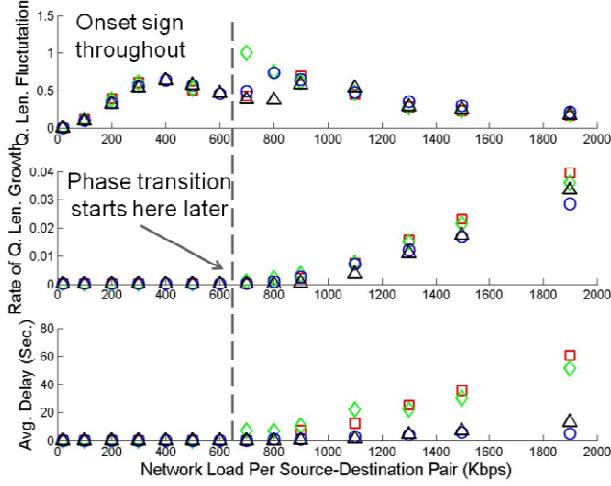


Figure 5. Criticality warning sign for Poisson traffic sources.

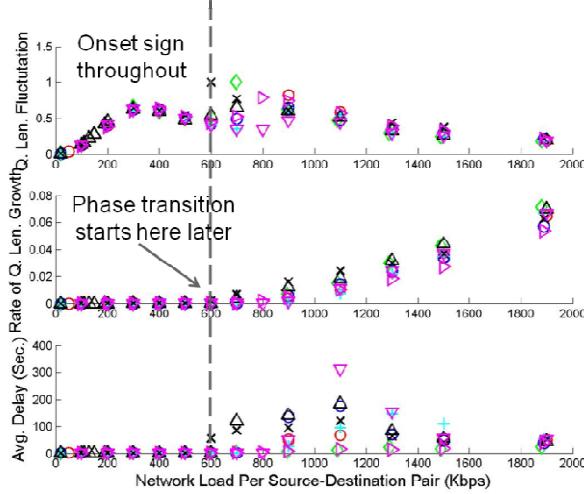


Figure 6. Criticality warning sign for long-range-dependent traffic sources, with Hurst parameter 0.7 for both the on and off periods.

The three figures show identical sets of simulations except for the traffic type. Burstiness increases from constant-bit-rate source to long-range-dependent (LRD) traffic source with the latter being a very realistic traffic model [21]. Testing warning signs with LRD traffic is important not only because it is realistic, but also because it has long correlation time. Our concern was that the inherently long-range and strong auto-correlation in LRD traffic can corrupt the warning signs as strong auto-correlation is one manifestation of proximity to criticality. Fortunately, Figure 6. demonstrates that at least the queue-occupancy fluctuation still acts as a good warning sign of criticality despite LRD sources (albeit with more noise near criticality).

Note that since queue-occupancy fluctuation is normalized to vary between zero and one, the interpretation of its value as closeness to criticality should be universal.

## V. DISCUSSIONS

After describing promises of congestion avoidance, potential drawbacks and comparisons to states of the art should be discussed. In such discussions, it is important to keep in mind that some potential drawbacks are common to all congestion-control methods, and the relative (to state of the art) susceptibilities of these potential drawbacks are more relevant in those cases.

Most potential drawbacks of congestion avoidance are similar to those of the general warning signs discussed in [17] with an important difference: The effects of drawbacks might be less serious for congestive CPT than for the general criticality. This is because the former is much better understood than the latter. For instance, the order parameter, and hence what drives the approach to criticality, for network congestion is known, with its critical behavior well studied [15]; while it tends to be unidentified for the general, and much more complex cases, such as global financial, ecological and climatic systems, considered in [17]. Further, the general criticality might be discontinuous rather than continuous. Finally, observability is presumably much higher for network congestion (queue-occupancies) than for those considered in [17]. In fact, without the knowledge of order parameter, one is likely observing some non-trivial function of it, which can introduce distortion and noise.

The first potential drawback raised by [17] is the robustness of warning signs in highly complex systems governed by complex spatial patterns, chaos and stochastic perturbations. However, there are evidences that the complexity of the dynamics of one such system, a geological system that had an earth quake, reduces when it is close to criticality [9].

The second concern is false detections, especially false negatives [17], that can be caused by rapid movements in order parameter (rapid changes in source-traffic rates) and poor observability. The former is a serious problem common to all congestion controls. Its only remediation is timely detection. Timeliness has two components: timely generation of detection signal and short delay in transmission of such signal. Timely warning generation has inherent tradeoff with false-detection probabilities. Whether CPT-based control is superior would depend on comparative receiver-operating characteristic (true positive probability vs. false positive probability) analyses against the state of the arts. The “loudness” of queue-occupancy fluctuation in our results should help; but the noise around its peak could hurt.

In terms of transmission delay of warning sign, there is no reason why our method should be inferior compared to the state of the art, given that detection metrics are inherently local in both space and time and control actions can be made local as well. This is especially true when compared against end-to-end congestion controls such as equation-based, AIMD and TCP [1], [2], for which detection is non-local. Even when CPT is used for non-local control, such as admission control, its multi-resolution warning signs could offer advantages in that the lower-resolution warnings associated with a larger region protect the sub-regions within it. This feature is unavailable from end-to-end controls.

As for observability, our results thus far suggest that the fluctuation warning sign is obvious even at far-from-critical

source rates. This is clearly superior to loss-based controls, such as TCP Reno [3], [4], as congestion loss tends to occur only near criticality; how this compares to delay-based controls, such as TCP Vegas [5], [6], remains to be seen.

A further concern for general criticality is the necessary delay associated with measuring temporal- or auto-correlation [17]. However, this concern is more serious for systems in which spatial warning signs are unavailable, which is not the case for queue-occupancies.

Finally, the phase transition is only sharp when queue buffers are infinite. For finite queue buffers, transition will be “soften” and warning signs should become more muted. The extent to which this occurs and how it affects performance should be studied.

## VI. SUMMARY AND ONGOING WORK

In this article, we argue for validity of viewing network congestion as a continuous phase transition (CPT) and the significant potential benefits of doing so. Initial simulation evidence supporting the utility of queue-length fluctuation as advanced onset warning sign under realistic traffic conditions is presented.

Our ongoing work includes exploring the other two onset warning signs and implementing a close-loop congestion control mechanism utilizing them. Despite good evidence for the universality of congestion criticality [15] (and undisputed evidence for general CPT universality [18], [19]), the advanced and universal properties of congestion warning signs should be more thoroughly verified across more network scenarios. An important question to answer is whether the resulting control mechanisms can act sufficiently rapidly in face of realistic dynamic congestion conditions. In other words, we need to find out whether the advanced and universal properties of warning signs translate to similar properties for the resulting controls, in realistic network scenarios. Finally, control performances should be compared to the state of the art. To these ends, our results, albeit being limited, are nonetheless encouraging.

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