

# Evolutionary Model of an Innovative and Differentiated Industry

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**Abstract.** In this paper, we propose an agent based model that describes the spatial and temporal evolution of an industry composed of a set of heterogeneous firms distributed in different regions. The model formalizes a particular hypothesis about spatial agglomeration and industrial concentration phenomena in which innovation occupies the central place of economic and geographical growth explanation. Each company owns one or more manufacturing divisions that produce an exclusive variety of product. Economic selection is modeled as a monopolistic competition market where competitive pressure depends on consumers' preference for variety. Moreover, firms may enjoy more competitive advantages innovating in processes, product characteristics and new commodities. The purpose of the model is integrating theories which come from research areas traditionally separated into a single formal proposal.

**Keywords:** evolutionary economics, industrial dynamics, economic geography.

## 1 Introduction

Traditionally, Economics theory has left aside the spatial dimension of economic phenomena, and in particular agglomeration and industrial concentration phenomena<sup>1</sup>, with the aim of developing a tractable and analytic framework [1]. However, industrial dynamics are usually characterized by economic forces that tend to agglomerate firms in particular locations and concentrate market power in a few group of companies. Because of this, different scientific disciplines, not exclusively economic, have given increased attention to these phenomena in the last two decades.

Due to the complexity of these phenomena, there are few and quite heterogeneous formal models that address the problem. A significant contribution comes from some economists of the new Economic Geography, who propose a new family of microeconomic models [1]. The main assumption of these models of spatial economic

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<sup>1</sup> Economic Geography often uses the term agglomeration to describe the spatial distribution of an industry over regions and countries. In this paper, we keep this assumption and use the term concentration to refer to the distribution of the market shares within an industry.

agglomeration is based primarily on the effect of local market size and the relationship between scale economies and transportation costs.

However, new Economic Geography models intentionally forget the dynamic dimension of industries to make the problem analytically tractable. In contrast, selection and innovation processes are of fundamental importance to other scholars [2]. For example, Audretsch and Feldman [3] demonstrate how R&D activities of many knowledge-based industries tend to be agglomerated in a few locations. Similarly, Baptista and Swann [4] show empirically the greater intensity of innovation activity in UK industrial clusters.

Evolutionary dynamics of industries are characterized by complex spatial-economic interdependence, insofar as firms' economic performance is conditioned by geographical factors, which at the same time are influenced by firms' performance. Industrial dynamics evolve simultaneously in both economic and geographic dimensions, making quite difficult, or even impossible, any intent to separate one from another.

Frenken and Boschma [5] integrate evolutionary theories of industrial dynamics with other geographical theories of regional growth and innovation to propose a new theoretical framework for developing geographic and evolutionary formal models, and this theoretical approach is the starting point of our work. We try to develop an evolutionary formal model of industrial dynamics in which innovation plays a key role as the engine of economic and geographical growth.

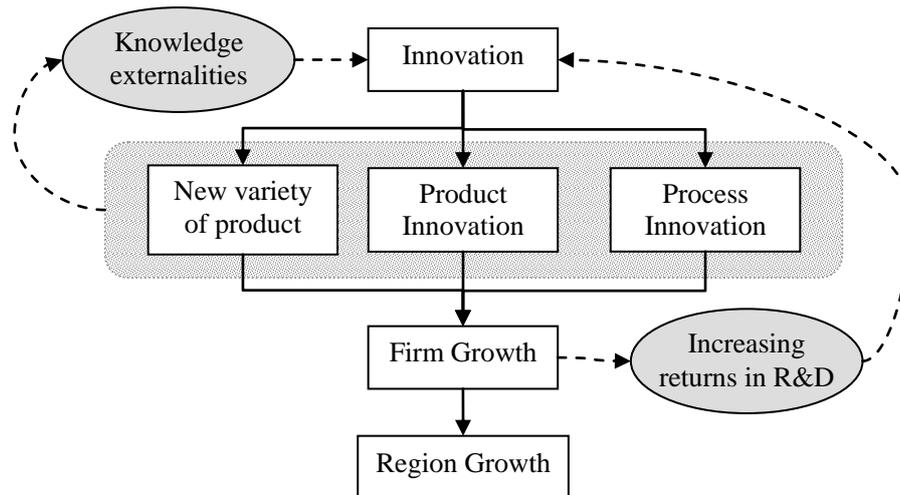
## **2 Cumulative Causality of Industrial Agglomeration and Concentration Phenomena**

The model is built on the following assumptions:

- The industry is formed by heterogeneous firms which are different in their capacities, knowledge and routines [6].
- Selection processes operate through a monopolistic competition market in which firms enjoy an imperfect monopolistic position producing differentiated products [7].
- Innovation is the source of diversity and competitive advantages. Firms do not only seek to improve processes and products, but also develop new products, and hence they can grow and strengthen their position in the industry [5].
- The synergy of knowledge within the firm as well as the knowledge diffusion (knowledge spillovers) outside the firm through labor mobility and personal contacts influence innovation results, reducing uncertainty and increasing the chances of success [3].

All these assumptions can be summarize in a hypothesis about spatial agglomeration and industrial concentration phenomena that we call it cumulative causality: a positive reinforcing chain of causes and effects. Figure 1 depicts graphically this cumulative causality: (1) innovation generates improvements in manufacturing processes (process innovation) and in the characteristics of products (product innovation), and therefore provides firms with competitive advantages; (2)

innovation also makes possible the creation of new products (product differentiation) that promote the expansion of firms; (3) the growth of firms may have a positive effect on their R&D activities through scale economies; (4) the growth of firms entails consequently the growth of the region where they are located; (5) the geographical proximity between firms facilitates knowledge externalities (knowledge spillovers) that influence all types of innovative activities.



**Fig. 1.** Main hypothesis of the model: cumulative causality of spatial agglomeration and industrial concentration phenomena [5].

### 3 Agent Based Model of a Differentiated Industry

We use the Agent-Based Modeling (ABM) paradigm to formalize the theoretical hypothesis outlined before. The model<sup>2</sup> describes the spatial and temporal evolution of an industry initially composed of a group of firms distributed in different locations (regions). Each company owns one or more manufacturing divisions that produce an exclusive variety of product. Economic selection is modeled as a monopolistic competition market, where consumers' preferences are determined by an aggregate utility function with constant elasticity of substitution (CES) between varieties. The income that consumers spend on products evolves exogenously according to a logistic curve, characteristic of the life cycle of the industry. Firms may enjoy competitive advantage innovating in processes and product characteristics or creating new products.

<sup>2</sup> The model has been implemented in Repast [8].

### 3.1 Monopolistic Competition Market

Selection processes in the industry is modeled as a competition between imperfect substitute varieties of products, each one is a monopoly of a single division of a firm. We incorporate a particular abstraction of imperfect monopolistic competition borrow from Dixit and Stiglitz [9] where consumers' behavior is fully described by an aggregate utility function  $U$  (Eq.1) with CES between varieties  $j \in 1, 2, \dots, m$

$$U = \left( \sum_{k=1}^m f_k(t) q_k(t)^\theta \right)^{1/\theta} = \left( \sum_{k=1}^m f_k(t) q_k(t) \frac{\sigma-1}{\sigma} \right)^{\frac{\sigma}{\sigma-1}} \quad 0 < \theta < 1 \quad (1)$$

where  $q_k$  denotes the quantity of commodity  $k$  and  $f_k(t)$  the consumers' predilection for it. The parameter  $\theta \in (0, 1)$  governs consumers' preference for variety, and therefore the degree of monopolistic competition in the market. Higher values of  $\theta$  denote weaker preferences for diversity. In particular,  $\theta = 1$  represents the situation where consumers do not distinguish between products, and the competition is similar to a homogeneous product market.

The family of CES utility functions in Eq.1 facilitates the derivation of the demand curve for each product  $j$  in Eq.2, where  $Y(t)$  represents consumers' income. We assume that consumer's income evolves exogenously according to a logistic equation that characterizes the life cycle of the industry [10].

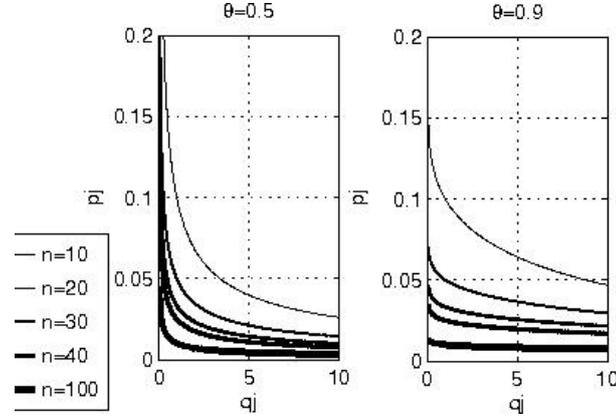
$$p_j(t) = \frac{f_j(t)^\theta}{q_j(t)^{1-\theta} \sum_{k=1}^m f_k(t) q_k(t)^\theta} Y(t) \quad (2)$$

It is interesting to observe how the demand for each commodity is affected by the total number of products in the industry (see Fig. 2), due to the particular abstraction of monopolistic competition used in the model. Note that new commodities in the market push down demand curves of all products, and this competitive pressure is higher as consumers' preference for variety declines, i.e., as  $\theta \rightarrow 1$ .

### 3.2 Firm behavior

The firm  $i$  is formed by one or more divisions  $j \in i$  responsible for a variety of product, all them located in a region  $r$ . At time period  $t$ , the division  $j$  belonging to the firm  $i$  produces  $Q_{ij}(t)$  units of the corresponding commodity according to the production function of Eq.3, which relates the output with the productivity of the division  $A_{ij}(t)$  and its stock of physical capital  $K_{ij}(t)$ .

$$Q_{ij}(t) = A_{ij}(t) K_{ij}(t) \quad (3)$$



**Fig. 2.** Family of demand curves of products (Eq.2) for two scenarios of a monopolistic competition market,  $\theta = 0.5$  on the left and  $\theta = 0.9$  on the right, where  $Y = 1, f_i(t) = 1 \forall i, q_i(t) = 1 \forall i \neq j$ . In each graph, the demand curve of a product  $j$  has been drawn for different values of the number of products  $n$  in the market. Looking at both graphs simultaneously we observe that the sensitivity of the demand curve to changes in the number  $n$  is modulated by consumers' preference for variety  $\theta$ .

The division  $j$  invests part of its capital  $r_j^{pc}(t)$  in process innovations, which may improve the productivity  $A_{ij}(t)$ , and other part  $r_j^{pd}(t)$  in product innovations, which may improve consumers' predilection  $f_k(t)$ . Similarly, the firm  $i$  invests part of its capital (the sum of capital over its divisions)  $r_i^{nv}(t)$  in innovation of new varieties of products.

We assume that the total production of the division  $j$ ,  $Q_{ij}(t)$ , is sold at price  $p_j(t)$ , which is determined by Eq.2. Division  $j$ 's profits  $\Pi_{ij}(t) = p_j(t)Q_{ij}(t) - c_{ij}(t)K_{ij}(t)$  are the difference between revenues and costs, where  $c_{ij}(t)$  represents the cost per unit of capital of the division (Eq.4); the parameter  $c$  quantifies the cost of capital, which is identical for all divisions.

$$c_{ij}(t) = c + r_j^{pc}(t) + r_j^{pd}(t) + r_i^{nv}(t) \quad (4)$$

At time period  $t$ , the division  $j$  estimates the capital investment rate  $I_{ij}(t)$  according to a simple adaptive rule [6], expressed in Eq.5.

$$I_{ij}(t) = \max \{0, \min \{I_{ij}^{\max}(t), I_{ij}^{des}(t)\}\} \quad (5)$$

The maximum capital investment rate  $I_{ij}^{\max}(t)$  is limited by division  $j$ 's profits in the period, and the desired investment rate  $I_{ij}^{des}(t)$  depends on how far division  $j$  is from the optimal situation in which its marginal revenue is equal to marginal costs. This deviation from the optimum can be expressed easily, Eq.6, by means of a desired

margin  $mg^{des}(t)$ , which depends on the elasticity of the demand curve, and an expected margin  $mg^{exp}(t)$ , which depends on expected price and marginal costs.

$$I_{ij}^{des}(t) = 1 + \delta - \frac{mg^{des}}{mg^{exp}(t)} \quad (6)$$

Then, division  $j$ 's stock of capital is updated according to Eq.7, where  $\delta$  modulates the depreciation of capital. Whenever the division  $j$  gets an expected margin higher than the desired margin, it decides to increase its stock of capital and consequently the quantity to be produced and sold in the next period.

$$K_{ij}(t+1) = K_{ij}(t) 1 - \delta + I_{ij}(t) \quad (7)$$

### 3.3 Innovation and Knowledge Spillovers

We model innovation as a stochastic process of two steps. First, a division/firm has a chance of innovating according to a probability of innovation. Second, if it achieves success, it assesses the scope of the innovation. The probability of innovation (for the three types of innovations considered in the model) is defined by Eq.8.

$$P^{in} = P_{max}^{in} - (P_{max}^{in} - P_{min}^{in}) \exp(-\alpha^{in} X^{in}) \quad (8)$$

The parameters  $P_{max}^{in}$  and  $P_{min}^{in}$ , which represent the maximum and minimum probability of innovating, determine the technological opportunity regime in the model –innovating can be more difficult in some industries than in others [11]–. The parameter  $\alpha^{in}$ , which governs the growth rate of the probability function, represents the productivity of innovative efforts  $X^{in}$ . Not every division/firm invests the same resources in innovative activities, and they are not equally efficient due to returns to scale and knowledge externalities [12].

The innovative effort  $X^{in}$  is modeled as a Cobb-Douglas function of increasing returns (Eq.9), where  $X_{division}^{I+D}$  represents the division's capital expenditure in R&D,  $X_{firm}^{I+D}$  the corresponding one in the firm, and  $X_{region}^{I+D}$  the total expenditure in the region in which the firm is located.

$$X^{in} = X_{division}^{I+D} X_{firm}^{I+D} \beta_{firm} X_{region}^{I+D} \beta_{region} x_{scala}^{1+\beta_{firm}+\beta_{region}} \quad (9)$$

$$0 < \beta_{region} < \beta_{firm} \leq 1$$

The parameters  $\beta_{firm}$  and  $\beta_{region}$  govern the importance of knowledge externalities within the firm and the region respectively. We assume that the former are more important than the latter, insofar as firms have more tools to manage them, and that both do not condition excessively the probability of innovation ( $\beta_{region} < \beta_{firm} \leq 1$ ), which depends mainly on each division's innovative effort.

In short, the probability of innovating is positively reinforced by: (1) returns to scale in the division's expenditure in R&D; (2) knowledge externalities within the firm [5]; and (3) knowledge spillovers in the region [3].

The scope of innovations in processes and products are modeled as a random leap forward from the current state of the division's productivity  $A_{ij}(t)$  and consumers' predilections  $f_j(t)$  according to a uniform distribution defined in Eq.10.

$$\begin{aligned} A_{ij}^{in}(t) &= U \left[ A_{ij}(t), (1 + \gamma^{pc})A_{ij}(t) \right] \\ f_j^{in}(t) &= U \left[ f_j(t), (1 + \gamma^{pd})f_j(t) \right] \end{aligned} \quad (10)$$

The parameters  $\gamma^{pc}$  and  $\gamma^{pd}$  are considered to be constant and equal to all firms in the industry. Moreover, we suppose that any innovation in processes or products gives the same order of competitive advantage, so it is not difficult to demonstrate a relation between them (Eq.11).

$$(1 + \gamma^{pc}) = (1 + \gamma^{pd})^\theta \quad (11)$$

A firm can also create a new product, starting a new division responsible for its manufacturing. The new division  $j^*$  replicates the capacities of its parent according to a stochastic process (Eq.12) that is function of all divisions' features of the firm. Furthermore, there is a probability  $p^{spinoff}$  that the new division becomes a new independent spin-off [13].

$$\begin{aligned} A_{j^*}(t) &= U \left[ \min(A_{ik}(t), \max(A_{ik}(t))) \quad k \in i \right] \\ f_{j^*}(t) &= U \left[ \min(f_k(t), \max(f_k(t))) \quad k \in i \right] \end{aligned} \quad (12)$$

We incorporate a noise in the model with the entry of new firms at each time period according to a Poisson distribution of parameter  $\lambda_{entry}$ , which it is assumed to be constant throughout the simulation. Finally, if a division reduces its stock of capital below a minimum threshold, it exits the industry; consequently, if all divisions of a firm exit, the company leaves de industry too.

## 5. Conclusions

We have proposed a formal model of an innovative and differentiated industry where a set of heterogeneous firms localized in different regions compete in an imperfect monopolistic market. The model is built on a particular hypothesis about spatial agglomeration and industrial concentration phenomena –that we call it cumulative causality– in which innovation occupies the central place of economic and geographical growth explanation. Innovation is modeled as a stochastic process where firm's probability of innovating depends not only on its innovative effort, but also on knowledge externalities within the firm and knowledge spillovers in the region.

The main contribution of the model is that we integrate theories which come from research areas traditionally separated into a single formal proposal. Thus, we incorporate the concepts of diversity, development and selection from Evolutionary Economics, and some theories about innovation and knowledge spillovers from Economic Geography and Geography of Innovation.

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