

A Data-Fusion Method using Bayesian Approach to Enhance Raw Data Accuracy of Position and Distance Measurements for Connected Vehicles

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Abstract— Accurate positioning of vehicles is a critical element of autonomous and connected vehicle systems. Most of other studies heavily focused on enhancing simultaneous localization and mapping (SLAM) methods, i.e., constructing or updating a map of an unknown environment and tracking an object within the map. This paper provides a method that can, in addition to existing SLAM or relevant methods, enhance the raw measurements of position and distance. The basic idea of this study is to identify and update the error distribution of each data source by combining all available information. A Bayesian approach was incorporated to estimate and update the error distribution of individual data sources or sensors. The proposed method can be conducted in real-time environments, and a self-learning scheme determines whether enough data has been collected to further improve the accuracy of such measurements. The simulated experiments show that the proposed model noticeably improves the accuracy of position and distance measurements. Especially, the estimated biases of position coordinates and distance measures are very close to the biases of true error distributions, with the R-squared over 0.98. A similar approach can also be utilized to enhance accuracy of other sensors or measurements in connected vehicle or relevant systems, where multi-data sources are available.

Keywords—connected & autonomous vehicle (CAV), GPS, data fusion, Bayesian approach

I. INTRODUCTION

Autonomous vehicles (AVs) and/or connected vehicles (CVs) will become available in the near future, most likely will be the both, since CVs are expected to add a cost of only several hundred dollars or less [1]. There are a variety of types of sensors that could be embedded in AVs and CVs, but one of inevitable functionality is positioning and measuring distances between vehicles and objects. The sensors and their measurements will affect the accuracy and the reliability of overall connected vehicle systems, as well as the road safety.

This importance of the measurement accuracy brought the attention of industry and government agencies that the sensor errors need to be controlled in a certain range. Although there are regulations that define the range of errors allowed in applications, how we can measure the errors is a remaining question. A wide range of experiments and benchmark data sources with high precision can lead to more trustful conclusions, but cost more.

Another challenge is determining the benchmark source to evaluate the designated measurements or data source. This

requires a method to evaluate the benchmark data source or at least to justify why it is used as an alternative to the ground truth. Furthermore, even if sufficient tests and experiments were conducted to define the error distribution of each data source for a certain area or time period, it may be changed over the operations in the real field, due to their installation, maintenance, geographical circumstance, intervention with other sensors, and so on.

The motivation of this paper is to identify and update the error distribution of multi-source raw data measurements by combining all available information. Then, the error distribution is updated, as operated, to further improve the accuracy of the measurements in real-time environments. Before this study, most other studies heavily focused on enhancing simultaneous localization and mapping (SLAM) methods, i.e., computationally constructing or updating a map of an unknown environment and tracking an object within the map.

Note that the proposed method is to enhance the accuracy of raw measurements and therefore it can be used before or after other post-processing techniques. In other words, the paper represents how much the proposed method can improve raw data of position and distance measurements, while other existing smoothing and filtering techniques can still be used to further improve the estimation results in practice. To this end, there is no comparison between the proposed method and other methods, but the improvements from raw data measurements are described.

II. LITERATURE REVIEW

In recent years, smart driving technologies, such as advanced driver-assistance systems and self-driving cars, have been put forth as a promising future that will improve our mobility and safety. The U.S. Department of Transportation's National Highway Traffic Safety Administration (NHTSA) reported that these technologies may reduce car crashes drastically, almost 90% of all the crashes that are caused by human error [2].

Although there will be a significant need for discussions on policies and regulations related to CVs and AVs, we will still need to confront and address technological challenges related to different kinds of safety and mobility issues, such as hacking, malfunctions of computer systems, and low accuracy of sensor measurements. The impact of the potential outcomes from implementing the new systems under unreliable

circumstances could be far more serious than types of car crashes with conventional human-driving systems, since vehicles in the new systems are likely to be “connected”.

Positioning sensors, secure telecommunications systems, cameras, automatic transmissions, navigation and security services are all key elements for CAVs [3]. Among all the essential elements, the accurate position and distance measures must be inevitable features of CAVs, as the system will require far more reliable navigation capabilities.

Global Positioning Systems (GPS), which is a constant position tracking method based on global location and time references of objects from satellites, is an indispensable element of CAVs. Typically, the error range of position measurements from GPS-enabled smartphones, under an open sky, is within 5 meter, and the accuracy can be improved by using dual-frequency receivers and/or augmentation systems [4]. Recently, centimeter-accurate (or even millimeter-accurate) GPS has been proposed and developed with its increasing demand, but often requiring additional modules or infrastructures for local referencing points to improve the accuracy, and thereby extra costs are necessary [5-8].

Another key sensor of CAVs is a Light Detection and Ranging (LiDAR), which is a laser detection sensor to identify surrounding objects and precisely measure distances to the objects. The accuracy of LiDAR varies a lot depending on their cost and measuring environment, like GPS. Bowen and Waltermire stated that the accuracy for LiDAR data ranges from the root mean square error of 1 to 2 meters horizontally and 15 to 20 centimeters vertically [9]. Csanyi and Toth also mentioned, “*State-of-the-art lidar systems can achieve 2 to 3 cm ranging accuracy under ideal conditions*”, but they also pointed out that the accuracy range is not realistic for typical navigation-based direct sensor platform orientations [10].

To improve the accuracy of vehicle position measurements, most papers have focused on localization methods of tracking objects. Simultaneous Localization and Mapping (SLAM) is a commonly used term for the methods of improving position estimations by using sensor measurements, landmarks, maps, and an estimator, such as extended Kalman filter and particle filter [11-30]. The extended Kalman filter has been used widely, especially in robotics, to address the limitation of linearity assumption of Kalman filter for an estimate of the current mean and covariance [22, 23, 26, 27]. However, Huang and Gamini have raised issues of convergence and consistency in the extended Kalman filter applications [31, 32]. Also, the extended Kalman filter has limitations on data association problem and an assumption of Gaussian distribution for sensor measurement noises. To overcome this issue, the particle filter has been introduced by estimating state from processing raw data without feature detection [14]. However, each particle in the particle filter represents a trajectory point, and thus increases computational load. Then, Montemerlo et al. applied Rao-Blackwellized Particle Filter (RBPF), which reduces memory usage by sharing a map between particles [33]. Although the RBPF method requires predetermined landmarks, it has become more popular recently [11-14, 29, 30]. Unfortunately, the performance of both methods varies much depending on those assumptions and limitations, study sites, and sensor errors, and by far no agreement has been made to which method is better in general.

Instead of using all features of localization, Lee et al. proposed a localization method based on GPS and dead reckoning error estimation, which is from a lane detection with curved lane models, stop line detection, and curve matching [34]. The result of their experimental site shows that the error of the estimated position stayed within a meter. There are also a decent number of researches on Mono-SLAM, localization, and mapping using a singular data source, mostly in visual data [35-39].

Several papers have utilized multi-source data to further improve sensor measurement accuracy [40-44]. In 2006, Mahlich et al. provided an approach to cross-calibrate vision and ranging sensors by a spatio-temporal alignment [44]. They have shown the proposed model could be applicable for real-time operation by using a low-level fusion of multibeam LiDAR and vision sensor measurements. Although most studies in SLAM also use multi-source data, focuses on SLAM are more on localization and mapping, rather than the data source type and their error distributions.

One of the main challenges of the smoothing and approximation techniques, used in SLAM, is that error distributions of raw measurements are assumed to be a certain type, e.g., Gaussian, or the performance is affected by the error distributions and other assumptions. The authors noticed that the error estimation of each individual data source seems not considered enough in most of the studies. Lee et al.’s paper, relatively, focused more on the error estimation, but their approach relies on the accuracy of lane detection [34].

More fundamentally, almost all proposed SLAM methods in this literature review need a certain degree of accuracy from raw data measurements, although the impact of such accuracy could vary by the smoothing techniques and purpose of uses.

This study uses a Bayesian approach, which has been used in estimating parameters and states for decades, to improve raw data measurements of position and distance by estimating the error distribution of each data source. Even the Kalman filter and the particle filter methods are based on Bayesian statistical inference, estimating a joint probability distribution of unknown variables or a conditional probability of the states of some processes. In transportation, it has become more popular than before, not just because of their benefits of performance, but also due to the introduction of easier approaches such as the Markov Chain Monte Carlo (MCMC) based Bayesian approach [45].

While the aforementioned Bayesian approaches typically focus on estimating designated parameters or states, the proposed method estimates error distributions of each data source or sensor before the final estimation of those parameters. This is the major contribution of proposed method, which can be applied in real-time operations based on continuous self-evaluation and updates of the error distributions.

III. METHODOLOGY

A. Overall Estimation Process of Proposed Method

The input for the proposed method involves a mixture of one- and two-dimensional variables, i.e., the distance and position measurements. Specifically, for each timestamp or data point, we assume that a vehicle will have the information from seven different sensors: (1) position of the designated vehicle; (2) distance measured from the designated vehicle to the preceding vehicle; (3) distance measured from the

designated vehicle to the following vehicle; (4) position of the preceding vehicle; (5) distance measured from the preceding vehicle to the designated vehicle; (6) position of the following vehicle; and (7) distance measured from the following vehicle to the designated vehicle.

If there is no error on all the measurements, the distance and position measurements must be consistent. For instance, the calculated distance from the measured positions between the preceding and the designated vehicle should be equal to the measured distance. In this case, the number of data sources at a single time frame will be seven, and the estimated error distribution of each source will contribute to calculate the likelihood of the candidate set of true vehicle positions and update its own error distributions.

Fig. 1 describes the overall flow of the estimation process from obtaining sensor measurements to updating the estimated error distribution and estimating the true position. In this study, we assume that each vehicle has a module described here and get the sensor measurements from two adjacent (preceding and following) vehicles as they transmit such information within the connected environment. Although the scope of this study does not cover data missing, the estimation of the error distribution process could still work on those interruptible situations since the learning process can just skip those missing data points. Furthermore, the estimation process can still be implemented with limited information if only several measurements are missing.

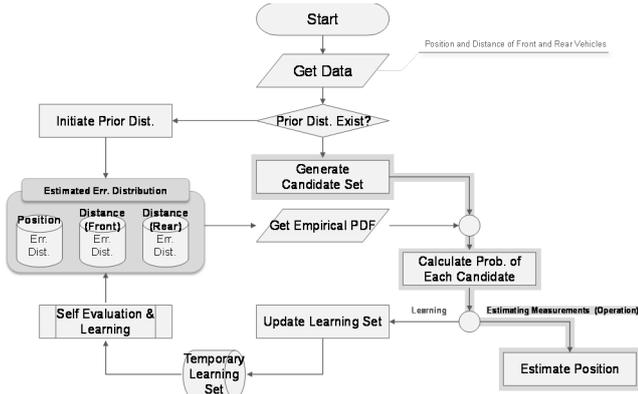


Fig. 1. Overall procedure of the proposed model.

If an initial prior distribution is unknown or not set up because of uncertainty, one can gather a decent number of samples to estimate the initial error distribution. The prior distribution can be either manually setup from other information or created by the initial sample data collection. Then, the estimated distribution will be used in the rest of the process to generate candidates, calculate the log-likelihood of the candidates, and therefore update the estimated error distribution and improve estimates of the true vehicle positions.

B. Generating Prior Distribution (Initial Learning)

At the beginning of the proposed model, we may or may not have enough information (or prior knowledge) about the error distribution of each sensor measurement. The approach described here is to be applied only where the given prior knowledge is considered to be not enough.

The basic idea of building a prior distribution of measurement error is to consider external information as benchmark data sources and calculate measurement errors as

compared to the estimated true measurements based on the benchmark data. For instance, distance measurements from the preceding and following vehicles to the designated vehicle are used as the benchmark data, and the errors can be estimated as a difference between the benchmark data and the collected measurements of the designated vehicle. The assumption of using external data sources as benchmark data would definitely not hold to be true, but it could be enough to build a prior distribution if we use various external data sources, i.e., position/distance measurements from many vehicles.

C. Generating Candidate Set

Basically, a grid search is used to generate a local candidate set of vehicle positions. The estimated true position will be determined by a candidate point that has maximum likelihood estimate (MLE) among the all trial set. The detailed calculation of this process is explained in the following section ‘Calculating Log-likelihood of Candidates’.

To estimate the true position more precisely, i.e., to have a higher resolution for a local optimum, the size of grid search is reduced as shown in Fig. 2, by a condition where the local MLE position is inside of the current searching boundary. The grid size remains same if the local optimum is on the boundary. This is to consider the cases where the local MLE position is far from the starting point of the search, without excessive use of computational resources. Each candidate point will yield a likelihood calculated by the given information and will be used to update the learning set, which is explained in the following section.

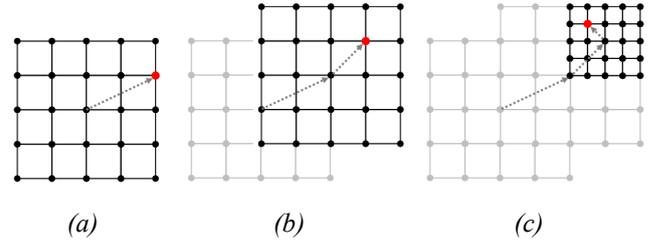


Fig. 2. Candidate set generation using dynamic grid search.

If we have N consecutively connected vehicles and test all combinations of M candidates for each vehicles’ position simultaneously, the number of computations should be made M^N times. To reduce the computational load, we only look at local MLE positions of the preceding and following vehicles to estimate the position of the designated vehicle and update its learning set. Since the local MLE position of a vehicle affects the estimation of adjacent vehicles, estimating the local MLE positions could be done iteratively by using MLE positions of the previous iteration each time. This reduces the computation load from $O(M^N)$ to $O(M \times N \times K)$, where K is a number of iterations.

D. Calculating Log-likelihood of Candidates

First, the log-likelihood of each candidate position can be calculated by the following equations.

$$\ln L_{\text{position}} = \text{Log}(P_{\text{position}}(\text{err}_x, \text{err}_y)) \quad (1)$$

$$\text{err}_x = \text{measurement}_x - \text{candidate}_x, \quad (2)$$

$$\text{err}_y = \text{measurement}_y - \text{candidate}_y, \quad (3)$$

Consequently, the associated distances, which are based on the Euclidean distances between the candidate positions of the vehicles, are obtained from the candidate positions. The log-likelihood of having a certain distance measurement error for each candidate can be calculated as the following:

$$\text{Ln}L_{\text{distance}} = \text{Log}(P_{\text{distance}}(\text{err}_{\text{dist.}})) \quad (4)$$

$$\text{err}_{\text{dist.}} = \text{measurement}_{\text{dist.}} - \text{candidate}_{\text{dist.}} \quad (5)$$

For both the position and distance measurement, the probability of having a certain error is determined based on the most recently updated (or learned) error distribution.

E. Self-Evaluation and Learning

We could update the estimated error distribution of measurements very frequently, in an extreme case, every time when a new data set is collected. However, too frequent updates might result in poor prediction of the error distribution for each update, and then also affect the accuracy of posterior distribution. Therefore, we need to set up a criterion to make the system updates the estimated error distribution only when enough information are obtained. The remaining question is how we determine “enough.”

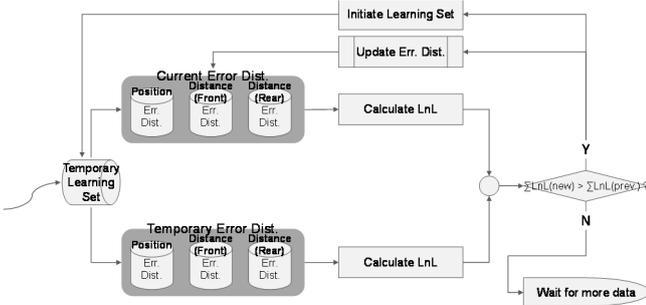


Fig. 3. Procedure of self-evaluation and learning for proposed model.

Fig. 3 illustrates the procedure of the evaluation and update of the error distribution. In the proposed methodology, the evaluation is conducted to see whether the updated error distribution with a newly learned set can better explain the collected data measurements.

The evaluation criterion here is the sum of log-likelihood between the one based on the most currently updated error distribution and a temporarily estimated error distribution based on the newly learned set. Since the learned set is generated based on the most currently updated error distribution, the temporarily estimated error distribution will tend to have a lower log-likelihood, like having a penalty, if there is no improvement on the current learning set. In other words, the update of error distribution is conducted only when we confidently expect the temporary learning set improves the estimation of the error distribution. Also, a minimum of 30 sample size is used to avoid some randomness, when determining whether the temporarily estimated error distribution is better than the current one.

One of the most benefits of using the suggested evaluation approach is that it does not require additional training data set to evaluate the model performance. The self-evaluation will ensure that numerous updates would direct to improve the estimation of error distribution and so does on the estimation of true positions. However, too strict evaluation criteria, e.g., too large minimum sample size or having a too large threshold

on the improvement of the log-likelihoods, can reduce the update frequency and slow down the learning speed.

IV. SIMULATED EXPERIMENTS

The main purpose of simulated experiments is to evaluate the model performance and its limitations. Although the vehicle trajectory data are based on the field data, the sensor data and associated measurement errors are generated by simulation. The key benefit of utilizing the simulated data for sensor measurements is an ability to evaluate the model estimates compared to the ground truth, whereas the actual “ground truth” data is almost impossible to obtain from the field data with numerous limitations, e.g., precision, accuracy, sample size, etc.

A. Data

The ‘highD’ data is a drone-based vehicle trajectory dataset, collected from German highways [46]. In this study, the x/y coordinates of the highD data are assumed to be the ground truth position of the collected vehicle trajectories.

The highD vehicle trajectory information were collected in six different locations for more than 100,000 vehicles. To minimize the impact of specific location and time period of the data collection, the study randomly selected 30,000 frames of data as for each iteration of new training dataset. Therefore, in the result section, each iteration represents additional learning or updates of error distributions and the consequent model performance based on the newly obtained 30,000 data points, which goes up to 100 iterations (total 3M data points).

B. Scenarios

Measurement errors of sensors could vary depending on many factors, such as manufacturer, device, environment, signal interruptions, etc. To evaluate the impact of proposed method based on different circumstances, multiple scenarios were prepared based on the following three error ranges (in meters):

- Bias: [-0.1 ~ +0.1], (Co)-Variance: [+0.1 ~ +0.5]
- Bias: [-0.5 ~ +0.5], (Co)-Variance: [+0.1 ~ +1.0]
- Bias: [-1.0 ~ +1.0], (Co)-Variance: [+0.1 ~ +3.0]

Consequently, total 9 scenarios (3 scenarios by distance × 3 scenarios by GPS) were evaluated based on the above range of bias and variance. The bias and the variance of true error distribution for each sensor were randomly selected from the uniform distribution within the range.

C. Results

Table I shows the overall improvement of mean absolute error (i.e., the average Euclidean distance from true vehicle positions) for the position measurements based on the different scenarios of sensor error ranges. Overall, the accuracy of position has been improved by 5 to 23%. Relatively, the accuracy improvement appears to be more significant when the distance measure has less bias/variance and the position measure has higher bias/variance.

TABLE I. IMPROVEMENT OF POSITION MEASUREMENTS

| Distance | Position (X/Y coordinate) | | |
|--|--|--|--|
| | $e: [-0.1 \sim +0.1]$ $v: [+0.1 \sim +0.5]$ | $e: [-0.5 \sim +0.5]$ $v: [+0.1 \sim +1.0]$ | $e: [-1.0 \sim +1.0]$ $v: [+0.1 \sim +3.0]$ |
| $e: [-0.1 \sim +0.1]$ $v: [+0.1 \sim +0.5]$ | 0.67 → 0.58 (- 13%) | 0.98 → 0.75 (- 23%) | 1.70 → 1.29 (- 24%) |
| $e: [-0.5 \sim +0.5]$ $v: [+0.1 \sim +1.0]$ | 0.68 → 0.60 (- 12%) | 0.99 → 0.79 (- 20%) | 1.71 → 1.34 (- 22%) |

| Distance | Position (X/Y coordinate) | | |
|--|--|--|--|
| | $\varepsilon: [-0.1 \sim +0.1]$ $v: [+0.1 \sim +0.5]$ | $\varepsilon: [-0.5 \sim +0.5]$ $v: [+0.1 \sim +1.0]$ | $\varepsilon: [-1.0 \sim +1.0]$ $v: [+0.1 \sim +3.0]$ |
| $\varepsilon: [-1.0 \sim +1.0]$ $v: [+0.1 \sim +3.0]$ | 0.68 \rightarrow 0.64 (- 5%) | 0.99 \rightarrow 0.87 (- 12%) | 1.68 \rightarrow 1.35 (- 20%) |

[MAE before. \rightarrow MAE after 100 iterations, ε : bias, v : (co-)variance, unit: meters]

Fig. 4 shows the overall mean absolute error of position over the number of iterations, where both the distance and the position have error distributions with the bias of range between -0.5 and 0.5 (the mid-range error scenario). In terms of the learning speed, the mean absolute error of position has decreased significantly from the first several iterations, and it seems converging after about 20 iterations. Note that the learning speed could vary depending on the detailed settings, such as number of sample size for each update on the error distribution.

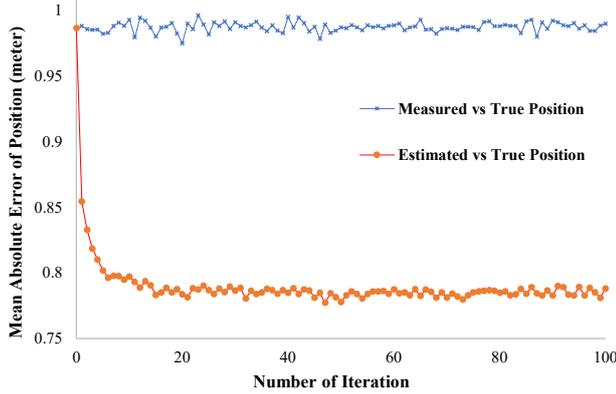


Fig. 4. Mean absolute error of position over number of iterations.

After the 100 iterations of learning, the biases of x/y coordinates and two (preceding/following) distance measures are very closely estimated, with the R-squared over 0.98, to the biases of true error distributions, as shown in Fig 5.

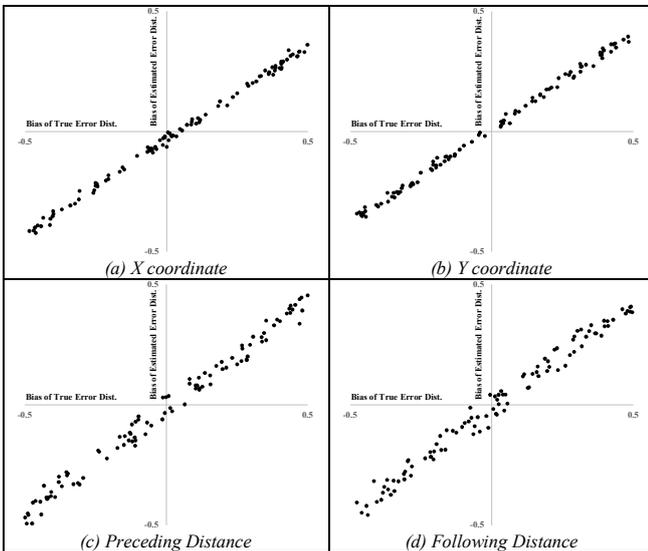


Fig. 5. Estimated bias of position and distance errors.

V. CONCLUSION

This study incorporated the Bayesian approach and estimated the error distributions of position and distance measurements from each vehicle in connected vehicle environment. The results show that the proposed model noticeably improves the accuracy of position and distance measurements. The key idea of the proposed method is to estimate the error distribution of each data source, especially

where the shapes of the distributions are not pre-defined. The estimated error distribution can also be used for calibrating biases of sensor measurements and improving the results of other post-processing techniques, which often require assumptions of a certain type of error distributions.

Furthermore, the estimated position and distance measurements appear to be more accurate than the raw measurements from the initial time of learning, and it becomes more accurate as more data are captured. It is expected that the proposed model could improve further than what is shown in this study, as more data (vehicles) will be available in the fields.

A similar approach can also be utilized to enhance the accuracy of other sensors or measurements in connected vehicle or relevant systems, where multi-data sources are available. For instance, vehicle speeds can be obtained from multiple sources, e.g., GPS, odometer, Bluetooth, and roadside detectors. The integration of such information could improve the accuracy of the speed and, it can be further enhanced by knowing the error distribution of the data sources.

Another possible benefit of the proposed approach is that it enables to update the estimated error distribution as a new set of data is gathered. This could be critically important to where the error distribution of sensor measurements is likely to be changed over time or affected by its local environments.

However, it is also important to know that the proposed model is not able to estimate the exact true values of measurements even if the learning time goes infinite. It does provide a likelihood of potential true values, and therefore the true values can be estimated using maximum likelihood, but the variances of error between the true values and the estimated values still exist. In other words, the proposed model makes an effort to reduce uncertainties of measurements using multi-source information, but those inherent variations will not be completely eliminated.

Another limitation of the proposed model is that it assumes overall error distributions of all sensors tend to be unbiased. After a decent amount of learning process, the bias of the estimated error distribution of each data source will be significantly contributed by the average error of all the data sources. Therefore, if the most (or all) of data sources are biased to one direction, the estimated error distributions might be biased as well. If the bias of the population is known or can be estimated from external features (e.g., stationary sensors, local infrastructures, buildings, etc.), the individual error distributions can be calibrated by the known or estimated bias.

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