QoS Guarantees in Multimedia CDMA Wireless Systems with Non-precise Network Parameter Estimates

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Abstract. Overwhelming majority of the next generation wireless cellular networks are based on CDMA (Code Division Multiple Access) technologies, with various flavors such as CDMA2000, WCDMA and their variations. Compared to the existing cellular networks which are designed for voice-only applications, the upcoming networks will be capable of providing high throughput and QoS dependent applications such as gaming, real-time streaming media, video conferencing etc., without significant improvements in the overall network capacity. Therefore for satisfactory end-user experiences, efficient use of the scarce network resources is vital. An efficient such algorithm has to depend on the estimates of information gathered from the network, such as the channel gains, received power levels, intercell/intracell interference etc. In such a scheme there are inherent sources for inaccuracies for the estimated values, such as the measurement errors, the delay in the estimates and the inaccuracies in models used for estimation. Implementing a resource allocation scheme which is robust to such measurement errors is important. In this paper, we study an optimum resource allocation scheme in a CDMA based cellular network, which is capable of allocating network resources to end-users of multimedia type applications, with QoS guarantees which are robust to inaccuracies in the estimated values, provided that those estimates can be bounded within a certain neighborhood of the real values.

1 Introduction

In cellular wireless communications, the recent worldwide trend has been overwhelmingly towards CDMA (Code Division Multiple Access) based technologies, namely, CDMA-2000 1x, CDMA-2000 HDR, CDMA-2000 EV-DO, CDMA-2000 EV-DV, WCDMA, HSDPA etc. As the deployments of these next generation technologies become more wide-spread, the importance of the efficient use of precious spectrum resources become increasingly vital. Part of this urgency comes with the introduction and penetration of various applications other than simple voice. Downloading/uploading images, online gaming, video conferencing, streaming media and many upcoming new applications can only succeed if the underlying infrastructure is capable of cramming in enough end-users with minimum network resource expenditure so as to reach and sustain viable economic models for network operators.

With the industry success of Qualcomm's IS-95 CDMA voice system, the resource allocation of voice-only applications has been mainly focused on "perfect-power-control", in the reverse-link which is targeting the lowest level of received powers at the base-station provided that the QoS level of the users are still satisfied. In the last decade, many researchers have attacked various research problems related to power control, majority of which were centered around the idea of achieving perfect-power-control either in a centralized or in a distributed fashion ([1], [2], [3] and references therein). In particular, when communications for real-time applications are considered, the majority of power control research papers have been focused on how to better achieve equal received powers in the uplink, to achieve efficient use of bandwidth. Some distributed algorithms were discussed to achieve the equal received power without a centralized control and the convergence of such methods were analyzed. In [2], an algorithm for handling mobiles' transmitter power levels while explicitly handling their time-varying transmission rates is introduced for future CDMA networks. In [3], the performance of the step size closed loop power control algorithm that is implemented in IS-95 is studied and a new predictive power control algorithm is suggested. In [4] and [5] a truncated channel inversion is studied for non-real time connections where the improvement in throughput and energy efficiency is achieved at the cost of extra queueing delay. In [6], the tradeoff between fairness and throughput is addressed and an extension is given to transmitting a single user at a time to transmitting more than one user at a time. In [7], the power management is done to minimize the power consumption of the wireless network and it is compared to traditional cellular networks.

The limited capacity of wireless telecommunication media, with the expectation of upcoming multimedia applications with different QoS has attracted attention to resource allocation issues in wireless systems. Some of the approaches are listed in [6]-[13]. Specifically, in [8] and [9], the wireless users are classified into real-time users and non-real time users. Then, the optimum spreading gain control is implemented to increase the spectral efficiency and satisfy the QoS of real-time users. The left-over capacity is shared among the non-real time users. Resource allocation in wireless LAN's with QoS requirements is studied in [10]. The coexistence of real-time voice users and high data rate non-real time users is studied in [11]. In [12], the throughput of CDMA-HDR is analyzed.

In recent years, some studies have proven that theoretically the perfect-power-control was indeed not optimum, even for voice-only applications and performed poorly for other QoS depended applications [15], [16].

In practical systems, the chosen resource allocation algorithm will make use of the best available estimates of the parameters in the network which the resource allocation algorithm uses to carry out the resource allocations. As an example, the parameter FER (Frame Error Rate) or the received power levels are both estimated values of the real values which are delayed in time, since estimating some of those values necessitate time averaging so-named values over time. There are also inherent estimation errors attached to each of the parameters caused by the inaccuracies of the methods and circuitries used to obtain those estimates. Given this framework, implementation of a resource allocation algorithm for cellular networks must be able to cope with the inherent inaccuracies of the estimated parameters to be able to implement the resource allocation algorithm it is targeting.

In this paper, we will describe an optimum resource allocation algorithm for a CDMA based wireless multimedia network which achieves the QoS requirements of the end-users even if the estimated parameters are inaccurate, provided that the inaccuracies are bounded within a certain neighborhood of the estimated value. In section II, we will describe the resource allocation algorithm. In section III, we will describe the resource allocation with the assumption of precise parameter estimates. In section IV, we will relax our assumptions to have non-precise parameter estimates and introduce the modified resource allocation algorithm. In section V, we will discuss our results and we will conclude in section VI.

2 Optimum Resource Allocation with Precise parameter estimates

2.1 System Model and Analysis

We will focus on the downlink (forward link) in a CDMA based cellular Wireless Wide Area Network (WWAN). We will assume that every wireless station is assigned to a single base station in the network and stays assigned to that base station throughout its connection lifetime. Every wireless station has some latency sensitive data to transmit with different QoS requirements during its connection lifetime. We will further assume without loss of generality that every wireless station has a single connection and the QoS requirement of that connection is not changing over time.

Let M be the number of wireless stations in a particular cell site of interest within a larger network. Let $\mathbf{p(t)} \doteq (p_1(t), p_2(t), \ldots, p_M(t))$ be the powers of the transmitted signals to the wireless stations from the basestation with time dependencies. Also define $\mathbf{g(t)} \doteq (g_1(t), g_2(t), \ldots, g_M(t))$ as the downlink channel gains vector from the base station to the wireless stations.

Define $\mathbf{P} \doteq (P_1, P_2, \ldots, P_M)$ as the maximum allowed transmission powers vector, where P_i is the maximum allowed power from the basestation to wireless station i. Notice that such a power restriction non-explicitly exists for real-time connections with latency requirements. As an example consider a 9.6Kbps realtime connection with 100msec latency tolerance (i.e. voice connection). Then, every 100msecs, there is only 960 bits to be sent. If the smallest granularity of packets is 10 msecs then the maximum needed peak rate is limited by 96Kbps which in turn creates a soft limit on the peak power level needed.

Let I_i denote the intercell interference (interference caused by the neighbor basestations in the adjacent cells) plus the background noise experienced by the wireless station i. In general the value of I_i will be time dependent, but during the initial part of this analysis, for short time durations of our interest, we will assume that I_i is a constant. Later, in the final algorithm, we will relax this assumption as we let I_i vary with time.

There is a unique mapping from the BER requirement of a downlink connection to the required E_b/N_0 value at the wireless station, where E_b is the energy per bit and N_0 is the total noise experienced at the wireless station for that connection. This mapping depends on factors such as the modulation scheme, interleaving method and error-correction scheme. Therefore we will assume that the wireless stations define their QoS requirements in terms of their E_b/N_0 needs. Let $(\kappa_1, \kappa_2, \ldots, \kappa_M)$ be that QoS requirements vector. Then the SNR equation for the wireless station i is:

$$(\frac{E_b}{N_0})_i = \frac{W}{R_i}\frac{g_i p_i}{\sum_{j\neq i} g_j p_j + I_i} = \frac{W}{R_i}\frac{p_i}{\sum_{j\neq i} p_j + \frac{I_i}{g_i}}, \qquad \forall i \in \{1, 2, \ldots, M\} \tag{1}$$

where R_i is the throughput of the i^{th} wireless station for a unit time duration ($rate$) and W is the bandwidth of the downlink. The last equation follows because $g_i = g_j$ for all $j \in \{1, 2, \ldots, M\}$ in a downlink from the base station to the i^{th} wireless station, in other words both the signal aimed for the i^{th} user and all the other communication that is only interference to user i traverses the same channel.

Notice that the right-hand side of the equation is the processing gain multiplied by the user's received power divided by the total noise power that the user is experiencing. $(E_b/N_0)_i$ and R_i are inversely proportional, therefore the QoS requirements should be met with equality, $(E_b/N_0)_i = \kappa_i$, for throughput maximization; since we can always lower the E_b/N_0, increase R_i and keep every other value constant in equation (1) as long as the E_b/N_0 requirement is satisfied. Therefore the throughput of the i^{th} wireless station in the $[0, t]$ time interval is given by:

$$h_i(0,t) = \int_0^t R_i(t)dt = \frac{W}{\kappa_i}\int_0^t \frac{p_i(t)}{\sum_{j\neq i} p_j(t) + I_i/g_i(t)}dt \tag{2}$$

Let $\Lambda \doteq (\rho_{\mathbf{1}}, \rho_{\mathbf{2}}, \ldots, \rho_{\mathbf{M}})$ be the minimum required rates vector. Define the sets $\Phi = \{\mathbf{R} \mid \frac{\int_0^t R_i(t)dt}{t} \geq \rho_i, \forall i = 1, 2, \ldots, M\}$ and $\Upsilon(B) = \{\mathbf{p} \mid \mathbf{0} \leq \mathbf{p} \leq \mathbf{P}\ \mathit{and}\ \mathbf{1}^T \cdot \mathbf{p} = B\}$ as the required rates and feasible powers vector sets respectively[3], where B is the maximum power level that the basestation can transmit. Assume that the connection topology stays the same during $[0, t]$ and further assume that $[0, t]$ is short enough so that the channel gains are constant. If we denote $H(0,t) \doteq \sum_i h_i(0,t)$ as the total throughput then the total downlink throughput maximization problem of the cell is given by:

$$\sup_{\mathbf{p}\in\Upsilon(B):\ \mathbf{R}\in\Phi} H(0,t) = \sup_{\mathbf{p}\in\Upsilon(B):\ \mathbf{R}\in\Phi} \sum_i \frac{W}{\kappa_i}\int_0^t \frac{p_i(t)}{\sum_{j\neq i} p_j(t) + I_i/g_i}dt \tag{3}$$

[3] Vector relations are componentwise.

Clearly the set that defines the domain of the optimization (3) is infinite and there is no clear feasible method of finding the best solution.

For a practical system, the transmitted powers are selected from a discrete (quantized) set of power levels. Therefore with a dense enough quantization, the continuous time power allocation and scheduling can be approximated with a power allocation and scheduling with fixed power levels, with arbitrary proximity. For quantization level k (i.e., a transmitted power can take one of the k discrete values), there are only k^M different power allocations possible. Let's associate the time durations $\Gamma_n = (t_n, t_{n+1})$, $n = 1, 2, \ldots, k^M$ to each such distinct power allocation and find the optimum set of Γ_n's which maximizes the total throughput of the cell site, i.e.:

$$\max_{\mathbf{p}\in\Upsilon(B):\ \mathbf{R}\in\Phi} H(0,t) \cong \max_{\Gamma:\ \sum|\Gamma_{\mathbf{n}}|=\mathbf{t},\ \ \mathbf{R}\in\Phi} \sum_i \frac{W}{\kappa_i} \sum_{n=1}^{K} \frac{p_{in} \mid \Gamma_n \mid}{\sum_{j\neq i} p_{jn} + \frac{I_i}{g_i}} \quad (4)$$

where the set $\Upsilon(\alpha) = \{\mathbf{p} \mid \mathbf{0} \leq \mathbf{p} \leq \mathbf{P} \ \mathit{and}\ \mathbf{1}^T \cdot \mathbf{p} = \alpha\}$ and B is the maximum total power the base station is allowed to transmit, $\mid \Gamma_n \mid = t_{n+1} - t_n$ and p_{in} is the fixed transmitted power level of wireless station i in the n^{th} subinterval Γ_n, i.e. $p_i(t) = p_{in}$ for $t \in \Gamma_n$. Γ_n's form a partition of the $[0, t]$ time interval. Notice that the above maximization problem is done over all possible sets of Γ_n's and the p_{in} values are constants. Therefore the condition $\mathbf{R} \in \Phi$ becomes the constraint on the decision variables Γ_n's. It is also worth nothing that the RHS of equation (4) can be made arbitrarily close to LHS for large enough k, which in turn requires a large enough K.

Definition 1. Vertex: A transmitted powers vector is a vertex in a time interval if $p_i = 0$ or $p_i = P_i$ for all $i = 1, 2, \ldots, M$ in that time interval.

Definition 2. Vertex-restricted-by-B: A transmitted powers vector is a vertex-restricted-by-B in a time interval if $p_i = 0$ or $p_i = P_i$ for all $i = 1, 2, \ldots, M$ except one $i = k \in \{1, 2, \ldots, M\}$ for which $0 \leq p_k \leq P_i$ and $\Sigma_i p_i = B$ in that time interval.

Proposition 3. *In the solution of the optimization problem (4), the transmitted powers vector in subinterval Γ_i is either a vertex or a vertex-restricted-by-B, for all $i = 1, 2, \ldots, M$.*

Proof. Assume there exists at least one subinterval Γ_j in the optimum solution such that the transmitted power vector is not a vertex nor a vertex-restricted-by-B. This means at least two of the transmitted power values, p_{ij} is neither 0 nor P_i and p_{kj} is neither 0 nor P_k. Assume $p_{ij} > p_{kj}$ without loss of generality. Then one can divide Γ_j into two subintervals such that the new value of p_{ij} is $p^1_{ij} = p_{ij} - q > 0$ and the new value of p_{kj} is $p^1_{kj} = p_{kj} + q$ in the first λ portion of Γ_j and the new values are $p^2_{ij} = p_{ij} + q$ and $p^2_{kj} = p_{kj} - q$ in the remaining $1 - \lambda$ portion of Γ_j. Let all the other transmitted power values stay unchanged for both subintervals. Then the throughput of the downlinks other than i and

k are unchanged in the new power allocation scenario. But for the i^{th} and k^{th} downlink we have an increased throughput in the new power allocation scenario if $\lambda \in (\frac{a+b_2-q}{2(a+b_2)}, \frac{c+b_1+q}{2(c+b_1)})$ where $a = p_{ij}$, $b_1 = (\Sigma_{n \neq i,k} p_{nj}) + \frac{I_i}{g_i}$, $b_2 = (\Sigma_{n \neq i,k} p_{nj}) + \frac{I_k}{g_k}$ and $c = p_{kj}$ since:

$$\lambda < \frac{c+b_1+q}{2(c+b_1)} \Rightarrow \lambda \frac{a-q}{b_1+c+q} + (1-\lambda)\frac{a+q}{b_1+c-q} > \frac{a}{c+b_1} \tag{5}$$

where the intermediate steps are mundane arithmetics and therefore are skipped. Notice that multiplying each side of the last inequality by $\mid \Gamma_j \mid$ proves that the throughput of the i^{th} downlink is improved in the new scenario. Similarly:

$$\lambda > \frac{a+b_2-q}{2(a+b_2)} \Rightarrow \lambda \frac{c+q}{a+b_2-q} + (1-\lambda)\frac{c-q}{a+b_2+q} > \frac{c}{a+b_2} \tag{6}$$

Again notice that multiplying each side of the last inequality by $\mid \Gamma_j \mid$ proves that the throughput of the k^{th} downlink is improved in the new scenario. Finally one can easily verify that $\frac{a+b_2-q}{2(a+b_2)} < \frac{c+b_1+q}{2(c+b_1)}$ to complete the proof.

□

Let $\Gamma \doteq (\mid \Gamma_\mathbf{1} \mid, \mid \Gamma_\mathbf{2} \mid, \ldots, \mid \Gamma_\mathbf{L} \mid)$, then by proposition 3, the optimization problem in (4) becomes:

$$\max_{\Gamma:\ \sum |\Gamma_\mathbf{n}| = \mathbf{t},\ \mathbf{R} \in \Phi} \sum_i \frac{W}{\kappa_i} \sum_{n=1}^{L} \frac{\tilde{p}_{in} \mid \Gamma_n \mid}{\sum_{j \neq i} \tilde{p}_{jn} + \frac{I_i}{g_i}} \tag{7}$$

where the vectors $(\tilde{p}_{1n}, \tilde{p}_{2n}, \ldots, \tilde{p}_{Mn}) \in \varphi$, $n = 1, 2, \ldots, L$ are all distinct, and φ denotes the set of all possible vertices and vertices-restricted-by-B. Also without loss of generality we renamed the partition in which the vertex $(\tilde{p}_{1n}, \tilde{p}_{2n}, \ldots, \tilde{p}_{Mn})$ is employed, to Γ_n, for all $n = 1, 2, \ldots, L$. Unlike the original optimization problem (over the infinite set $\Upsilon(B)$) , after restricting the solution set considerably (to the finite set φ) , we can now solve the optimization problem with Linear Programming (LP). The output of the optimization algorithm will be the $\mid \Gamma_1 \mid, \ldots, \mid \Gamma_L \mid$ values.

Let $\mathbf{A} = ((a_{ij}))$ where $a_{ij} = \frac{W}{\kappa_i} \frac{\tilde{p}_{in}}{\sum_{j \neq i} \tilde{p}_{jn} + \frac{I_i}{g_i}}$ then the optimization problem can be written as:

$$\textit{maximize } \mathbf{1^T A}\Gamma \textit{ with the constraints}^4\textit{: } \mathbf{A}\Gamma \geq \Lambda \tag{8}$$

Notice that although we have found the throughput maximizing scheduling for the duration $(0, t)$, this scheduling will satisfy all the QoS requirements of each wireless user but the extra capacity will be transferred to users with better channel gains which is unfair and unpractical. If we introduce the additional condition that each user will share the extra capacity proportional to their QoS

[4] the inequalities are componentwise

requirements, then it is easy to show that the throughput maximization problem is equivalent to finding the minimum feasible value t, by when all the QoS requirements are satisfied. Therefore we transform the throughput optimization problem to the following minimization problem:

$$minimize \sum_{i=1}^{L} \mid \Gamma_i \mid \ \ with\ the\ constraints^4\!: \mathbf{A}\Gamma \geq \Lambda \tag{9}$$

It is not hard to see that the inequality in the constraint above can be replaced by equality since the optimum solution will satisfy the rate requirements with equality. Let $\Gamma^* = (\mid \Gamma^*_\mathbf{1} \mid, \mid \Gamma^*_\mathbf{2} \mid, \ldots, \mid \Gamma^*_\mathbf{L} \mid)$ be the solution to the last optimization problem which can be solved by LP. The optimum solution Γ^* will have at least $L - M$ zero values and at most M non-zero values. The efficient LP techniques like *simplex method* can be used in order to achieve fast results. Since in the algorithm we would only have M *basic feasible solutions* at any iteration, we will have $O(ML)$ worst-case time and $O(M^2)$ best-case time. The memory need is only $O(M^2)$.

We can now construct the power allocation and scheduling scheme for precise channel gain and interference estimates. The proposed power allocation scheme will work as follows: As soon as one of the values of measured values of $\mathbf{g(t)}$ or I_i's changes, the base station will run the optimization algorithm and will assign the powers according to the optimization output, meaning for a period of $\mid \Gamma^*_1 \mid$, the powers vector $\mathbf{V}_1$ will be transmitted, then for a period of $\mid \Gamma^*_2 \mid$, the powers vector $\mathbf{V}_2$ will be transmitted and so forth. As soon as the duration $\mid \Gamma^*_L \mid$ where the powers vector $\mathbf{V}_L$ is assigned elapses, the base station will repeat the exact same assignments until either the value of $\mathbf{g(t)}$ or I_i changes. Remember that there are only at most M non-zero Γ^*_i values, meaning that there will be a time-division round robin between at most M vertices. We named the generic family of this resource allocation method as FiGARO.

For uplink (reverse link) resource allocation problem, most of the analysis follows in a similar fashion. For sake of completeness of the analysis, we will list the results for uplink with some notation abuse. For uplink the Γ only consists of the 2^M vertices. The minimization problem corresponding to equation (9) is:

$$minimize \sum_{i=1}^{2^M} \mid \Gamma_i \mid \ \ with\ the\ constraints^4\!: \mathbf{A}\Gamma \geq \Lambda \tag{10}$$

where $\mathbf{A} = ((a_{ij}))$ is the matrix with entries $a_{ij} = \frac{W}{\kappa_i} \frac{g_i \tilde{p}_{ij}}{\sum_{k \neq i} g_k \tilde{p}_{kj} + I_0}$.

3 Optimum Resource Allocation with Non-Precise parameter estimates

In practical systems the channel gain and interference estimations of the network may not be perfect. If the estimation of the channel gain is different than the actual value, the FiGARO algorithm may result some of the connections to fail to satisfy their QoS guarantees. If the real channel gain is lower than the estimated value, the QoS of the channel will suffer and the connection may eventually be lost. In order to have a robust resource allocation, we will formulate and solve the resource allocation algorithm such that the QoS requirements will be met even if they are off by some amount. To serve this aim we will formulate the resource allocation problem with inexact constraints for the estimated values, namely, the channel gains and the interference estimates will lie within a certain interval, rather than being exact. Then we will use the Inexact Linear Programming (ILP) method to reduce that stochastic LP problem to an ordinary LP and solve it.

3.1 Inexact Linear Programming

In an inexact LP, the usual convex inequalities, $\mathbf{A}\mathit{\Gamma} \geq \mathit{\Lambda}$, are replaced by the constraint that the sum of a finite number of convex sets is contained in another convex set, in our case, we will restrict the latter to the special form of a polyhedral convex set, i.e.:

$$\textit{maximize } \mathbf{1A}\mathit{\Gamma} \textit{ with the constraints}^{5}\textit{: } \mathit{\Gamma}_{\mathbf{1}}\mathbf{G_1}+\mathit{\Gamma}_{\mathbf{2}}\mathbf{G_2}+\ldots+\mathit{\Gamma}_{\mathbf{L}}\mathbf{G_L} \subseteq \mathbf{G} \textit{ and } \mathit{\Gamma}_{\mathbf{j}} \geq \mathbf{0} \tag{11}$$

where G_j is a convex set containing $\mathbf{a}_j$, the j^{th} column of the matrix $\mathbf{A}$, and $G = \{\mathbf{y} \in \mathbf{R}^{\mathbf{2^M}} \mid \mathbf{y} \geq \mathit{\Lambda}\}$. In (11), $\mathit{\Gamma}$ is only a feasible solution if and only if $\mathit{\Gamma}_1\mathbf{a}_1 + \mathit{\Gamma}_2\mathbf{a}_2 + \ldots + \mathit{\Gamma}_L\mathbf{a}_L \geq \mathit{\Lambda}$ and $\mathit{\Gamma}_i \geq 0$ for all possible sets of activity vectors in G_i's.

Notice that unlike (11), in generalized LP there is freedom to choose any vector $\mathbf{a}_j \in G_j$ for each j to maximize the objective function, i.e.,

$$\textit{maximize } \mathbf{1A}\mathit{\Gamma} \textit{ with the constraints: } \mathit{\Gamma}_{\mathbf{1}}\mathbf{a_1}+\mathit{\Gamma}_{\mathbf{2}}\mathbf{a_2}+\ldots+\mathit{\Gamma}_{\mathbf{L}}\mathbf{a_L} \geq \mathit{\Lambda} \textit{ and } \mathit{\Gamma}_{\mathbf{j}} \geq \mathbf{0}, \mathbf{a_j} \in \mathbf{G_j} \tag{12}$$

In the generalized LP the activity vectors $\mathbf{a}_j$ are decision quantities as are the $\mathit{\Gamma}_j$'s.

If the convex sets G_i's are equal to single vector, than the inexact LP coincides with regular LP. Therefore the inexact LP [14] applies to problems where the constraint vectors $\mathbf{a}_j$'s are not exactly known but are known to be in a convex set G_j.

Proposition 4. *$S = \{\mathit{\Gamma} \mid \mathit{\Gamma}$ is feasible for (11) $\}$ is a convex set.*

Proof. Let $(\tilde{\mathit{\Gamma}}_1, \tilde{\mathit{\Gamma}}_2, \ldots, \tilde{\mathit{\Gamma}}_L)$ and $(\hat{\mathit{\Gamma}}_1, \hat{\mathit{\Gamma}}_2, \ldots, \hat{\mathit{\Gamma}}_L) \in S$ and for any $\mathbf{a}_j$, $\forall j = 1, \ldots, n$, and $\lambda \in (0,1)$ we have $\mathbf{s}_1 = \tilde{\mathit{\Gamma}}_1\mathbf{a}_1 + \tilde{\mathit{\Gamma}}_2\mathbf{a}_2 + \ldots + \tilde{\mathit{\Gamma}}_L\mathbf{a}_L$ and $\mathbf{s}_2 = \hat{\mathit{\Gamma}}_1\mathbf{a}_1 + \hat{\mathit{\Gamma}}_2\mathbf{a}_2 + \ldots + \hat{\mathit{\Gamma}}_L\mathbf{a}_L \in S$, so $(\lambda\hat{\mathit{\Gamma}}_1 + (1-\lambda)\tilde{\mathit{\Gamma}}_1)\mathbf{a}_1 + (\lambda\hat{\mathit{\Gamma}}_2 + (1-\lambda)\tilde{\mathit{\Gamma}}_2)\mathbf{a}_2 + \ldots + (\lambda\hat{\mathit{\Gamma}}_L + (1-\lambda)\tilde{\mathit{\Gamma}}_L)\mathbf{a}_L = \lambda\mathbf{s}_2 + (1-\lambda)\mathbf{s}_1 \geq \mathit{\Lambda}$, which means $\lambda\mathbf{s}_2 + (1-\lambda)\mathbf{s}_1 \in S$. □

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[5] the inequalities are componentwise and the + refers to addition of sets in this equation

Definition 5. The support functional of the convex set G_j, denoted as $\delta(\mathbf{z} \mid G_j)$, is equal to $\inf_{a_j \in G_j} \mathbf{z} \cdot \mathbf{a}_j$.

For each j define the vector $\bar{\mathbf{a}}_j$ where its i^{th} entry is equal to $\hat{\delta}(\mathbf{e}_i \mid G_j) \doteq \inf_{\mathbf{a}_j \in G_j} a_{ij}$, where $\mathbf{e}_i$ is the vector with its i^{th} entry as 1 and has entry 0 elsewhere. Notice that if the set G_j includes a vector who has an entry equal to $-\infty$, say the i^{th} entry , then $\hat{\delta}(\mathbf{e}_i \mid G_j) = -\infty$ and therefore $\Gamma_1 G_1 + \Gamma_2 \cdot G_2 + \ldots + \Gamma_L \cdot G_L \geq \Lambda$ necessarily implies that $\Gamma_j = 0$. Therefore we can omit the activity set G_j from our LP without loss of any generality. Therefore we will assume that $\hat{\delta}(\mathbf{e}_i \mid G_j) > -\infty$ for all i and j, from now on. The same restriction is automatically achieved if it is assumed that the sets $\{G_j\}$ are compact [14].

After forming $\bar{\mathbf{A}} = (\bar{\mathbf{a}}_1, \bar{\mathbf{a}}_2, \ldots, \bar{\mathbf{a}}_L)$, consider the following artificial LP problem:

$$\textit{maximize } \mathbf{1}\bar{\mathbf{A}}\Gamma \textit{ with the constraints: } \Gamma_\mathbf{1}\bar{\mathbf{a}}_\mathbf{1} + \Gamma_\mathbf{2}\bar{\mathbf{a}}_\mathbf{2} + \ldots + \Gamma_\mathbf{L}\bar{\mathbf{a}}_\mathbf{L} \geq \Lambda \textit{ and } \Gamma_\mathbf{j} \geq \mathbf{0} \tag{13}$$

In the following paragraphs we will argue that the optimal solution to (13) is also the optimal solution to (11). Let's define the set H, as the set of all possible matrices formed from the convex sets G_i's; i.e.:

$$H = \{(\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_L) \mid \mathbf{a}_i \in G_i \ , \forall i\} \tag{14}$$

After this definition, it's appropriate to claim that Γ is a feasible solution to problem (11) if and only if $\mathbf{A}\Gamma \geq \Lambda$, $\forall \mathbf{A} \in H$ and $\Gamma \geq \mathbf{0}$.

Proposition 6. *If $\bar{\Gamma}$ is a feasible solution to (13) (the artificial ordinary linear optimization problem) , then $\bar{\Gamma}$ is a feasible solution for (11) (the inexact linear optimization problem), and vice versa.*

Proof. Let $\bar{\Gamma}$ be a feasible solution to our artificial LP (13), then since $\bar{\mathbf{A}}\bar{\Gamma} \geq \Lambda$ and $\bar{\Gamma} \geq 0$, and by construction we have $\mathbf{A} \geq \bar{\mathbf{A}}$. But then $\bar{\Gamma}$ is a feasible solution for our original inexact LP (11) since $\mathbf{A}\bar{\Gamma} \geq \bar{\mathbf{A}}\bar{\Gamma} \geq \Lambda$ for all $\mathbf{A} \in H$.

Conversely, if $\bar{\Gamma}$ is a feasible solution for (11), then, $\bar{\Gamma}_1 \mathbf{a}_{i1} + \cdots + \bar{\Gamma}_L \mathbf{a}_{iL} \geq \rho_i$ (where ρ_i is the i^{th} component of the vector Λ), where $\mathbf{a}_j \in G_j$, for $i = 1, 2, \ldots, M$. Therefore, for all $i = 1, 2, \ldots, M$ we have

$$\bar{\Gamma}_1 \inf_{a_1 \in K_1} a_{i1} + \bar{\Gamma}_2 \inf_{a_2 \in K_2} a_{i2} + \cdots + \bar{\Gamma}_L \inf_{a_L \in K_L} a_{iL} \geq \rho_i, \tag{15}$$

which is indeed equivalent to $\bar{\Gamma}$ being a feasible solution to (13).

□

Corollary 7. *As an immediate result of proposition 6, the sets of the feasible solutions to the two problems are identical, i.e., the solution of (11) can be directly obtained by solving (13), which is a ordinary linear optimization problem.*

3.2 FiGARO with Inexact Estimates

Assume that the FiGARO engine does not have the exact values of the channel gains and the interference values, but rather inaccurate estimates of those variables. Therefore the real values of the estimated variables are only guaranteed to be within a neighborhood of the estimate, i.e. we have:

$$g_i \in [\hat{g}_i - \Delta_i, \hat{g}_i + \Delta_i] \text{ and } I_i \in [\hat{I}_i - \bar{\Delta}_i, \hat{I}_i + \bar{\Delta}_i] \text{ for all } i = 1, \ldots, M \tag{16}$$

These relations translate into the following condition that a_{ij} is guaranteed to be in the interval θ_{ij} where the θ_{ij} is defined by

$$\theta_{ij} = \left[\frac{W}{\kappa_i} \frac{\tilde{p}_{ij}}{\sum_{k \neq i} \tilde{p}_{kj} + \frac{\hat{I}_i + \bar{\Delta}_i}{\hat{g}_i - \Delta_i}}, \frac{W}{\kappa_i} \frac{\tilde{p}_{ij}}{\sum_{k \neq i} \tilde{p}_{kj} + \frac{\hat{I}_i - \bar{\Delta}_i}{\hat{g}_i + \Delta_i}}, \right] \tag{17}$$

If we define the set G_j's, $j = 1, 2, \ldots, M$ such that:

$$G_j = \{(a_{1j}, a_{2j}, \ldots, a_{Lj})^T \mid \forall\ i\ a_{ij} \in \theta_{ij}\}, \tag{18}$$

to satisfy the individual connection requirements under FiGARO, no matter what channel gains and interference values we get (given that they will lie within their allowed intervals) we have to solve (11) with the new values of G_j's. But by corollary, solving that LP is equivalent to solving the following artificial LP:

$$\textit{maximize } \mathbf{1}\bar{\mathbf{A}}\Gamma \textit{ with the constraints: } \Gamma_\mathbf{1}\bar{\mathbf{a}}_\mathbf{1} + \Gamma_\mathbf{2}\bar{\mathbf{a}}_\mathbf{2} + \ldots + \Gamma_\mathbf{L}\bar{\mathbf{a}}_\mathbf{L} \geq \Lambda \textit{ and } \Gamma_\mathbf{j} \geq \mathbf{0} \tag{19}$$

where the vectors $\bar{\mathbf{a}}_j = (\bar{a}_{1j}, \bar{a}_{2j}, \ldots, \bar{a}_{Mj})$, the matrix $\bar{\mathbf{A}} = (\bar{\mathbf{a}}_1, \bar{\mathbf{a}}_2, \ldots, \bar{\mathbf{a}}_L)$ and

$$\bar{a}_{ij} = \hat{\delta}(\mathbf{e}_i \mid G_j) = \inf_{\mathbf{a}_j \in G_j} a_{ij} = \inf_{\mathbf{a}_{ij} \in \theta_{ij}} a_{ij} = \frac{W}{\kappa_i} \frac{\tilde{p}_{ij}}{\sum_{k \neq i} \tilde{p}_{kj} + \frac{\hat{I}_i + \bar{\Delta}_i}{\hat{g}_i - \Delta_i}} \tag{20}$$

Notice that at this point, we can assign the desired values to Δ_i's and $\bar{\Delta}_i$'s and calculate the output of FiGARO, namely, the vector Γ.

4 Discussions

Since it leads to a very practical implication, which is a slightly more interesting case, in this section we will present our results for the uplink scenario. In the uplink the θ_{ij}'s are given by

$$\theta_{ij} = \left[\frac{W}{\kappa_i} \frac{(\hat{g}_i - \Delta_i)\tilde{p}_{ij}}{\sum_{k \neq i} (\hat{g}_k + \Delta_k)\tilde{p}_{kj} + (\hat{I}_0 + \Delta_I)}, \frac{W}{\kappa_i} \frac{(\hat{g}_i + \Delta_i)\tilde{p}_{ij}}{\sum_{k \neq i} (\hat{g}_k - \Delta_k)\tilde{p}_{kj} + (\hat{I}_0 - \Delta_I)} \right] \tag{21}$$

For the sake of analysis, let's assume that we can foresee and guarantee estimation of the channel gains and the intercell interference values with the same percentage accuracy, i.e.:

$$\frac{\Delta_1}{\hat{g}_1} = \frac{\Delta_2}{\hat{g}_2} = \cdots = \frac{\Delta_M}{\hat{g_M}} = \frac{\Delta_I}{\hat{I}_0} = c \tag{22}$$

Then,

$$\bar{a}_{ij} = \frac{W}{\kappa_i} \frac{(\hat{g}_i - c\hat{g}_i)\tilde{p}_{ij}}{\sum_{k \neq i}(\hat{g}_k + c\hat{g}_k)\tilde{p}_{kj} + (\hat{I}_0 + c\hat{I}_0)} = \frac{1-c}{1+c} \frac{W}{\kappa_i} \frac{\hat{g}_i \tilde{p}_{ij}}{\sum_{k \neq i} \hat{g}_k \tilde{p}_{kj} + \hat{I}_0} = \frac{1-c}{1+c} \hat{a}_{ij} \tag{23}$$

and therefore, $\bar{\mathbf{A}} = \frac{1-c}{1+c}\hat{\mathbf{A}}$, where $\hat{\mathbf{A}} = (\hat{\mathbf{a}}_1, \hat{\mathbf{a}}_2, \ldots, \hat{\mathbf{a}}_{2^M})$. To interpret what this means, we will go over an example. Assume we have the channel gain estimates, $\hat{g}_1, \cdots, \hat{g}_M$, and an estimate for the intercell interference value,$\hat{I}_0$. Additionally, assume that we know that the estimates are guaranteed to be within 5% of our estimated values. FiGARO would calculate the output, Γ with the input set $\hat{g}_1, \cdots, \hat{g}_M, \hat{I}_0$. But the Quality of Service requirements will not be satisfied unless we are lucky and get exactly the estimated values. If we want the outcome scheduling be optimum and satisfy the individual Quality of Service requirements as long as the estimation errors are within the 5% error margins, then FiGARO should calculate Γ as if the rate requirements of each of the connections were higher by a multiple of $\frac{1+c}{1-c} = \frac{1+0.05}{1-0.05} \cong 1.105$ and implement the modified output. But this will result in an output of $\Gamma' = \frac{\mathbf{1+c}}{\mathbf{1-c}}\Gamma$, where Γ is the solution to the original situation where we weren't seeking any guarantees for the estimation errors. So if (22) is satisfied, we have a very simple modification to the FiGARO algorithm and that is:

run FiGARO with the input set $(\hat{g}_1, \cdots, \hat{g}_M, \hat{I}_0)$ *to get output* Γ

implement $\Gamma' = \frac{\mathbf{1+c}}{\mathbf{1-c}}\Gamma$ *where* c *is the percentage error that can be tolerated for estimates*

Table 1. INCREASE IN TIME REQUIREMENTS WITH ESTIMATION ERRORS IN UPLINK

Estimation Error	1%	2%	3%	5%	10%	20%	25%	33%	50%
Γ'/Γ	1.020	1.041	1.062	1.105	1.222	1.500	1.667	2.000	3.000

In table 1, we tabulated how the time requirements increase with respect to the error margin tolerance. As we become closer to 100% estimate error range, the $\Gamma^{'}/\Gamma$ value reaches ∞ which is expected. The solution of (13), actually provides an ultraconservative strategy for the stochastic LP of the form (11). If we have different estimation error percentages for the different connections, i.e. for example the better channel may be estimated more accurately, then those values should be used to calculate the corresponding $\bar{a}_{ij}$ values. This will improve the performance loss for rate requirement guarantees.

We have simulated a CDMA2000 EV-DV like system in Mathematica. Our system model allows $5msec$ packets on a 1.25MHz channel with Rayleigh distribution on top of a lognormal fading for the channel model. We have soft-limited the peak rates to $200Kbps$ for the multi media type application with a $10Kbps$ minimum rate requirements. We assume each terminal have the same application with the same QoS requirements. Our model also incorporates some technical details such as the need to be connected to the basestation with a bare-minimum rate ($1200Kpbs$) even in case the transmitter is scheduled to be silent, in order to keep the connection alive. The suggested resource allocation scheme with over provisioning for 2%, 5%, and 10% error margins are compared to a theoretical upperbound where the scheduler has perfect knowledge of the parameters and implements the optimum corresponding to those values. Our simulations over many iterations show that the overprovisioning in time was 1.0414, 1.1063, and 1.2266 times more than the theoretical optimum respectively for 2%, 5%, and 10% error margins for the estimated parameters. The theoretical optimum is calculated such that for every iteration, the scheduler calculates the optimum allocation with the perfect knowledge of the parameters. The average additional time required to provision for the estimation errors in simulations are in perfect agreement with the tabulated theoretical data in table1.

5 Conclusion

We proposed a novel power allocation and scheduling scheme for multimedia CDMA based wireless wide area networks with non-precise parameter estimates. Unlike traditional CDMA networks, our proposed algorithm transmits to the wireless stations with certain power levels and durations which is a result of an optimization problem we define and solve. The resulting algorithm dynamically adapts to the changes in the channel gains, intercell interference and background noise levels and guarantees QoS for end-users as along as the estimation errors are within a pre-defined range. Therefore the algorithm FiGARO provides an adaptive resource allocation scheme for cellular wireless networks with fault-tolerance towards parameter estimation errors.

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