

# Channel Frequency Optimization in Optical Networks Based on Gaussian Noise Model

Cao Chen<sup>†\*</sup>, Fen Zhou<sup>\*</sup>, Yuanhao Liu<sup>\*‡</sup>, Shilin Xiao<sup>†</sup>

<sup>\*</sup>Institut Supérieur d'Electronique de Paris (ISEP); CERI-LIA, University of Avignon, France.

<sup>†</sup>State Key Laboratory of Advanced Optical Communication Systems and Networks, Shanghai Jiao Tong University, China.

<sup>‡</sup>State Key Laboratory of Integrated Service Networks, School of Telecommunications Engineering, XiDian University, China  
Emails: cao.chen@isep.fr, fen.zhou@{isep, univ-avignon}.fr, yuanhao.liu@isep.fr, slxiao@sjtu.edu.cn

**Abstract**—To make the most of spectrum resources in optical fibers, we propose to improve the quality of transmission leveraging the optimization of channel center frequencies in optical networks with flex grid. For point-to-point communication, we first compute the optimal transmit power that minimizes the physical layer impairment (PLI) by using the Gaussian Noise (GN) model. We then derive the theoretical PLI-aware provisioning capacity in fixed grid optical networks and further formulate an optimization model that can estimate the maximum number of requests that can be provisioned in flex grid optical networks. Numerical simulation results reveal that with the help of channel center frequency optimization, 8.7% capacity improvement can be achieved in flex grid optical networks. The signal to noise ratio (SNR) margin improvement is also demonstrated in both point-to-point optical communication and optical rings.

**Index terms**— Flexible Optical Networks; Physical Layer Impairment, Frequency Optimization; Fixed Grid; Flex Grid;

## I. INTRODUCTION

The capacity demand for optical fiber communication continues to grow exponentially. Multi-core or multi-mode fibers enable to achieve a higher transmission capacity than conventional fibers. However, as the fiber spectrum resource becomes filled, it is urgent to make the spectrum usage as effective as possible [1, 2]. The evolution from wavelength division multiplexing to flexible optical networks (FONs) is accompanied by the development of optical equipment, such as waveshaper, optical tunable filter, and bandwidth variable transponder. Thus, each channel can work on spectral slot with 12.5GHz or even on continuous spectrum domain [3, 4]. Besides, it can choose various modulation formats, such as BPSK, QPSK, or 16QAM. With high order modulation formats, more traffic can be transported. As the adaptive spectral slot and multiple modulation formats enable to accommodate services of distinct granularity, we can make the spectrum usage efficient.

However, a dominating limitation of high order modulation format and dense channels is physical layer impairment (PLI). In order to successfully decode the received light signal, the

signal to noise ratio (SNR) of each channel should be higher than a certain threshold. A bigger SNR margin signifies a higher reliability and resilience against system aging and other transient events. Using the Gaussian Noise (GN) model, we can calculate the PLIs of channels with different bandwidth and modulation formats [5, 6]. The non-linear interference between channels can be also influenced by their center frequencies. Motivated by the fact that current spectral grid is evenly distributed, *i.e.* *fixed grid*, we propose a resource allocation model with flex grid where the center frequencies is continuous. The main objective is to improve the quality of transmission leveraging channel frequency optimization and the second objective is to maximize the PLI-aware service provisioning capacity. The main contributions of this paper can be summarized as follows,

- For point-to-point communication, we first present the PLI model using the GN model and give the optimal transmitting power that minimizes the PLI. With fixed grid, we further derive mathematically the upper bound of the PLI-aware service provisioning capacity.
- We propose a spectrum allocation method leveraging the center frequency optimization with flex grid. Simulation results demonstrate that our method outperforms the solution with fixed grid in terms of transmission capacity and SNR margin.

The rest of this paper is organized as follows. In Sec. II, we present a brief related work. Sec. III gives the PLI model and problem formulation. We derive the optimal transmit power and estimate the maximum number of requests that can be provisioned in Sec. IV. We then give the performance evaluation for point-to-point communication and optical rings in Sec. V and Sec. VI, respectively. Finally, Sec. VII concludes this paper.

## II. RELATED WORK

From a physical-layer perspective, the cross-channel interference is known in the literature as four-wave mixing. It can evaluate the non-linear distortion that other channels produce to the channel under evaluation. The traditional four-wave mixing model may face difficulties in FONs, where the channels can use variable bandwidth and modulation formats [7]–[9].

The work of F. Zhou is partially supported by the open project (2020GZKF017) of the State Key Laboratory of Advanced Optical Communication Systems and Networks, Shanghai Jiao Tong University, China.

The work is also supported by National Nature Science Fund of China (No.61775137, No.61431009, No.61271216 and No.61221001), China Scholarship Council (No. 201806093), and Eiffel Scholarship (No. 945226E).

Recently, the nonlinear interference models, *e.g.* the GN model based on spectral domain and the Volterra Series method based on time domain, have been proposed to adopt in FONs to quantify the nonlinear distortion [5]. It is worth mentioning that since the GN model can calculate the nonlinear distortion with different bandwidths, center frequencies, and light power, it has been widely adopted in the network design [8, 10].

From a network-wide perspective, routing and spectrum allocation for FONs has been widely studied in the literature [11]–[15]. For FONs, the spectrum continuity and contiguity constraints require that the spectrum indices in the fiber should be identical on its lightpath. Thus, one common method considering the PLIs is to assign the sufficient guard bands between the lightpaths. Each lightpath should work in the worst case assumption to obtain sufficient SNR margin but at the cost of wasting spectrum resources [11, 16]. Considering the different redundant SNR margins of each lightpath before spectrum assignment, the authors in [17] propose to distribute lightpath efficiently by pushing the lightpath with less SNR margin into the extreme edge of fibers, while more SNR margin in the center. Another randomly swapping method that swaps the frequency grids in the fibers has been also used to improve the SNR margin in the final spectrum assignment [8]. However, these researches on spectrum assignments are based on the ITU.T fixed grid, which cannot fully exploit the spectrum resource of fiber [4]. In fact, the GN model that can calculate the nonlinear distortion with different channel frequencies provides the basis of frequency optimization. Therefore, we investigate channel frequencies optimization based on the GN model to fully exploit the fiber spectrum resources.

### III. PHYSICAL LAYER IMPAIRMENT AND PROBLEM STATEMENT

#### A. PLI

The PLI of a signal in optical fibers can be calculated by the GN model. The advantage of GN model that can calculate the signal degradation by center frequencies, bandwidth, and transmit power independently, makes it more feasible for spectrum allocation [5, 6, 16]. For example, when transmit power spectral density (PSD)  $G_i$  (unit: W/GHz), bandwidth  $\Delta f_i$  (unit: GHz), and center frequency  $f_i$  (unit: GHz) are determined, the SNR after  $N_i$  spans can be denoted by the following equation,

$$\text{SNR}_i = \frac{G_i}{G_i^{\text{ASE}} + G_i^{\text{NLI}}} \quad (1)$$

where the parameter  $G_i^{\text{ASE}}$  is PSD of amplifier spontaneous emission (ASE) from optical amplifiers,  $G_i^{\text{NLI}}$  is PSD of nonlinear interference noise caused by the Kerr effect of fiber. In general, if we neglect the minor impact of multi-channel interference, the nonlinear interference noise can be expressed as the sum of *self-channel interference* (SCI),  $G_i^{\text{SCI}}$ , and *cross-channel interference* (XCI),  $G_i^{\text{XCI}}$ ,  $G_i^{\text{NLI}} = G_i^{\text{SCI}} + G_i^{\text{XCI}}$ . According to GN model, these interference  $G_i^{\text{ASE}}$ ,  $G_i^{\text{SCI}}$ , and  $G_i^{\text{XCI}}$  can be written as,

$$\begin{cases} t_i^{\text{ASE}} = \frac{G_i^{\text{ASE}}}{G_i} = N_i G_i^{\text{ASE}} / G_i \\ t_i^{\text{SCI}} = \frac{G_i^{\text{SCI}}}{G_i} = N_i \mu G_i^2 \text{asinh}(\rho \Delta f_i^2) \\ t_i^{\text{XCI}} = \frac{G_i^{\text{XCI}}}{G_i} = \sum_{j:j \neq i} \mu N_{ij} G_j^2 \ln \left( \frac{|f_i - f_j| + \Delta f_j / 2}{|f_i - f_j| - \Delta f_j / 2} \right) \end{cases} \quad (2)$$

where  $N_{ij}$  denotes the number of common spans for channels  $i$  and  $j$ , the coefficient  $\mu = \frac{3\gamma^2}{2\pi\alpha\beta_2}$  and  $\rho = \pi^2\beta_2/\alpha$ . In (2), the ASE noise PSD  $G_i^{\text{ASE}} = 10^{\alpha L_{\text{span}}/10} h\nu n_{\text{sp}}$ , where  $\alpha$  is the fiber attenuation factor,  $L_{\text{span}}$  is the length per span,  $h$  is the Planck constant ( $6.63 \times 10^{-34}$  J·s),  $\nu$  is the optical carrier frequency (193.5 THz),  $n_{\text{sp}}$  is the EDFA noise figure,  $\gamma$  is the nonlinear coefficient, and  $\beta_2$  is the second-order dispersion of 1550 nm [6, 8, 16, 17].

A parameter  $k_{i,j}$  is introduced to define the ratio of frequency center distance between channel  $i$  and channel  $j$  as follows,

$$k_{i,j} = |f_i - f_j| / \Delta f_j \quad (3)$$

Therefore, the frequency distance  $f_{ij} = |f_i - f_j| = k_{i,j} \Delta f_j = k_{j,i} \Delta f_i$ . The variable  $f_i$  is replaced by  $k_{i,j}$  in the following text. By substituting (2) into (1), we can get a universal quality of transmission (QoT) in (4).

$$\begin{aligned} \text{QoT}_i : N_i \frac{G_i^{\text{ASE}}}{G_i} + N_i \mu G_i^2 \text{asinh}(\rho \Delta f_i^2) \\ + \sum_{j:j \neq i} \mu N_{ij} G_j^2 \ln \left( \frac{2k_{i,j} + 1}{2k_{i,j} - 1} \right) \leq \frac{1}{\text{SNR}^{\text{th}}(c_i)} \end{aligned} \quad (4)$$

#### B. Problem Statement

We use  $G(V, E)$  to denote the optical network, where  $V$  represents the set of optical cross-connect nodes and  $E$  represents the set of links. Each link  $e \in E$  represents two fibers with available spectrum resource  $F$  (unit: GHz). To satisfy a demand with traffic rate  $r_i$  (unit: Gbps), the lightpath from source node  $s_i$  to destination node  $d_i$  should be assigned with continuous spectrum channel  $(f_i - \frac{\Delta f_i}{2}, f_i + \frac{\Delta f_i}{2})$ . The bandwidth  $\Delta f_i = r_i / \text{SE}(c_i)$ , where  $\text{SE}(c_i)$  is the spectral efficiency of a modulation format that can be selected from the set {BPSK, QPSK, 8QAM, 16QAM} in Table I [18]. After the routing, modulation format, and spectrum assignment, the SNR margin of all lightpaths must satisfy the QoT expressed by (4). In order to further improve the channel's transmission performance, we propose to optimize the center frequencies  $f_i$  in the final spectrum assignment.

As an example, Fig. 1 shows a spectrum assignment of 5 channels in a single fiber. With fixed grid mode, *i.e.* no channel frequency optimization, at most four channels can be allocated. The fragile channel in the center is deteriorated because it experiences the nonlinear distortion from two sides. However, with flex grid mode, the second and the fourth side channels move away from the center, which reduces the

TABLE I  
SPECTRAL EFFICIENCY AND SNR THRESHOLD OF DIFFERENT  
MODULATION FORMATS [18].

Modulation Format $c$	Spectral efficiency (SE) (bits/s/Hz)	SNR <sup>th</sup> (dB)
PM-BPSK	2	3.56
PM-QPSK	4	6.52
PM-8QAM	6	10.98
PM-16QAM	8	13.2

nonlinear interference on the third channel. Thus, the fragile channel in the center can be established successfully.

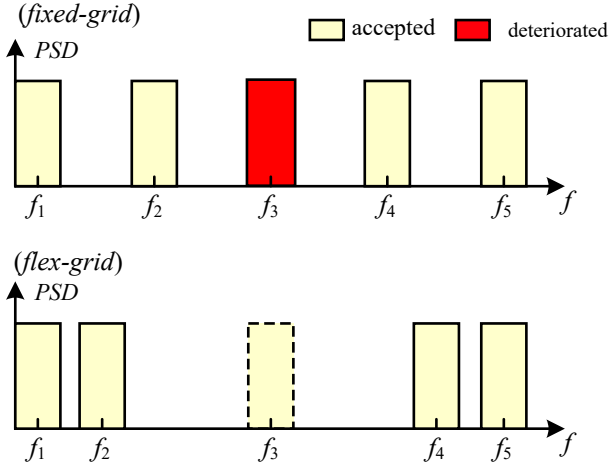


Fig. 1. Illustration of the fixed grid and flex grid in a fiber.

By channel frequency optimization, the primary goal of our optimization model is to improve the channel's transmission quality. Since the spectrum resource is limited, we also take the number of accepted channels as metrics, *i.e.* capacity. Therefore, the second goal is to maximize the total number of requests with a satisfactory QoT.

#### IV. FIXED GRID AND FLEX GRID RESOURCE ALLOCATION MODEL

In this section, we first compute the optimal transmit power minimizing the PLI. Then, we further formulate an optimization model that can estimate the provisioning capacity. To validate the perception of center frequency optimization, we first focus on the fiber transmission in a point-to-point communication with span length  $N_s$ , where each lightpath is with the identical transmit optical power  $G_i = G$ , bandwidth  $\Delta f_i = \Delta f$ , and modulation format  $c_i = c$ . The undetermined variables are the center frequencies  $\{f_1, f_2, \dots, f_n\}$ , denoted by  $\mathbf{f}$ . The QoT in (4) is updated as follows,

$$\sum_{j:j \neq i} \ln \left( \frac{2|f_i - f_j|/\Delta f + 1}{2|f_i - f_j|/\Delta f - 1} \right) \leq H(G), \quad \forall i \quad (5)$$

where  $H(G) = \frac{1}{\mu N_s \text{SNR}^{\text{th}}(c)} \left( \frac{1}{G^2} - \frac{N_s G^{\text{ASE}} \text{SNR}^{\text{th}}(c)}{G^3} \right) - \text{asinh}(\rho \Delta f^2)$ ,  $\text{SNR}^{\text{th}}(c)$  is the SNR threshold of modulation

format  $c$ . Since only the left term in (5) is controlled by variable  $\mathbf{f}$ , we can adjust a proper PSD  $G$  that maximizes the function  $H(G)$ .

The function  $H(G)$  with different span numbers is plotted in Fig. 2. By solving the first derivative order of  $H(G)$ , we can find the maximum value  $H_{max}$ ,

$$H_{max} = \frac{4G^{\text{ASE}}}{27\mu (N_s G^{\text{ASE}} \text{SNR}^{\text{th}})^3} - \text{asinh}(\rho \Delta f^2) \quad (6)$$

is obtained at  $G_{opt} = \frac{3}{2} N_s G^{\text{ASE}} \text{SNR}^{\text{th}}$ . It should be noted that the value of optimal PSD  $G_{opt}$  is related to the  $\text{SNR}^{\text{th}}$  and spans  $N_s$ . It is different from the well-known LOGON strategy, which can operate on the worst-case assumption without knowledge of spans and SNR threshold [16, 17].

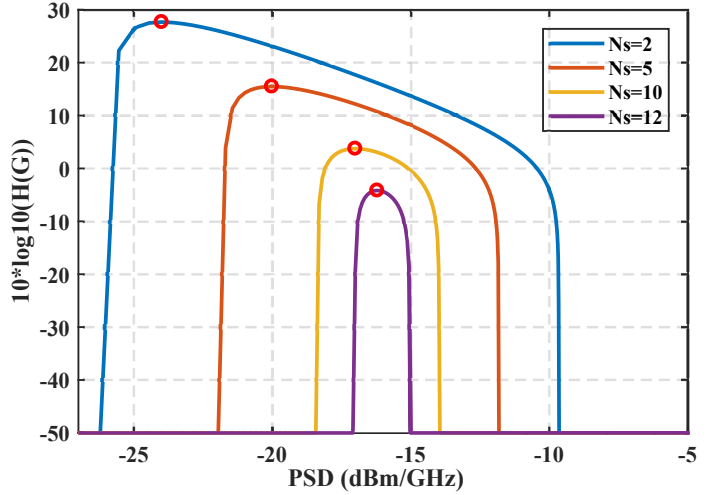


Fig. 2. The function  $H(G)$  as  $G$  varies.

On the left hand of (5), we denote by  $L_{\text{fixed}}^i(\mathbf{f})$  the XCI term of fixed grid, and by  $L_{\text{flex}}^i(\mathbf{f})$  the XCI term of flex grid.

##### A. Fixed Grid

In this case, a unit guard band ratio  $k_0$  is assumed. Thus, the center frequency  $\mathbf{f}$  can be determined with one variable  $k_0$ . By determining whether the XCI term  $L_{\text{fixed}}^{max}$  satisfies (5), we can know the status of all lightpaths and determine the best  $k_0$ . The model that we can use to obtain  $L_{\text{fixed}}^{max}$  is written as follows,

$$\min L_{\text{fixed}}^{max} \quad (7a)$$

$$s.t. L_{\text{fixed}}^i(f_1, f_2, \dots, f_n) \leq L_{\text{fixed}}^{max} \quad \forall i \quad (7b)$$

$$f_i + k_0 \Delta f = f_{i+1} \quad \forall i \quad (7c)$$

$$f_1 - \frac{\Delta f}{2} \geq 0, f_n + \frac{\Delta f}{2} \leq F \quad (7d)$$

Interestingly, due to the even frequency space  $k_0 \Delta f$ , the center channel must experience the most interference. In other word, we just need to calculate the XCI term  $L_{\text{fixed}}^{\frac{n+1}{2}}(\mathbf{f})$  or  $L_{\text{fixed}}^{\frac{n}{2}}(\mathbf{f})$  in (7b). We discuss two scenarios with odd and even number of channels, respectively.

1) *odd number channels*: The indices of center channel is  $\frac{n+1}{2}$ . Its XCI term  $L_{\text{fixed}}^{\frac{n+1}{2}}$  is written as follows,

$$\begin{aligned} L_{\text{fixed}}^{\frac{n+1}{2}} &= \sum_{j:j \neq i} \ln \left( \frac{2|f_i - f_j|/\Delta f + 1}{2|f_i - f_j|/\Delta f - 1} \right) \\ &= 2 \left( \ln \frac{2k_0 + 1}{2k_0 - 1} + \ln \frac{4k_0 + 1}{4k_0 - 1} + \dots + \ln \frac{2k_0 \frac{n-1}{2} + 1}{2k_0 \frac{n-1}{2} - 1} \right) \\ &= 2 \left( \ln \frac{\Gamma(\frac{n+1}{2} + \frac{1}{2k_0})}{\Gamma(\frac{n+1}{2} - \frac{1}{2k_0})} - \ln \frac{\Gamma(1 + \frac{1}{2k_0})}{\Gamma(1 - \frac{1}{2k_0})} \right) \end{aligned} \quad (8)$$

2) *even number channels*: The indices of center channel is  $\frac{n}{2}$ . Its XCI term  $L_{\text{fixed}}^{\frac{n}{2}}$  is,

$$\begin{aligned} L_{\text{fixed}}^{\frac{n}{2}} &= \sum_{j:j \neq i} \ln \left( \frac{2|f_i - f_j|/\Delta f + 1}{2|f_i - f_j|/\Delta f - 1} \right) \\ &= 2 \left( \ln \frac{2k_0 + 1}{2k_0 - 1} + \dots + \ln \frac{2k_0 \frac{n}{2} - 1 + 1}{2k_0 \frac{n}{2} - 1 - 1} \right) + \ln \frac{k_0 n + 1}{k_0 n - 1} \\ &= 2 \left( \ln \frac{\Gamma(\frac{n}{2} + \frac{1}{2k_0})}{\Gamma(\frac{n}{2} - \frac{1}{2k_0})} - \ln \frac{\Gamma(1 + \frac{1}{2k_0})}{\Gamma(1 - \frac{1}{2k_0})} \right) + \ln \frac{k_0 n + 1}{k_0 n - 1} \end{aligned} \quad (9)$$

The maximum XCI term  $L_{\text{fixed}}^{\text{max}} = L_{\text{fixed}}^l$ ,  $l = \frac{n}{2}$  or  $\frac{n+1}{2}$ . Therefore, the constraints (7b) can be replaced by either (8) or (9).

Without SNR limitation, the maximum number of requests is  $\frac{F - \Delta f}{k_0 \Delta f} + 1$ . Thus, with (6), (8) and (9), we can also give the number of requests that can be accommodated as follows,

$$\left\{ n \in \mathbb{N}^+ \mid L_{\text{fixed}}^{\text{max}}(k_0, n) \leq H_{\text{max}}, \text{ and } n \leq \frac{F - \Delta f}{k_0 \Delta f} + 1 \right\} \quad (10)$$

### B. Flex Grid

No frequency grid is assumed. Compared to fixed grid, the variables change from one-dimensional  $k_0$  to  $n$ -dimensional  $\mathbf{f}$ . The model that we use to determine the status of lightpaths is presented as follows,

$$\min L_{\text{flex}}^{\text{max}} \quad (11a)$$

$$\text{s.t. } L_{\text{flex}}^i(f_1, f_2, \dots, f_n) \leq L_{\text{flex}}^{\text{max}} \quad \forall i \quad (11b)$$

$$f_i + \Delta f \leq f_{i+1} \quad \forall i \quad (11c)$$

$$f_1 - \frac{\Delta f}{2} \geq 0, f_n + \frac{\Delta f}{2} \leq F \quad (11d)$$

The model in (11) is a nonlinear model. The fixed grid constraints of (7c) are replaced by the inequality in (11c). It can be proved that the Hessian matrix  $\nabla^2 L_{\text{flex}}(f_1, f_2, \dots, f_i)$  is positive,  $\forall i \leq n$ . We solve the model (11) by the Optimization Toolbox of MATLAB™.

We summarize the frequency optimization procedure based on the previous two modes. With fixed grid mode, by solving  $L_{\text{fixed}}^{\text{max}} \leq H_{\text{max}}$ , we can determine the best  $k_0$  that ensures all channels' quality. With flex grid mode, we can find the maximum XCI term  $L_{\text{flex}}^{\text{max}}$  by solving the model (11). The number of channels is increased incrementally until the maximum XCI term  $L_{\text{fixed}}^{\text{max}}$  or  $L_{\text{flex}}^{\text{max}}$  exceeds the  $H_{\text{max}}$ . Thus, the provisioning capacity is taken as the largest number of requests for which the frequency optimization is possible.

## V. EVALUATIONS FOR POINT-TO-POINT COMMUNICATION

In our simulations, the parameters related to the PLIs are stated as follows,  $\alpha = 0.22$  dB/km,  $L_{\text{span}} = 100$  km,  $n_{\text{sp}} = 7$  dB,  $\gamma = 1.3$  W · Km<sup>-1</sup>, and  $\beta_2 = 21.7$  ps<sup>2</sup>/km [18].

### A. Capacity Improvement

By setting each channel with PM-16QAM and traffic rate of 250 Gbps,  $N_s=10$ , and  $F=2000$  GHz, we calculate the number of accepted requests based on fixed grid and flex grid. The results are shown in Fig. 3. For fixed grid, we vary the even frequency grid from 40 GHz to 150 GHz. The curves that with only spectrum resource constraint and with only QoT constraint are also illustrated. The former adopts the constraints (7c) and (7d), while the latter adopts the constraints (7b) and (7d).

In Fig. 3, the number of accepted requests increases with the frequency grids before it reaches to its maximum 23 at 87 GHz. It can be explained that when only SNR is considered, large frequency space can reduce the interference on all requests. As the frequency grid continues to grow, the number of accepted requests decreases, because the spectrum resource  $F$  runs out. However, with flex grid that optimizes the channel frequencies, we observe that the number of requests can further increase to 25. The capacity improvement is about 8.7%.

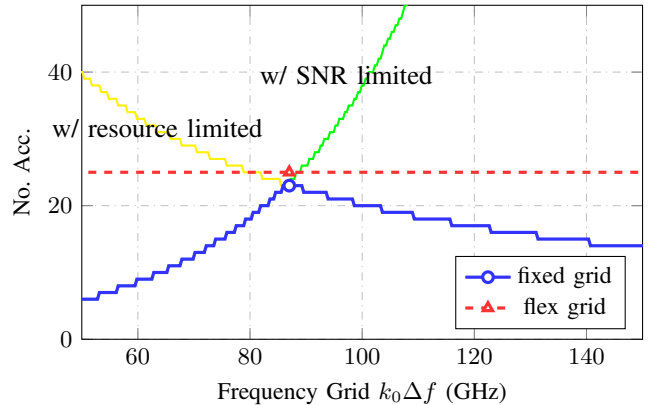


Fig. 3. Number of accepted channels with the configuration of 16QAM and 10 spans as frequency grid increases.

The channel distribution of these two examples are plotted in Fig. 4. In fixed grid example, the extreme edge channels' SNR is larger than the center channel. Besides, the SNR of center channel gets more close to the threshold than other channels. In flex grid example, the SNR distributes uniformly on the fiber. In other words, by optimizing the frequency space, model (11) can balance all channels' SNR. It can be seen in Fig. 4(b), the channel in the center requires more space than those at the extreme edge to reduce XCI interference. Compared to the fixed grid example with even space, model (11) uses the frequency space from 50 to 90 GHz, rather than the even frequency space of fixed grid 87 GHz.

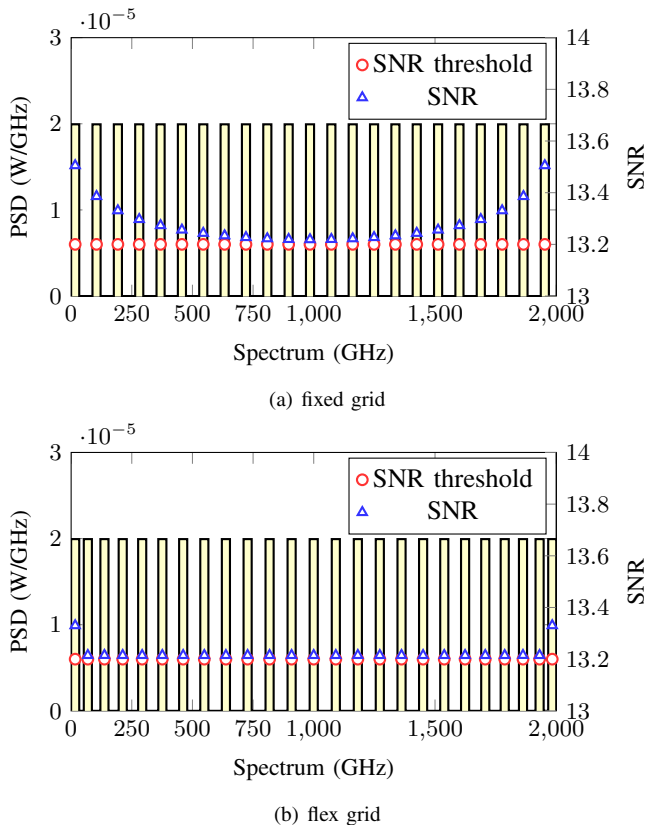


Fig. 4. Channel distribution and SNR of the example in fixed grid and flex grid. (a) blue circle in Fig. 3; (b) red triangle in Fig. 3.

### B. Minimum Margin Improvement

As mentioned previously, flex grid model in (11) can balance SNR margin of the channels. Therefore, we compare the SNR margin improvement of two grid modes with the same number of channels, *e.g.* 23. The results are shown in Fig. 5. The SNR distribution of flex grid is flat in all channels, while the fixed grid is of bowl shape. The minimum SNR margin improvement of flex grid is 0.094 dB. Compared to the optimized power allocation with 0.043 dB margin improvement, the frequency optimization is also small [2].

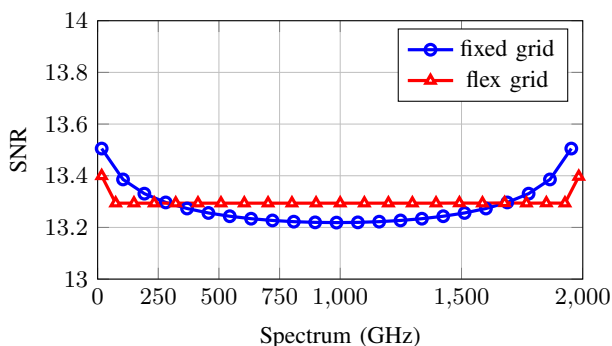


Fig. 5. SNR of 23 channels with modulation format PM-16QAM,  $F=2000\text{GHz}$ .

## VI. EVALUATIONS FOR OPTICAL RINGS

In the previous section, we have observed the performance enhancement by optimizing channel's center frequencies in point-to-point communication, including capacity improvement and margin improvement. The advantage of flex grid in optical network will be stated in this section.

Our frequency optimization is based on a specified routing and spectrum assignment result, in which the route and spectrum indices have been determined. We denote by  $x_{il}$  whether request  $i$  uses link  $l$ , and by  $T_{l\lambda}$  the link spectrum usage table.  $T_{l\lambda}=i$  means that request  $i$  uses both link  $l$  and slot  $\lambda$ . Other parameters can refer to the Table II. Since the cross-interfering spans and optimal power rely on the routing path in network, we cannot use the frequency optimization in point-to-point communication. Instead, we use the original QoT form of (4) and set the objective with maximum network margin,

$$\max L \quad (12a)$$

$$N_i \frac{G_i^{\text{ASE}}}{G_i} + N_i \mu G_i^2 \text{asinh}(\rho \Delta f_i^2) \quad (12b)$$

$$s.t. \quad + \sum_{j:j \neq i} \mu N_{ij} G_j^2 \ln \left( \frac{2k_{i,j} + 1}{2k_{i,j} - 1} \right) \leq \frac{1}{\text{SNR}^{\text{th}}(c)} - L \quad \forall i \quad (12c)$$

$$k_{i,j} = (f_j - f_i) / \Delta f, k_{i,j} = k_{j,i} \quad \forall \lambda_i < \lambda_j \quad (12d)$$

$$f_i + \Delta f \leq f_j, \Delta f + f_j - f_i \leq F \quad \forall N_{ij} \neq 0 \& \lambda_i < \lambda_j \quad (12d)$$

TABLE II  
PARAMETERS IN THE SIMULATION OF OPTICAL RING

Network sets & Parameters	
$T_{l\lambda}$	Link spectrum table of routing and spectrum results. $T_{l\lambda} = i$ means request $i$ uses both link $l$ and slot $\lambda$ .
$x_{il} \in \{0, 1\}$	Equals 1 if request $i$ uses link $l$ .
$N_i$	Number of span traversed by request $i$ .
$N_{ij}$	Number of common spans traversed by request $i$ and $j$ .
$\lambda_i \in \mathbb{N}$	Frequency slot of request $i$ .
Variables	
$f_i$	Frequency center of request $i$ .
$L$	Network margin, $L \geq 0$ .

We validate the channel frequency optimization through a small number of lightpaths in optical ring network, which has less connectivity. The reason is that the complexity of routing and spectrum assignment in the resource provisioning problem can be minimized.

Figure 6(a) shows four lightpaths in a 4-node ring network topology, including four requests R1~R4. Each link is with  $F=1000\text{GHz}$ . The requests carry traffic 250 Gbps and use the same modulation format 16QAM. Different lightpaths are marked with different colors. In this optical ring network, the unique route between two nodes can be determined with a given direction, *e.g.* counterclockwise, but the spectrum assignment may vary with their different spectrum slots ( $\lambda_i$ ). Hence, we enumerate 16 possible spectrum orders of four lightpaths, which is shown in Fig. 6(b). For example, the first spectrum assignment assigns four lightpaths on spectrum slot ( $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ ) on ( $f_1, f_2, f_3, f_4$ ), while the last spectrum assignment assigns four lightpaths on ( $f_4, f_3, f_2, f_1$ ).

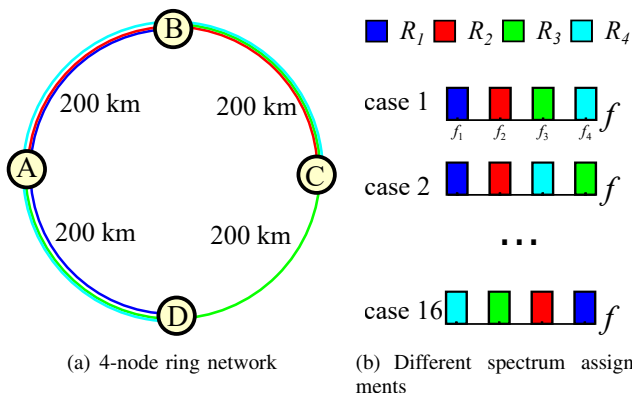


Fig. 6. Illustration of the topology and the different routing and spectrum assignment results. The lightpaths of four requests are marked with different colors, R1(B-A-D,blue), R2(C-B-A,red), R3(A-D-C-B,green), and R4(C-B-A-D,cyan).

By solving (12), we can find the maximum SNR margin  $L$  of the network with flex grid. Then, we calculate the SNR margin of each lightpath with the channel frequency assignment results, as illustrated in Fig. 7. Moreover, to avoid the impact of different spectrum assignments, we also optimize the frequencies with the other 15 different spectrum assignments. When we compare the SNR margin with other fixed grid mode in Fig. 7, we observe that the minimum margin of each lightpath increases with the frequency grid. However, it cannot achieve the same performance of flex grid.

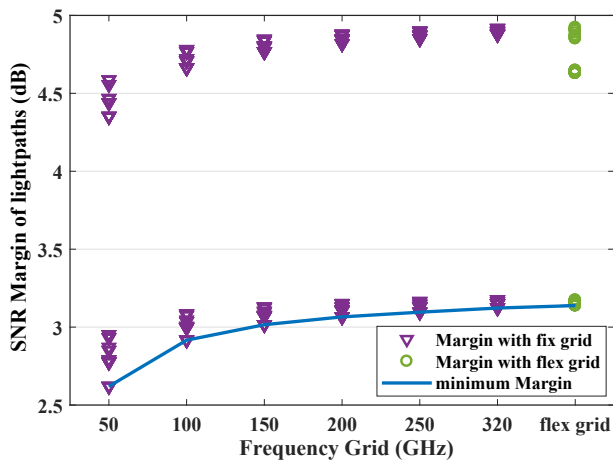


Fig. 7. Margin improvement of the lightpath in Fig. 6. Each dot represents the relative SNR margin improvement to the threshold.

## VII. CONCLUSION

In this paper, we have proposed a resource allocation model that optimizes the channel center frequencies based on GN model. Compared with the existing fixed grid with even frequency space, our scheme can improve minimum SNR margin and increase the number of accepted channels. The PLI-aware service provisioning capacity of fixed grid is given, which is controlled by the guard band ratio and the spectrum resources. In point-to-point communication with flex grid,

8.7% capacity improvement is achieved. Besides, in optical ring, the SNR margin of all lightpaths can be improved.

## REFERENCES

- [1] S. J. Savory, "Approximations for the nonlinear self-channel interference of channels with rectangular spectra," *IEEE Photon. Technol. Lett.*, vol. 25, no. 10, pp. 961–964, Apr. 2013.
- [2] I. Roberts, J. M. Kahn, and D. Boertjes, "Convex channel power optimization in nonlinear WDM systems using Gaussian Noise model," *IEEE/OSA J. Lightw. Technol.*, vol. 34, no. 13, pp. 3212–3222, May 2016.
- [3] D. Xie, D. Wang, M. Zhang, Z. Liu, Q. You, Q. Yang, and S. Yu, "LCoS-based wavelength-selective switch for future finer-grid elastic optical networks capable of all-optical wavelength conversion," *IEEE Photonics J.*, vol. 9, no. 2, pp. 1–12, Feb. 2017.
- [4] *Spectral grids for WDM applications: DWDM frequency grid*, ITU-T Recommendation G.694.1 Std., Feb. 2012. [Online]. Available: <https://www.itu.int/rec/T-REC-G.694.1-201202-1/en>
- [5] P. Poggiolini, "The GN model of non-linear propagation in uncompensated coherent optical systems," *IEEE/OSA J. Lightw. Technol.*, vol. 30, no. 24, pp. 3857–3879, Dec. 2012.
- [6] P. Johannisson and E. Agrell, "Modeling of nonlinear signal distortion in fiber-optic networks," *IEEE/OSA J. Lightw. Technol.*, vol. 32, no. 23, pp. 3942–3950, Oct. 2014.
- [7] W. Zeiler, F. Di Pasquale, P. Bayvel, and J. E. Midwinter, "Modeling of four-wave mixing and gain peaking in amplified wdm optical communication systems and networks," *IEEE/OSA J. Lightw. Technol.*, vol. 14, no. 9, pp. 1933–1942, Sep. 1996.
- [8] D. J. Ives, P. Bayvel, and S. J. Savory, "Routing, modulation, spectrum and launch power assignment to maximize the traffic throughput of a nonlinear optical mesh network," *Photonic Netw. Commun.*, vol. 29, no. 3, pp. 244–256, Jun. 2015.
- [9] R. Proietti, X. Chen, K. Zhang, G. Liu, M. Shamsabardeh, A. Castro, L. Velasco, Z. Zhu, and S. J. B. Yoo, "Experimental demonstration of machine-learning-aided QoT estimation in multi-domain elastic optical networks with alien wavelengths," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 11, no. 1, pp. A1–A10, Jan. 2019.
- [10] A. Alvarado, D. J. Ives, S. J. Savory, and P. Bayvel, "On the impact of optimal modulation and FEC overhead on future optical networks," *IEEE/OSA J. Lightw. Technol.*, vol. 34, no. 9, pp. 2339–2352, Jan. 2016.
- [11] H. Wu, F. Zhou, Z. Zhu, and Y. Chen, "On the distance spectrum assignment in elastic optical networks," *IEEE/ACM Trans. Netw.*, vol. 25, no. 4, pp. 2391–2404, Aug. 2017.
- [12] Y. Yin, H. Zhang, M. Zhang, M. Xia, Z. Zhu, S. Dahlfors, and S. B. Yoo, "Spectral and spatial 2D fragmentation-aware routing and spectrum assignment algorithms in elastic optical networks," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 5, no. 10, pp. A100–A106, Oct. 2013.
- [13] L. Gong, X. Zhou, X. Liu, W. Zhao, W. Lu, and Z. Zhu, "Efficient resource allocation for all-optical multicasting over spectrum-sliced elastic optical networks," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 5, no. 8, pp. 836–847, Aug. 2013.
- [14] L. Gong and Z. Zhu, "Virtual optical network embedding (VONE) over elastic optical networks," *IEEE/OSA J. Lightw. Technol.*, vol. 32, no. 3, pp. 450–460, Dec. 2013.
- [15] Z. Zhu, W. Lu, L. Zhang, and N. Ansari, "Dynamic service provisioning in elastic optical networks with hybrid single-/multi-path routing," *IEEE/OSA J. Lightw. Technol.*, vol. 31, no. 1, pp. 15–22, Nov. 2012.
- [16] P. Poggiolini, G. Bosco, A. Carena, R. Cigliutti, V. Curri, F. Forghieri, R. Pastorelli, and S. Piciaccia, "The LOGON strategy for low-complexity control plane implementation in new-generation flexible networks," in *Proc. Opt. Fiber Commun. Conf. (OFC)*, Anaheim, CA, USA, Mar. 2013, p. OW1H.3.
- [17] L. Yan, E. Agrell, M. N. Dharmaweera, and H. Wymeersch, "Joint assignment of power, routing, and spectrum in static flexible-grid networks," *IEEE/OSA J. Lightw. Technol.*, vol. 35, no. 10, pp. 1766–1774, May 2017.
- [18] C. Chen, F. Zhou, and S. Xiao, "Maximizing revenue with adaptive modulation and multiple FEC in flexible optical networks," in *Proc. Int. Conf. on High Perform. Comput. and Commun.*, Zhangjiajie, China, Aug. 2019, pp. 2476–2481.