

# Interference Aware RSA Problem

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**Abstract**—Several studies have been proposed for the Routing and Slot Allocation (RSA) problem in optical networks. However, while some of them integrate the Optical Signal-to-Noise Ratio (OSNR) to quantify the degree of optical noise interference on optical signals, very few of them do it with an exact modelling, in the context of mathematical programming network provisioning models. In the present study, we propose a mathematical model that integrates it without the recourse to linearization techniques, hence approximations, thanks to a Dantzig-Wolfe decomposition scheme combined with a Tabu Search algorithm. Therein, while taking into account the OSNR interference ratio, we optimize the routing and the assignment of the frequency slots in order to maximize the throughput.

Computational results illustrate the impact on the throughput when OSNR is taken into account. We also show that we can solve accurately data instances with 400 slots and up to 700 requests in reasonable computing times, i.e., significantly larger data sets than in the literature.

## I. INTRODUCTION

With the growth of Internet and network traffic demands, the efficient and cost-effective usage of bandwidth and spectrum in optical networks plays an important role in improving service provisioning. Elastic Optical Networks (EONs) define the new generation of optical networks with a higher flexibility and scalability in spectrum allocation and data rate accommodation to support different traffic types. Indeed, EONs are based on orthogonal frequency division multiplexing (OFDM). Therein, the spectrum is divided into finer spectrum intervals, called Frequency Slots (FSs), with a bandwidth of 12.5GHz or less, so that narrower spectrum slots can be allocated to lower bit rate traffic. As a result, the use of spectral resources is improved. Routing is done with lightpaths, i.e., optical paths established between a given source–destination node pair with a predefined data rate. Frequency slots assigned to a lightpath must satisfy two conditions: continuity and contiguity of spectrum resources. According to the continuity constraint, the same frequency slots need to be used from source to destination for a given demand request. According to the contiguity constraint, the allocated frequency slots must be pairwise contiguous in any given lightpath.

Many of the proposed RSA solutions consider the optical fiber as an ideal channel and do not consider the QoS requirements in their optimization analysis. Indeed, fiber is a non-ideal channel and optical fiber communication requires special attention to channel effects such as Amplified Spontaneous Emission (ASE) noise and Non Linear Interferences (NLI). Consequently, in this study, we focus on the design of an exact

optimization RSA model with a decomposition structure that includes the OSNR (Optical Signal-to-Noise Ratio).

Contributions of this study is therefore an original exact mathematical programming model for the interference aware RSA problem. It also includes an implementation and a validation of this model, with numerical experiments on medium to large optical networks.

The paper is organized as follows. In Section II, we summarize several references related to the interference aware RSA problem. In III, we describe the RSA problem statement, introduce the GN (Gaussian Noise) model for the OSNR ratio and a reach table. We develop our mathematical model in Section IV and expose the solution process in Section V. Numerical results are presented in Section VI. Conclusions are drawn in the last section.

## II. LITERATURE REVIEW

As mentioned earlier, only few authors got interested in including the OSNR into their RSA network provisioning mathematical models.

Ives *et al.* [5] propose an algorithm to maximize the throughput in a Dense Wavelength Division Multiplexing (DWDM) network, with a 2-step solution, first the solution of the Routing, Modulation and Spectrum Assignment (RMSA) problem and secondly the optimization of the launch power optimization separately. They next investigate how the increase in OSNR margin can be utilized through modulation format adaption to increase the overall network throughput. The authors report an improvement of up to 300% in network throughput on the Google B4 network (12 nodes, 19 links).

Bhar *et al.* [1] propose a solution to the channel ordering on a single link using graph theory and traveling salesman problem. Interferences are modeled using the GN formula.

Yan *et al.* [10], [8] and Hadi *et al.* [4] proposed solutions for RSA in EONs. In [10], [8], the authors solved a nonlinear model with the embedding of the Gaussian Noise (GN) formula to approximate the optical signal to noise ratio (OSNR), and conduct experiments on a set of two links with multiple spans. In [9], the authors proposed a linearization of their previous model, but scalability issues remain with their resulting Mixed Integer Linear Program (MILP) and the authors conduct their numerical experiments with a heuristic. In [4] the authors proposed a 2-stage heuristic: routing and channel ordering, power and spectrum assignment, each stage of the heuristic is formulated as a geometric program. The results were compared with the MILP proposed in [10] and

shown to be much faster. Unlike [9], they approximate the nonlinear term using a polynomial function, which was more economical than linearization in terms of the number of variables.

Salani *et al.* [7] use a machine learning (ML) approach to assess whether noise in networks is above its threshold, with the help of a linear program model integer (ILP) for RSA problem. Their approach solves the issue of inaccurate parameters of the network infrastructure, by letting the ML model learn the characteristics of the network through data. The resulting algorithm saves up to 30% (around 20% on average) compared to traditional ILP-based RSA approaches with reach constraints based on margined analytical models. However, the authors face some scalability issues when running on large data instances.

Different formulas have been proposed for modelling the OSNR. While they are few variations, most formulas differ in their coefficient numerical values. Our study used Poggiolini *et al.* [6] as reference for the GN formula. Accurate estimation of the OSNR by a formula is indeed a difficult modelling and then optimization problem, and some authors, e.g., Caballero *et al.* [2], have investigated its estimation using machine learning techniques.

### III. PROBLEM STATEMENT

#### A. Routing and Spectrum Allocation Problem

Consider an optical network  $G = (V, L)$ , where  $V$  is the set of optical nodes (e.g., Reconfigurable Optical Add-Drop Multiplexer ROADMs) and  $L$  the set of fiber links. Spectrum in each fiber link is divided into a set of frequency slots with a bandwidth assumed to be 12.5 GHz in this paper. In addition, each fiber link is divided into a number of spans, with amplifying stations at both ends, see Figure 1 below.

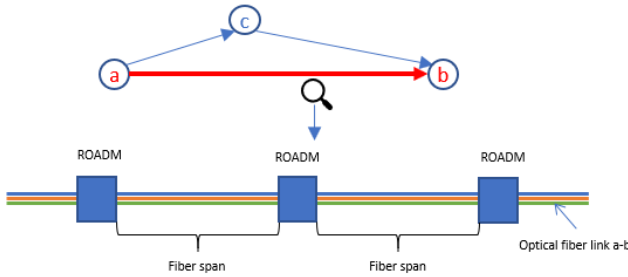


Fig. 1. Span vs. Link

The set of traffic requests is denoted by  $K$ , and indexed by  $k$ . Each request is characterized by its source and destination nodes, and its data rate  $r_k$ . We are interested in the solution of the Routing and Spectrum Allocation (RSA) problem in which we consider the link noise level as estimated by the OSNR, with the objective of maximizing the throughput.

Each granted request  $k$  is associated with a lightpath  $\pi_k$ , consisting of a path and a set of frequency slots which must

satisfy both the contiguity and continuity constraints: the FSs allocated to  $k$  must be adjacent to each other, and the same on every optical fiber link along the optical path/route of the request.

Furthermore, each granted request  $k$  must satisfy the OSNR constraint, that is for the channel  $c$  that is allocated to  $k$ , its OSNR value must be greater or equal its OSNR threshold calculated using the Shannon formula. This constraint is explained in detail in Section (III-C).

#### B. GN Model for the OSNR

Recent studies with an OSNR analytical formula rely on the GN (Gaussian Noise) model. In this study, we use the original GN model of [6] (formula (41)) under the assumption that the Power Spectral Density (PSD) of a channel per fiber span, and the parameters of the fiber are identical on every fiber span. Under these assumption, expression (41) of [6] can be written:

$$G_{\text{NLI}}(f_c) = \frac{16}{27} \sum_{n_{\text{span}}=1}^{N_{\text{span}}} \gamma^2 L_{\text{eff, a}}^2 \times (\Gamma^3 e^{-6\alpha L_{\text{span}}})^{n_{\text{span}}-1} \times (\Gamma e^{-2\alpha L_{\text{span}}})^{N_{\text{span}}-n_{\text{span}}-1} \sum_{i=1}^{N_{\text{ch}}} G_i^2 G_c \times (2 - \delta_{i,c}) \psi_{i,c,n_{\text{span}}} \quad (1)$$

where

$G_{\text{NLI}}(f_c)$	non linear interference (NLI) at the center frequency $f_c$ of channel $c$
$G_c$	PSD of channel $c$
$N_{\text{span}}$	number of spans of channel $c$
$\gamma$	non linear coefficient
$L_{\text{eff, a}}$	asymptotic effective length
$L_{\text{span}}$	span length
$\alpha$	power attenuation
$\Gamma$	Erbium-Doped Fiber Amplifier (EDFA) gain
$\delta_{i,c}$	= 1 if $i = c$ , 0 otherwise
$\gamma = 1.3 \cdot 10^{-3} \text{ mW}^{-1} \text{ km}^{-1}$	$\alpha = 0.023 \text{ km}^{-1}$
$L_{\text{eff, a}} = 1/(2\alpha)$	$\Gamma = e^{2\alpha L_s}$
$\psi_{i,c,n_{\text{span}}}$	$\approx \frac{1}{4\pi(2\alpha)^{-1} \beta_2 } \ln \left( \frac{ f_i - f_c  + B_i/2}{ f_i - f_c  - B_i/2} \right)$ for $i \neq c$ $\approx \frac{\text{asinh}(\frac{\pi^2}{2}  \beta_2  (2\alpha)^{-1} B_c^2)}{2\pi(2\alpha)^{-1} \beta_2 }$ for $i = c$ .

Using the mathematical expression of  $\psi_{i,c,n_{\text{span}}}$ , formula (1) then becomes:

$$G_{\text{NLI}}(f_i) = \frac{16}{27} \frac{\gamma^2 L_{\text{eff, a}}^2 \alpha}{\pi |\beta_2|} \times \left[ \sum_{j=1, j \neq i}^{N_{\text{channel}}} G_j^2 G_i N_{\text{span}}^{ij} \ln \left( \frac{|f_j - f_i| + B_j/2}{|f_j - f_i| - B_j/2} \right) + N_{\text{span}} G_i^3 \text{asinh} \left( \frac{\pi^2 |\beta_2| B_i^2}{4\alpha} \right) \right], \quad (2)$$

where  $N_{\text{span}}^{ij}$  is the number of spans shared between channel  $i$  and  $j$ . Next is the ASE noise:

$$G_{\text{ASE}}(f_i) = N_s (e^{2\alpha L_{\text{span}}} - 1) h n_{sp} f_i \quad (3)$$

where  $h = 6.62607004 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1}$  is the Planck's constant,  $n_{sp} = 5.01$  is the factor of spontaneous noise. The OSNR is then written as follows:

$$\text{OSNR}_i = \frac{G_i}{G_{\text{ASE}}(f_i) + G_{\text{NLI}}(f_i)}. \quad (4)$$

From (2), we have:

$$G_{\text{SCI}}(f_i) = \frac{16 \gamma^2 L_{\text{eff},a}^2 \alpha}{27 \pi |\beta_2|} N_{\text{span}} G_i^3 \text{asinh} \left( \frac{\pi^2 |\beta_2|}{4\alpha} B_i^2 \right) \quad (5)$$

$$G_{\text{XCI}}(f_j, f_i) = \frac{16 \gamma^2 L_{\text{eff},a}^2 \alpha}{27 \pi |\beta_2|} \times G_j^2 G_i N_{\text{span}}^{ij} \times \ln \left( \frac{|f_j - f_i| + B_j/2}{|f_j - f_i| - B_j/2} \right) \quad (6)$$

$$\rightsquigarrow G_{\text{XCI}}(f_i) = \sum_{j=1, j \neq i}^{N_{\text{channel}}} G_{\text{XCI}}(f_j, f_i), \quad (7)$$

where  $G_{\text{SCI}}(f_i)$  is Self-Channel Interference (SCI) of channel  $i$ ,  $G_{\text{XCI}}(f_j, f_i)$  is Cross-Channel Interference (XCI) caused by channel  $j$  to channel  $i$ , and  $G_{\text{XCI}}(f_i)$  is total cross interference of channel  $i$ .

### C. Reach Table

In this section, we establish the reach table, i.e, the maximum reach optical signals can propagate, expressed in terms of spans, for a given rate  $r$  and channel  $c$ , for a single optical fiber lightpath  $\pi$ , so that  $N_{\text{span}}^{c,c} = N_{\text{span}}$  in formula (2). We assume that the set of channel widths to be  $B = \{37.5, 62.5, 87.5, 112.5\}$  GHz, and that the set of required bit rates is  $R = \{100\text{Gbps}, 200\text{Gbps}, 400\text{Gbps}\}$ . We use the Shannon's formula and (4) to compute the bandwidth that can be assigned to a channel. Shannon's formula is as follows:

$$r_\pi = B_\pi \log_2(1 + T_\pi), \quad (8)$$

where  $r_\pi$  and  $T_\pi$  are the bit rate and the OSNR threshold of channel  $c$  in  $\pi$ , respectively, assuming the OSNR constraint being written:

$$\text{OSNR}_\pi \geq T_\pi. \quad (9)$$

From (8), we have:

$$T_\pi = 2^{r_\pi/B_\pi} - 1. \quad (10)$$

In practice, a factor is applied to fine tune the value:

$$T_\pi = (2^{r_\pi/B_\pi} - 1) \times \frac{1}{0.85}. \quad (11)$$

We used a fulfilled scenario to calculate the reach table, that is, for a single optical fiber with C-band spectrum, it consists of 380 frequency slots, each slot is 12.5 GHz, and the spectrum is filled with the same type of channels, i.e., with the same bandwidth and bit rate. For positioning a channel in the spectrum, we need guardbands on the left and right side of each channel. In practice, the width of each guardband is 6.25 GHz, that means each channel takes another frequency slot as

guard band next to the required one(s). Looking at the formula (2), (3) and (4), when the variables  $B_c, f_c$  are known, and all the launch powers of each channel  $P_c$  are identical (which lead to all channels having the same PSD  $G_c = P_c/B_c$ ), then the only variable left is  $N_s$ , i.e., number of spans. As our scenario consists of a single optical fiber link, we need to find the maximum value of  $N_s$  that still satisfies the OSNR constraint. The noise of the middle channel of the spectrum is the one with the highest noise (or the lowest OSNR) value. All of the  $\text{OSNR}_{T,c}$  channel thresholds are identical because of the same value of  $B_c, f_c$ . Satisfying the OSNR constraint of the median channel is equivalent to ensuring the OSNR constraint of each spectrum channel in the spectrum. Computational results are summarized below in Table I.

TABLE I  
REACH TABLE

Bit rate $r_c$ Gbps	Bandwidth $B_c$ GHz	Bandwidth + guardbands GHz	Number of spans $N_{\text{span}}$
100	37.5	50	57
	62.5	75	137
	87.5	100	222
	112.5	125	307
200	37.5	50	7
	62.5	75	34
	87.5	100	69
	112.5	125	107
400	62.5	75	3
	87.5	100	11
	112.5	125	24

## IV. MATHEMATICAL MODEL

We describe here our proposed optimization model for the routing and slot allocation problem, in which we include the modelling of the Optical Signal-to-Noise Ratio (OSNR).

### A. Variables and Parameters

For any given link  $\ell$ , we introduce the concept of  $\ell$  lightpath configuration, denoted by  $\gamma$ , as a set of lightpaths such that the first link in their path is  $\ell$ . The overall set of lightpath configuration is denoted by  $\Gamma_\ell$  and the overall set of configurations is:

$$\Gamma = \bigcup_{\ell \in L} \Gamma_\ell.$$

Figure 2 depicts an example of a lightpath configuration associated with link  $\tilde{\ell}$ .

We now define the set of parameters and variables for our optimization model.

#### Parameters

- $\gamma$  a lightpath configuration
- $K$  set of requests, indexed by  $k$
- $S$  set of frequency slots, indexed by  $s$
- $\pi_k$  set of lightpaths associated with request  $k$ , indexed by  $\pi$
- $r_k$  data rate of request  $k$

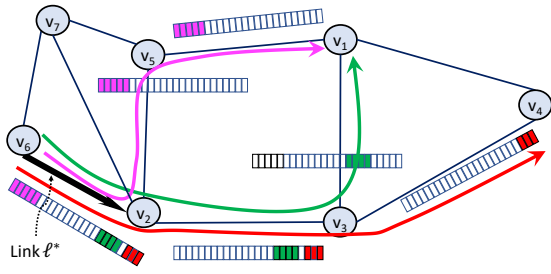


Fig. 2. Lightpath configuration example

Parameters of configuration  $\gamma$

- $a_k^\gamma = 1$  if request  $k$  is granted in  $\gamma$ , 0 otherwise
- $a_{\ell s}^\gamma = 1$  if slot  $s$  on link  $\ell$  in  $\gamma$  is occupied, 0 otherwise
- $y_\pi^\gamma = 1$  if  $\pi$  appears in  $\gamma$ , 0 otherwise
- $\theta_\pi^\gamma$  total cross interference (XCI) caused by all the lightpaths in  $\gamma$  to  $\pi$ .

Variables

- $z_\gamma = 1$  if  $\gamma$  is selected, 0 otherwise
- $x_k = 1$  if request  $k$  is granted, 0 otherwise

### B. OSNR Constraint

Before setting the optimization model, we discuss the OSNR constraint. It is written as follows:

$$\text{OSNR}_\pi = \frac{G_\pi}{G_{\text{ASE},\pi} + G_{\text{SCI},\pi} + G_{\text{XCI},\pi}} \geq T_\pi. \quad (12)$$

It can be equivalently rewritten:

$$\frac{1}{T_\pi} \geq \frac{G_{\text{ASE},\pi} + G_{\text{SCI},\pi} + G_{\text{XCI},\pi}}{G_\pi},$$

$$\text{or } \frac{G_{\text{XCI},\pi}}{G_\pi} \leq \frac{1}{T_\pi} - \underbrace{\frac{G_{\text{ASE},\pi} + G_{\text{SCI},\pi}}{G_\pi}}_{C_\pi}. \quad (13)$$

The last inequality (13) is the expression of the OSNR constraint that we will use in the optimization model, see next section.

### C. Master problem

$$\max \sum_{k \in K} r_k x_k \quad (\text{Throughput}) \quad (14)$$

subject to:

$$\sum_{\gamma \in \Gamma_\ell} z_\gamma \leq 1 \quad \ell \in L \quad (15)$$

$$\sum_{\gamma \in \Gamma} a_{\ell s}^\gamma z_\gamma \leq 1 \quad \ell \in L, s \in S \quad (16)$$

$$x_k \leq \sum_{\gamma \in \Gamma} a_k^\gamma z_\gamma \quad k \in K \quad (17)$$

$$\sum_{\gamma \in \Gamma} z_\gamma (\theta_\pi^\gamma + (M - C_\pi) y_\pi^\gamma) \leq M \quad \pi \in \Pi_k, k \in K \quad (18)$$

$$z_\gamma \in \{0, 1\} \quad \gamma \in \Gamma \quad (19)$$

$$x_k \in \{0, 1\} \quad k \in K. \quad (20)$$

Constraints (15) ensure that for each link, there is at most one selected configuration. Constraints (16) ensure that each slot on each link is used in at most 1 configuration. Constraints (17) check whether request  $k$  is granted. Constraints (18) check the OSNR constraint of lightpath  $\pi$ . Note that  $C_\pi$  is a coefficient.  $M$  is a constant big enough to be an upper bound of  $G_{\text{XCI},\pi}/G_\pi$ . In our implementation, we set  $M = 1,000$ .

## V. SOLUTION PROCESS: COLUMN GENERATION & TABU SEARCH

Following the exponential number of link configurations, it is required to solve the optimization model using column generation techniques.

### A. Column Generation Techniques

Column generation provides a decomposition of a compact mathematical model into the so-called master and pricing problems (see, e.g., [3] if not familiar with it). In the context of this study, the master problem corresponds to the optimization model (14)-(20), and the pricing problem to the generation of profitable  $\ell$  configurations.

Column generation algorithm starts with the Restricted Master problem, i.e., (14)-(20) with an initial set of variables (or configurations). In each iteration, the continuous relaxation of the restricted master problem is solved, then its dual values feed the pricing problems associated with each link of the network. If all pricing problems return a solution with a positive reduced cost then the current Restricted Problem is solved as an ILP, and output an integer solution of value  $\tilde{z}_{\text{ILP}}$ . Otherwise, the continuous relaxation of the restricted master problem is solved again after being fed by the profitable configurations, i.e., those with a negative reduced cost. This process is repeated until no more profitable configurations can be generated, meaning that we have reached the optimal solution of the Master Problem. Denote by  $z_{\text{LP}}^*$  its value. We are then left with an  $\epsilon$ -optimal solution where  $\epsilon = \frac{z_{\text{LP}}^* - \tilde{z}_{\text{ILP}}}{\tilde{z}_{\text{ILP}}}$ .

### B. Pricing problem

For any given link  $\ell$ , each pricing problem is associated with the generation of a profitable configuration, if one exists, or show that none exists. The set of variables is as follows:

- $v_p = 1$  if path  $p$  is selected, 0 otherwise
- $a_k = 1$  if request  $k$  is granted, 0 otherwise
- $a_{s\tilde{\ell}}^p = 1$  if path  $p$  occupy slot  $s$  in link  $\tilde{\ell}$ , 0 otherwise
- $b_{s\tilde{\ell}}^p = 1$  if slot  $s$  is the starting slot of path  $p$  in link  $\tilde{\ell}$ , 0 otherwise.

Let  $n_p$  be the number of slots that path  $p$  requires.

$$\max \overline{\text{RCOST}}_\gamma = -u^{(15)} - \sum_{s \in S} \sum_{\ell \in L} u_{s\ell}^{(16)} \underbrace{\sum_{k \in K} \sum_{p \in P_k} \delta_\ell^p a_{s\tilde{\ell}}^p}_{a_{\ell s}} - \sum_{k \in K} \sum_{\pi \in \Pi_k} u_\pi^{(18)} (\theta_\pi + (M - C_\pi) y_\pi) + \sum_{k \in K} u_k^{(17)} a_k \quad (21)$$

subject to:

$$\sum_{p \in P_k} v_p = a_k \quad k \in K \quad (22)$$

$$a_{s\tilde{\ell}}^p \leq v_p \quad p \in P_k, k \in K, s \in S \quad (23)$$

$$\sum_{p \in P_k} \frac{1}{n_p} \sum_{s \in S} a_{s\tilde{\ell}}^p = a_k \quad k \in K \quad (24)$$

$$\sum_{p \in P_k} \sum_{s \in [1, |S| - n_p + 1]} b_{s\tilde{\ell}}^p = a_k \quad k \in K \quad (25)$$

$$\sum_{i=0}^{n_p-1} a_{t+i, \tilde{\ell}}^p \geq n_p b_{t\tilde{\ell}}^p \quad t \in [1, |S| - n_p + 1], \quad (26)$$

$$\sum_{k \in K} \sum_{p \in P_k} a_{s\tilde{\ell}}^p \leq 1 \quad s \in S \quad (27)$$

$$v_p \in \{0, 1\} \quad p \in P_k, k \in K \quad (28)$$

$$a_k \in \{0, 1\} \quad k \in K \quad (29)$$

$$a_{s\tilde{\ell}}^p \in \{0, 1\} \quad p \in P_k, k \in K, s \in S \quad (30)$$

$$b_{s\tilde{\ell}}^p \in \{0, 1\} \quad p \in P_k, k \in K, s \in S \quad (31)$$

Constraints (22) ensure that we select at most one path (routing) for request  $k$  if it is granted in the  $\gamma$  configuration under construction. Constraints (23) ensure that variable  $y_p = 1$  if path  $p$  occupies any slot  $s$  on link  $\tilde{\ell}$ . Constraints (24) ensure the total number of slots for  $p$  match with  $n_p$ . Constraints (25) ensure a unique starting slot for each request. Constraints (26) express the contiguity constraints on link  $\tilde{\ell}$ . Constraints (27) ensure that each slot is used at most once in the overall set of connection requests.

Term  $\theta_\pi$  in (21) expresses the interference caused by  $\gamma$  which is under construction to lightpath  $\pi$ , and is written as:

$$\begin{aligned} \theta_\pi &= \sum_{\substack{\pi' \in \gamma \\ \pi' \neq \pi}} \frac{G_{\text{XCI}, \pi}^{\pi'}}{G_\pi} = M \sum_{\substack{\pi' \in \gamma \\ \pi' \neq \pi}} N_s^{\pi, \pi'} \ln \left( \frac{|f_\pi - f_{\pi'}| + B_{\pi'}/2}{|f_\pi - f_{\pi'}| - B_{\pi'}/2} \right) \\ &= M \sum_{\substack{\pi' \in \gamma \\ \pi' \neq \pi}} N_s^{\pi, \pi'} \ln \left( 1 + \frac{1}{|f_\pi - f_{\pi'}|/B_{\pi'} - 1/2} \right) \end{aligned}$$

where  $f_{\pi'} = \sum_{s \in S} b_{s\tilde{\ell}}^{p'} * (2s + n_{p'})/2$ ,  $f_\pi$  is a known value. Expression  $|f_\pi - f_{\pi'}|$  can be easily linearized. With a linearization of this absolute value expression, the pricing problem becomes a convex optimization problem subject to linear constraints. However, due to the complex analytical expression of the reduced cost, the pricing problem remains difficult to solve even using convex solvers, so we resort to a Tabu search heuristic to solve it.

### C. Tabu Search Heuristic

Denote by  $\overline{\text{RCOST}}(\gamma)$  the reduced cost of configuration  $\gamma$ . The goal of the Tabu Search Heuristic is to check whether there exists  $\gamma$  such that  $\overline{\text{RCOST}}(\gamma) \geq 0$ . Denote by  $\text{TABU}(\pi)$

the tabu status of lightpath  $\pi$ ,  $\text{TABU}(\pi) \geq 0$ . If  $\text{TABU}(\pi) > 0$  then the status of  $\pi$  is Tabu (we cannot consider changing  $\pi$ ), regular status otherwise. Denote by  $f(\pi, \pi')$  the cross interference caused by  $\pi$  that affects  $\pi'$ .

Denote by  $\mathcal{N}(\gamma)$  the neighborhood of  $\gamma$ . The neighborhood of a given configuration is defined as follows:

- Shifting one lightpath in  $\gamma$  up or down by one frequency slot, if there is conflict with another lightpath, then shift them in the same direction, if there is a lightpath that get out of the spectrum, then delete that lightpath
- With a random selection, switch one of the lightpath  $\pi_{sd} = (p, s)$  (path, starting slot) in  $\gamma$  with another lightpath  $\pi'_{sd} = (p', s)$  in the pool
- Select randomly two lightpaths  $\pi = (p, s, \#slots(\pi))$  and  $\pi' = (p', s', \#slots(\pi'))$  in  $c$ . Assume wlog that  $s \geq s'$ . Exchange positions of  $\pi, \pi'$  (wrt to their frequency slots), if there are conflicts in the process, try to shift the lightpaths in between.
- With a random selection, add a path into biggest available chain of frequency slots
- With a random selection, remove a random lightpath
- Choose only 1 out of 5 strategies above to generate 1 neighbor. The created configuration will be abandoned if a lightpath  $\pi$  with  $\text{TABU}(\pi) > 0$  is moved in the process.

The Tabu Search. algorithm can be described as follows:

1. Find an initial solution  $\gamma_0$  (by greedy or by solving the ILP without the log terms)
2.  $\gamma_{\text{BEST}} \leftarrow c_0 : \gamma_{\text{current}} \leftarrow c_0$
3.  $\gamma_t \leftarrow \text{argmax} \{ \overline{\text{RCOST}}(\gamma) : \gamma \in \mathcal{N}(\gamma_{\text{current}}) \}$
4.  $\forall \pi \in \gamma_t$ , if  $\text{TABU}(\pi) > 0$  then  $\text{TABU}(\pi) \leftarrow \text{TABU}(\pi) - 1$
5. If  $\overline{\text{RCOST}}(\gamma_t) < \overline{\text{RCOST}}(\gamma_{\text{current}})$  (the solution did not improve): choose the lightpath  $\pi'$  in  $\gamma_t$  such that the value  $\sum_{\pi \in \Pi} f(\pi, \pi')$  is the smallest and  $\text{TABU}(\pi') = 0$ ,  $\text{TABU}(\pi') \leftarrow 20$
6.  $\gamma_{\text{current}} \leftarrow \gamma_t$
7. If  $\overline{\text{RCOST}}(\gamma_{\text{current}}) > \overline{\text{RCOST}}(\gamma_{\text{BEST}})$  then  $\gamma_{\text{BEST}} \leftarrow \gamma_{\text{current}}$
8. If  $\overline{\text{RCOST}}(\gamma_{\text{BEST}}) > 0$  then stop,  $\gamma_{\text{BEST}}$  is the solution. If the number of iteration exceed 100 then stop, there is no solution, else repeat step V-C.

## VI. NUMERICAL RESULTS

We now discuss the performance of the proposed decomposition model and its solution scheme.

### A. Data Sets

We use the Spain network, with the set of distances as reported in Figure 3. We consider different sets of requests and different number of frequency slots. Data rates of the requests generated within the set  $\{100, 200, 400\}$  Gbps with a distribution of 40%, 30%, 30%, respectively.

### B. OSNR Impact on the RSA Solutions

We summarize in Table II the impact of taking into account the OSNR threshold when solving the RSA problem. Network provisioning solutions are compared using the throughput and the spectrum usage (SU).

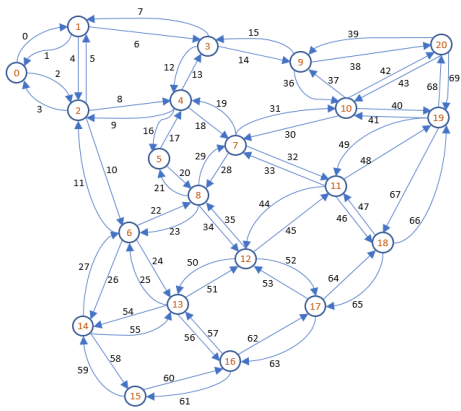


Fig. 3. Spain network

We also compare the results of our model against a first-fit algorithm. Results show that our model outperforms the First-Fit heuristic, especially when there is a large number of FSs and requests. When running on a small number of slots ( $\leq 100$ ), the spectrum usage (SU) decrease is around 5% when adding OSNR constraints and raises to more than 10% when the number of slots is higher. Where the provisioning is most packed (200 slots, 600 requests) the SU drops by 38%.

TABLE II  
NUMERICAL RESULT

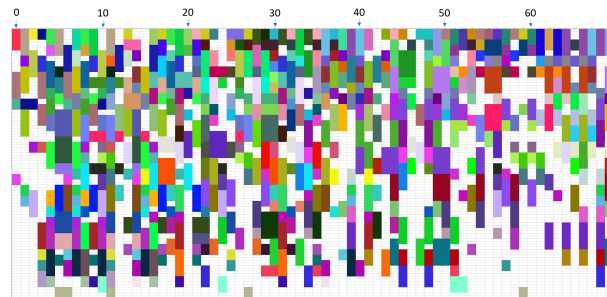
# slots	# requests	Total load	No OSNR		With OSNR		
			$\tilde{z}_{ILP}$	SU	First fit	$\tilde{z}_{ILP}$	SU
50	200	44,300	33,000	0.73	19,800	23,500	0.60
100	200	44,300	44,300	0.54	22,200	33,200	0.52
100	250	55,300	52,600	0.64	18,900	36,700	0.52
100	300	65,400	58,800	0.70	22,000	32,900	0.45
200	300	65,400	65,400	0.40	22,200	42,100	0.33
200	500	110,800	109,400	0.66	25,800	55,300	0.38
200	600	129,700	123,900	0.75	41,400	57,600	0.37
400	500	110,800	110,800	0.34	26,400	61,500	0.23
400	600	129,700	129,700	0.40	41,800	68,800	0.25

### C. Channel Spacing

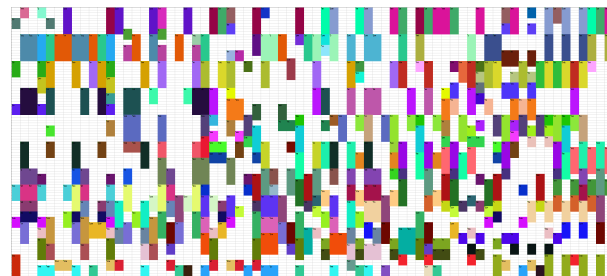
We plotted in Figure 4 the RSA provisioning solution as output by our model for the Spain network with 250 requests and 100 frequency slots. Each column is associated with a single link and each row with a frequency slot. Figure 4 provides then a visual look of how provisioning differs, mainly in terms of slot density as expected. Although there is some fragmentation, it is rather limited and no worse than for the Routing and Wavelength Assignment (RWA) problem.

## VII. CONCLUSIONS

OSNR constraints have a big impact on the request provisioning in Elastic Optical Networks, and therefore it is critical to consider them explicitly in the network provisioning optimization models. The proposed mathematical model of this paper perform much better than previously proposed models, although some improvements are still be to sought in order to improve its scalability.



(a) Without OSNR Constraints



(b) With OSNR constraints

Fig. 4. Provisioning of Spain network, 250 requests, 100 slots

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