Measurement Based Impromptu Deployment of a Multi-Hop Wireless Relay Network

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***Abstract*—We study the problem of optimal sequential ("as-you-go") deployment of wireless relay nodes, as a person walks along a line of random length (with a known distribution). The objective is to create an impromptu multihop wireless network for connecting a packet source to be placed at the end of the line with a sink node located at the starting point, to operate in the light traffic regime. In walking from the sink towards the source, at every step, measurements yield the transmit powers required to establish links to one or more previously placed nodes. Based on these measurements, at every step, a decision is made to place a relay node, the overall system objective being to minimize a linear combination of the expected sum power (or the expected maximum power) required to deliver a packet from the source to the sink node and the expected number of relay nodes deployed. For each of these two objectives, two different relay selection strategies are considered: (i) each relay communicates with the sink via its immediate previous relay, (ii) the communication path can skip some of the deployed relays. With appropriate modeling assumptions, we formulate each of these problems as a Markov decision process (MDP). We provide the optimal policy structures for all these cases, and provide illustrations of the policies and their performance, via numerical results, for some typical parameters.**

I. Introduction

We consider the problem of "as-you-go" deployment of relay nodes along a line, between a sink node and a source node (see Figure 1), where the deployment operative starts from the sink node, places relay nodes along the line, and places the source node where the line ends. The problem is motivated by the need for impromptu deployment of wireless networks by "first responders," for situation monitoring in an emergency such as a building fire or a terrorist siege. Such problems can also arise when deploying wireless sensor networks in large difficult terrains (such as forests) where it is difficult to plan a deployment due to the unavailability of a precise map of the terrain, or when such networks need to be deployed and redeployed quickly and there is little time in between to plan, or in situations where the deployment needs to be stealthy (for example, when deploying sensor networks for detecting poachers or fugitives in a forest).

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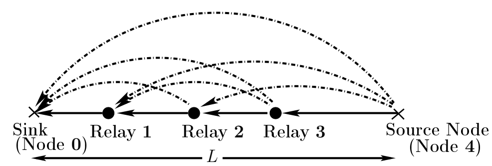

Fig. 1. A line network with one source, one sink and three relays.

Motivated by the above larger problem, we consider the problem of as-you-go deployment along a line of unknown random length, L, whose distribution is known. The transmit power required to establish a link (of a certain minimum quality) between any two nodes is modeled by a random variable capturing the effect of path-loss and shadowing. There is a cost for placing a relay, and the communication cost of a deployment is measured as some function of the powers required to communicate over the links. We consider two performance measures: the sum-power and the max-power along the path from the source to the sink, under two different relay selection strategies: (i) each relay communicates with the sink via its immediate previous relay, (ii) the communication path can skip some of the deployed relays. Under certain assumptions on the distribution of L, and the powers required at the relays, we formulate each of the sequential placement problems as a total cost Markov decision process (MDP).

The optimal policies for various MDPs formulated in this paper turn out to be *threshold* policies; the decision to place a relay at a given location involves the power required to establish a link to one or more previous nodes, and the distance to one or more previous nodes (depending on the objective and the relay selection strategy). Our analysis and numerical work also suggest that allowing the possibility of skipping some of the deployed relays may result in a reduction in the total cost.

A. Related Work

There has been increasing interest in the research community to explore the impromptu relay placement problem in recent years. Howard et al., in [1], provide heuristic algorithms for incremental deployment of sensors with the objective of covering the deployment area. Souryal et al., in [2], address the problem of impromptu deployment of static wireless networks with an extensive study of indoor RF link quality variation. However, there has been little effort to rigorously formulate the problem in order to derive optimal policies. Recently, Sinha et al. ([3]) have provided an MDP formulation for establishing a multi-hop network between a destination and an unknown source location by placing relay nodes along a random lattice path. They assume a given deterministic mapping between power and link length, and do not consider the statistical variability (due to shadowing) of the transmit power required to maintain the link quality over links of a given length. We, however, consider such variability, and therefore bring in the idea of measurement based impromptu placement.[1]

B. Organization

The system model and notation are discussed in Section II. In Section III, the problem of sequential relay placement (for the sum power and max power objectives) is addressed, under the assumption that a packet originating from the source makes a hop-by-hop traversal through all relay nodes. In Section IV, we address the same problems as in Section III, but we relax the restriction that the links of the path from the source to the sink must be between adjacent deployed relays. We conclude in Section V.

II. System Model and Notation

A. Length of the Line

The length L of the line is a priori unknown, but there is prior information (e.g., the mean length $\overline{L}$) that leads us to model L as a geometrically distributed number of steps.[2] The step length δ (whose values will typically be several meters, e.g., 2 meters in our numerical work) and the mean length of the line, $\overline{L}$, can be used to obtain the parameter of the geometric distribution, i.e., the probability θ that the line ends at the next step. In the problem formulation, we assume $\delta = 1$ for simplicity. All distances are assumed to be integer multiples of δ.

B. Deployment Process and Some Notation

As the person walks along the line, at each step he measures the link quality from the current location to one or more than one previous node and accordingly decides whether to place a relay at the current location or not. After the deployment process is complete (at the end of the line where the source is placed), we denote the number of deployed relays by N, which is a random number, with the randomness coming from the randomness in the link qualities and in the length of the line. As shown in Figure 1, the sink is called Node 0, the relay closest to the sink is called Node 1, and finally the source is called Node $(N+1)$. The link whose transmitter is Node i and receiver is Node j is called link (i, j). A generic link is denoted by e.

C. Traffic Model

We consider a traffic model where the traffic is so low that there is only one packet in the network at a time; we call this the "lone packet model." As a consequence of this assumption, (i) the transmit power required over each link depends only on the propagation loss over that link, as there are no simultaneous transmissions to cause interference, and (ii) the transmission delay over each link is easily calculated, as there are no simultaneous transmitters to contend with. This permits us to easily write down the communication cost on a path over the deployed relays.

It was shown in [5] that a network operating under CSMA/CA medium access, and designed for carrying any positive traffic, with some QoS (in terms of the packet delivery probability), must necessarily be able to provide the desired QoS to lone packet traffic. As-you-go deployment of wireless networks while meeting QoS objectives for specific positive packet arrival rates is a topic of our ongoing research.

D. Channel Model

For our network performance objective (see Section II-E), we need the transmit power required to sustain a certain quality of communication over a link. In order to model this required power, we consider the usual aspects of path-loss, shadowing, and fading. A link is considered to be in *outage* if the received signal strength drops below $P_{rcv-min}$ (due to fading) (e.g., below -88 dBm). The transmit power that we use is such that the probability of outage is less than a small value (say 5%). For a generic link of length r, we denote by Γ_r the transmit power required; due to shadowing, this is modeled as a random variable. Since practical radios can only be set to transmit at a finite set of power levels, the random variable Γ_r takes values from a discrete set, $\mathcal{S}$. The distribution function of Γ_r is denoted by $G_r(\cdot)$, and the probability mass function (p.m.f.) by $g(r, \cdot)$, i.e., $g(r, \gamma) := P(\Gamma_r = \gamma)$ for all $\gamma \in \mathcal{S}$; $g(r, \gamma)$ is the probability that at least the transmit power level γ is required to establish a link of length r. Since the required transmit power increases with distance, we assume that $\{G_r\}_{r=1,2,...}$ is a sequence of distributions *stochastically increasing* (for definition, see [6]) in r. We also need to talk about a specific link, say e; we will denote the transmit power required for this link by $\Gamma^{(e)}$. We assume that the powers required to establish any two different links in the network are independent, i.e., $\Gamma^{(e_1)}$ is independent of $\Gamma^{(e_2)}$ for $e_1 \neq e_2$. Spatially correlated shadowing will be considered in our future work.

E. Deployment Objective

In this paper, we do not consider the possibility of another person following behind, who can learn from the measurements and actions of the first person, thereby supplementing the actions of the preceding individual. Our objective is to design relay placement policies so as to minimize the sum of the expected sum power/ maximum power (to deliver a packet from the source node to the sink node) and the expected cost of placing the relays (the expected number of relays multiplied by the relay cost, ξ). By a standard constraint relaxation, this problem also arises from the problem of minimizing the expected sum/ max power, subject to a constraint on the mean number of relays. Such a constraint can be justified if we consider relay deployment for multiple source-sink pairs over several different lines of mean length $\overline{L}$, given a large pool of

[1] See [4], the detailed version of this paper, for additional references.

[2] The geometric distribution is the maximum entropy discrete probability mass function with a given mean. Thus, by using the geometric distribution, we are leaving the length of the line as uncertain as we can, given the prior knowledge of $\overline{L}$ and the step length δ.

relays, and we are only interested in keeping small the total number of relays over all these deployments.

The max-power objective is useful in a typical sensor network setting, since, to maximize network lifetime, each relay must use as little power as possible. The sum-power objective may be useful in a scenario where a mobile station (MS) tries to establish a *multihop* connection to a base station (BS) at an unknown distance by sequentially selecting relays from a continuum of nodes on the line joining the MS to the BS, but the MS has to pay a certain price to use each relay (see [4]). The sum power objective is also interesting in the context of global energy saving. Note that, our formulation is applicable if a relay can be set to a low power state except when it has to receive or transmit, so that the power consumptions in the transmit and the receive modes govern the lifetime.

F. Routing over the Deployed Relays

After node deployment, the routes could be constrained so as to allow transmissions only between adjacent nodes, i.e., the routes use solely the links represented by the solid lines in Figure 1; we consider this problem in Section III. However, after deployment, it may turn out that it is better that the route from the source to the sink skips some relays (e.g., in Figure 1, if the channel between the source node and relay 2 is very good, it could be better to directly transmit from the source node to relay 2 without using relay 3). Hence, while formulating the problem, it would be beneficial to permit the possibility that some of the dotted links in Figure 1 can be used after deployment; this problem is solved in Section IV.

III. Relaying via Adjacent Previous Node Only

In this section we allow relaying from the source to the sink only by each relay passing the packet to the immediate previous relay, in the order of deployment.

A. Sum-Power Objective

1) Problem Formulation: We formulate this problem as an MDP with state (r, γ), where r is the distance of the current location from the previous node and and γ is the transmit power required to establish a link to the previous node from the current location. Based on (r, γ) a decision is made whether to place a relay at the current position or not. $\mathbf{0}$ denotes the state at the beginning of the process (at the sink node). When the source is placed, the process terminates and the system enters and stays forever at a state $\mathbf{e}$. The action space is $\{$*place*, *do not place*$\}$. The randomness comes from the random length L and the randomness in $\Gamma^{(e)}$.

The problem we seek to solve here is:

$$\min_{\pi \in \Pi} \mathbb{E}_{\pi} \left(\sum_{i=1}^{N+1} \Gamma^{(i,i-1)} + \xi N \right) \tag{1}$$

where Π is the set of all stationary deterministic Markov placement policies. Here we have countable state space and finite action space. The single-stage costs are nonnegative. Hence, based on the theory in [7], we can concentrate only on stationary deterministic Markov policies.

Solving (1) also helps in solving the following constrained problem (see [8]):

$$\min_{\pi \in \Pi} \mathbb{E}_{\pi} \left(\sum_{i=1}^{N+1} \Gamma^{(i,i-1)} \right) \text{ such that } \mathbb{E}_{\pi} N \leq M, \tag{2}$$

where M represents the constraint on the mean number of deployed relays. In this paper, however, we consider only the unconstrained problem.

If the state is (r, γ) and a relay is placed, the relay cost ξ and the power cost γ is incurred at that step. We do not count the price of the source node, but include the power used by the source in our cost. *No cost is incurred if we do not place a relay at a certain location.* Note that, in this problem, $\mathbf{0}$ also denotes the state immediately after placing a relay, since the process regenerates whenever a relay is placed (this follows from the memoryless property of geometric distribution and the independence of $\Gamma^{(i,j)}$ and $\Gamma^{(k,l)}$ for $(i,j) \neq (k,l)$). Let us define $J_{\xi}(r, \gamma)$ and $J_{\xi}(\mathbf{0})$ to be the optimal expected cost-to-go starting from state (r, γ) and state $\mathbf{0}$ respectively.

2) Bellman Equation: By Proposition 3.1.1 of [7], $J_{\xi}(\cdot)$ satisfies the following Bellman equation, where c_p and c_{np} are the cost of placing a relay and the cost of not placing a relay at the state (r, γ), respectively:

$$\begin{aligned} J_{\xi}(r,\gamma) &= \min \Big\{ \overbrace{\xi + \gamma + J_{\xi}(\mathbf{0})}^{c_p}, \\ &\qquad \overbrace{\theta \mathbb{E}(\Gamma_{r+1}) + (1-\theta)\mathbb{E} J_{\xi}\left(r+1, \Gamma_{r+1}\right)}^{c_{np}} \Big\} \\ J_{\xi}(\mathbf{0}) &= \theta \mathbb{E}(\Gamma_1) + (1-\theta)\mathbb{E} J_{\xi}\left(1, \Gamma_1\right) \end{aligned} \tag{3}$$

If the current state is (r, γ) and the line has not ended yet, we can take either of the two actions. If we place a relay, a cost $(\xi + \gamma)$ is incurred; another cost $J_{\xi}(\mathbf{0})$ is also incurred since the decision process regenerates at that point. If we do not place a relay, the line will end with probability θ in the next step, in which case a cost $\mathbb{E}(\Gamma_{r+1})$ will be incurred. If the line does not end in the next step, the next state will be $(r+1, \gamma')$ where $\gamma' \sim G_{r+1}$ and a mean cost of $\mathbb{E} J_{\xi}(r+1, \Gamma_{r+1}) = \sum_{\gamma} g(r+1, \gamma) J_{\xi}(r+1, \gamma)$ will be incurred. Note that it is never optimal to place a relay at state $\mathbf{0}$. If it were so, then we would have placed infinitely many relays at the sink, leading to infinite relay cost. But if we place one relay at each step until the line ends, the expected cost will be less than $(\frac{1}{\theta} + 1)(\xi + \mathbb{E}(\Gamma_1)) < \infty$.

3) Value Iteration: The value iteration for (1) is given by:

$$\begin{aligned} J_{\xi}^{(k+1)}(\mathbf{0}) &= \theta \mathbb{E}(\Gamma_1) + (1-\theta)\mathbb{E} J_{\xi}^{(k)}\left(1, \Gamma_1\right) \\ J_{\xi}^{(k+1)}(r,\gamma) &= \min \Big\{ \xi + \gamma + J_{\xi}^{(k)}(\mathbf{0}), \theta \mathbb{E}(\Gamma_{r+1}) \\ &+ (1-\theta)\mathbb{E} J_{\xi}^{(k)}\left(r+1, \Gamma_{r+1}\right) \Big\} \end{aligned} \tag{4}$$

with $J_{\xi}^{(0)}(r, \gamma) = 0$ for all r, γ and $J_{\xi}^{(0)}(\mathbf{0}) = 0$.

Lemma 1: The value iteration (4) provides a nondecreasing sequence of iterates that converges to the optimal value function, i.e., $J_\xi^{(k)}(r,\gamma) \uparrow J_\xi(r,\gamma)$ for all r,γ, and $J_\xi^{(k)}(\mathbf{0}) \uparrow J_\xi(\mathbf{0})$ as $k \uparrow \infty$.

Proof: The proofs of all lemmas and theorems of this paper are available in [4]. ■

4) Policy Structure:

Lemma 2: $J_\xi(r,\gamma)$ is concave, increasing in γ and ξ and also increasing in r. $J_\xi(\mathbf{0})$ is concave, increasing in ξ. □

Theorem 1: Policy Structure: The optimal policy for Problem (1) is a threshold policy with a threshold $\gamma_{th}(r)$ increasing in r such that at a state (r,γ) it is optimal to place a relay if and only if $\gamma \leq \gamma_{th}(r)$. This corresponds to the condition $c_p \leq c_{np}$. □

Remark 1: If $\gamma = \gamma_{th}(r)$, either action is optimal.

Discussion of the Policy Structure: We do not place at an r if the required power is too high, as one might expect to get a better channel if one takes another step. For each r, there is a threshold on γ below which we place. This threshold increases with r since $G_r(\cdot)$ is stochastically increasing in r.

Note that the optimal policy in Theorem 1 can also be stated as follows: place a relay if and only if $r \leq r_{th}(\gamma)$ (i.e., $c_p \leq c_{np}$) where $r_{th}(\gamma)$ is some threshold on r, increasing in γ.

5) Computation of the Optimal Policy: Let us write $V_\xi(r) := \mathbb{E}J_\xi(r,\Gamma_r)$, i.e., $V_\xi(r) := \sum_\gamma g(r,\gamma) J_\xi(r,\gamma)$ for all $r \in \{1,2,3,\cdots\}$, and $V_\xi(\mathbf{0}) := J_\xi(\mathbf{0})$. Also, for each stage $k \geq 0$ of the value iteration (4), define $V_\xi^{(k)}(r) := \mathbb{E}J_\xi^{(k)}(r,\Gamma_r)$ and $V_\xi^{(k)}(\mathbf{0}) := J_\xi^{(k)}(\mathbf{0})$.

Observe that from the value iteration (4), we obtain:

$$\begin{aligned} V_\xi^{(k+1)}(r) &= \sum_\gamma g(r,\gamma) \min \Big\{ \xi + \gamma + V_\xi^{(k)}(\mathbf{0}), \\ &\qquad \theta \mathbb{E}(\Gamma_{r+1}) + (1-\theta) V_\xi^{(k)}(r+1) \Big\} \\ V_\xi^{(k+1)}(\mathbf{0}) &= \theta \mathbb{E}(\Gamma_1) + (1-\theta) V_\xi^{(k)}(1) \end{aligned} \tag{7}$$

Since $J_\xi^{(k)}(r,\gamma) \uparrow J_\xi(r,\gamma)$ for each r, γ and $J_\xi^{(k)}(\mathbf{0}) \uparrow J_\xi(\mathbf{0})$ as $k \uparrow \infty$, we can argue that $V_\xi^{(k)}(r) \uparrow \mathbb{E}J_\xi(r,\Gamma(r))$ for all $r \in \{1,2,3,\cdots\}$ (by Monotone Convergence Theorem) and $V_\xi^{(k)}(\mathbf{0}) \uparrow J_\xi(\mathbf{0})$. Thus, $V_\xi^{(k)}(r) \uparrow V_\xi(r)$ and $V_\xi^{(k)}(\mathbf{0}) \uparrow V_\xi(\mathbf{0})$. Hence, by the function iteration (7), we obtain $V_\xi(\mathbf{0})$ and $V_\xi(r)$ for all $r \geq 1$. Then, from (3), we can compute $\gamma_{th}(r)$. Thus, for this iteration, we simply need to keep track of $V_\xi^{(k)}(r)$ instead of $J_\xi^{(k)}(r,\gamma)$ for each r, γ.

6) A Numerical Example: We take $\delta = 2$ meters and $\theta = 0.025$, (i.e., $\overline{L} = 40$ steps, or 80 meters), and $\mathcal{S} = \{-25,-20,-15,-10,-5,0,3\}$ in dBm. Using a standard model, with transmit power P_T (mW), the received power (in mW) at a distance r from the transmitter is given by $P_T \alpha (\frac{r}{r_0})^{-\eta} H 10^{-\frac{\nu}{10}}$ where α is a constant and r_0 is a reference distance. H models Rayleigh fading, and is exponentially distributed with mean 1. ν is assumed to be distributed as $\mathcal{N}(0,\sigma^2)$ with $\sigma = 8$ dB; i.e., we have log-normal shadowing. The shadowing is assumed to be independent from link to link. For a commercial implementation of the PHY/MAC of IEEE 802.15.4 (a popular wireless sensor networking standard), -88 dBm received power corresponds to a 2% packet loss probability for 140 byte packets. Taking -88 dBm to be the minimum acceptable received power, we set the target received power (averaged over fading) to be $\psi = 10^{-7.5}$ mW (i.e., -75 dBm), which (under Rayleigh fading) yields an "outage" probability of 5%. Hence, the transmit power required to establish the link is:

$$P_{req} = \frac{\psi}{10^{-\frac{\nu}{10}}\alpha} \left(\frac{r}{r_0}\right)^{\eta} \tag{8}$$

Now, since the set $\mathcal{S}$ is bounded, at large distance the required power to achieve ψ will exceed 2 mW (i.e., 3 dBm) with high probability. To tackle this problem, *we modify the problem formulation by requiring that a relay is placed when $r = 10$ (in steps).* For $1 \leq r \leq 10$ (in steps), we obtain Γ_r from (8) and using the next higher power level in $\mathcal{S}$. For example, if the required transmit power to establish the link is in $(-5, 0]$ dBm, then we say that the power requirement is 0 dBm for that link. But, if the power requirement is more than 3 dBm, then we say that the required power is 3 dBm. It is easy to see that the distribution $G_r(\cdot)$ of Γ_r obtained in this fashion is stochastically increasing [3] in r. For the numerical work, we assume that $\eta = 2.5$, $\frac{\alpha}{r_0^{-\eta}} = 10^{-3}/m^{-2.5}$ (-30 dB). With these parameters, the probability of the transmit power required to achieve a target received power $\psi = -75$ dBm (averaged over fading) for a link of length 10 steps (20 meters) exceeding 3 dBm (the maximum transmit power) will be less than 2.65%, and it will be even less for links having smaller length. Clearly, there is a positive probability of deployment failure; we will quantify this later in Section III-A7.

Figure 2 shows the variation of $\gamma_{th}(r)$ with r and ξ. Here the unit of ξ is mW/relay, so that the unit of $J_\xi(\mathbf{0})$ is also mW. We see from Figure 2 that, for $\xi = 0.001$, if $r = 8$ meters, and the required power is -20 dBm or below, then a relay should be placed at this point. Also, note that -25 dBm is the smallest possible transmit power level, and for $\xi = 0.001$ we will never place a relay at $r = 2$ meters since there $\gamma_{th}(r)$ is 0 mW ($-\infty$ dBm). Figure 2 shows that $\gamma_{th}(r)$ is decreasing in ξ for each r. All it says is that as the price of a relay, ξ, increases, we will place the relays less frequently.

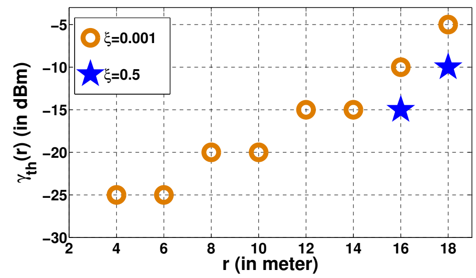

Fig. 2. Relaying only via last placed relay: $\gamma_{th}(r)$ vs. r, for various relay costs, ξ, for the numerical example described in Section III-A6.

[3]The results asserted for the formulation in Section III-A, e.g., threshold structure of the optimal policy, hold for this variation as well. The only change will be that at $r = 10$ steps the only feasible action is to place the relay. We can use similar iterations as (4) and (7) to compute the optimal policy.

$$J_\xi(r,\gamma,\gamma_{max}) = \min\left\{ \overbrace{\xi + \theta\mathbb{E}\max\{\gamma,\gamma_{max},\Gamma_1\} + (1-\theta)\mathbb{E}J_\xi(1,\Gamma_1,\max\{\gamma,\gamma_{max}\})}^{c_p}, \overbrace{\theta\mathbb{E}\max\{\gamma_{max},\Gamma_{r+1}\} + (1-\theta)\mathbb{E}J_\xi(r+1,\Gamma_{r+1},\gamma_{max})}^{c_{np}} \right\} \quad (5)$$

$$J_\xi^{(k+1)}(r,\gamma,\gamma_{max}) = \min\Big\{\xi + \theta\mathbb{E}\max\{\gamma,\gamma_{max},\Gamma_1\} + (1-\theta)\mathbb{E}J_\xi^{(k)}(1,\Gamma_1,\max\{\gamma,\gamma_{max}\}), \theta\mathbb{E}\max\{\gamma_{max},\Gamma_{r+1}\} + (1-\theta)\mathbb{E}J_\xi^{(k)}(r+1,\Gamma_{r+1},\gamma_{max})\Big\} \quad (6)$$

	$\xi = 0.001$	$\xi = 0.01$	$\xi = 0.1$
$\mathbb{E}(N)$	15.8754	10.3069	5.3225
Relay cost $\xi\mathbb{E}(N)$	0.01588	0.10307	0.53225
Power Cost	0.07513	0.08277	0.19291
$J_\xi(\mathbf{0})$	0.09101	0.18584	0.72516

TABLE I

RELAYING VIA THE LAST PLACED RELAY: BREAK-UP OF THE OPTIMAL COST FOR THE EXAMPLE IN SECTION III-A6, FOR THREE VALUES OF ξ.

Table I shows that as ξ increases, the mean number of relays $\mathbb{E}(N)$ decreases, and the power cost and $J_\xi(\mathbf{0})$ increase.

7) Deployment Failure: Because of the bounded transmit power (e.g., 3 dBm) of a node, there is a possibility that a deployment can fail. Here we provide a simulation estimate of the deployment failure probabilities under the threshold policies obtained in Section III-A6. A deployment failure can occur if: (i) for at least one relay, at $r = 10$ steps the required power exceeds 3 dBm, and (ii) the source at the end of the line requires more than 3 dBm power. By simulating 200000 deployments, we observe that the deployment failure probability for $\xi = 0.001, 0.01, 0.1$ and 1 are $0.025\%, 0.057\%, 0.555\%$ and 3.8% respectively. Evidently, there is a trade-off between the target link performance and the probability of link failure. In addition, by placing relays more frequently, we reduce the chance of being caught in a situation where the deployment operative has walked too far without placing a relay and is unable to get a workable link to the previous node. In future work, we propose to include deployment failure probability as a constraint in the optimization formulation. Another way to reduce deployment failure is to permit *back-tracking* by the deployment operative, which, of course, will require the placement algorithm to keep more measurement history; we propose to permit this in our future work as well.

B. Max-Power Objective

1) Problem Formulation: We aim to address the following problem:

$$\min_{\pi\in\Pi} \mathbb{E}_\pi\left(\max_{i\in\{1,2,\cdots,N+1\}} \Gamma^{(i,i-1)} + \xi N\right) \quad (9)$$

We formulate (9) as an MDP with state (r,γ,γ_{max}), where r and γ are the same as before. γ_{max} is the maximum power used in all the previously established links. The action space is $\{$*place*, *do not place*$\}$. The power cost is incurred only after the source node is placed.

2) Bellman Equation: By Proposition 3.1.1 of [7], $J_\xi(\cdot)$ satisfies the Bellman equation (5). At state $\mathbf{0}$, it is not optimal to place a relay. Hence, $J_\xi(\mathbf{0}) = \theta\mathbb{E}(\Gamma_1) + (1-\theta)\mathbb{E}J_\xi(1,\Gamma_1,0)$.

At state (r,γ,γ_{max}), if we place a relay, we incur a cost ξ. In the next step the line ends with probability θ, in which case a power cost of $\mathbb{E}\max\{\gamma,\gamma_{max},\Gamma_1\}$ is incurred. If the line does not end in the next step, the next state becomes $(1,\gamma',\max\{\gamma,\gamma_{max}\})$ where $\gamma' \sim G_1(\cdot)$, and a cost of $\mathbb{E}J_\xi(1,\Gamma_1,\max\{\gamma,\gamma_{max}\})$ is incurred. On the other hand, if we do not place a relay at state (r,γ,γ_{max}), the line ends in the next step with probability θ, in which case a power cost of $\mathbb{E}\max\{\gamma_{max},\Gamma_{r+1}\}$ is incurred. If the line does not end in the next step, the next state will be $(r+1,\Gamma_{r+1},\gamma_{max})$.

3) Value Iteration: The value iteration for this MDP is given by (6) with $J_\xi^{(0)}(r,\gamma,\gamma_{max}) = 0$ for all r, γ, γ_{max}.

Lemma 3: The iterates of the value iteration (6) converge to the optimal value function, i.e., $J_\xi^{(k)}(r,\gamma,\gamma_{max}) \uparrow J_\xi(r,\gamma,\gamma_{max})$ for all (r,γ,γ_{max}), as $k\uparrow\infty$. ☐

4) Policy Structure:

Lemma 4: $J_\xi(r,\gamma,\gamma_{max})$ is concave, increasing in ξ and increasing in r, γ, γ_{max}. ☐

Theorem 2: Policy Structure: The conditions for optimal relay placement are:

(i) *If* $\gamma \le \gamma_{max}$*,* place the relay when $r \ge r_{th}(\gamma_{max})$ where $r_{th}(\gamma_{max})$ is a threshold value.

(ii) *If* $\gamma > \gamma_{max}$*,* place the relay when $\gamma \le \gamma_{th}(r,\gamma_{max})$, where $\gamma_{th}(r,\gamma_{max})$ is a threshold increasing in r and γ_{max}. ☐

Discussion of the policy structure: When $\gamma \le \gamma_{max}$, we can postpone placement until the point beyond which the chance of getting a worse value of power becomes significant. For $\gamma > \gamma_{max}$, waiting to place the relay may result in a better channel; there is a threshold $\gamma_{th}(r,\gamma_{max})$ such that $\gamma_{th}(r,\gamma_{max})$ may cross γ_{max} for large r. If γ is between these two values then we place.

5) Computation of the Optimal Policy: Let us define $V_\xi(r,\gamma_{max}) := \mathbb{E}J_\xi(r,\Gamma_r,\gamma_{max})$. We can again argue that the following function iteration (similar to that used in Section III-A) will yield $V_\xi(r,\gamma_{max})$ for all r, γ_{max}, from which we can compute $r_{th}(\gamma_{max})$ and $\gamma_{th}(r,\gamma_{max})$:

$$\begin{aligned} V_\xi^{(k+1)}(r,\gamma_{max}) &= \sum_\gamma g(r,\gamma)\min\Big\{\xi + \theta\mathbb{E}\max\{\gamma,\gamma_{max},\Gamma_1\} \\ &+ (1-\theta)V_\xi^{(k)}(1,\max\{\gamma,\gamma_{max}\}), \\ &\theta\mathbb{E}\max\{\gamma_{max},\Gamma_{r+1}\} \\ &+ (1-\theta)V_\xi^{(k)}(r+1,\gamma_{max})\Big\} \end{aligned} \quad (10)$$

with $V_\xi^{(0)}(r,\gamma_{max}) = 0$ for all r, γ_{max}.

6) A Numerical Example: Figure 3 (see next page) shows the variation of $r_{th}(\gamma_{max})$ with γ_{max} and ξ, for the same setting as Section III-A6. The plot shows that $r_{th}(\gamma_{max})$ increases with γ_{max}. Let us consider the situation $\gamma < \gamma_{max}$. If r is small, then it is more likely that in the next step also the power

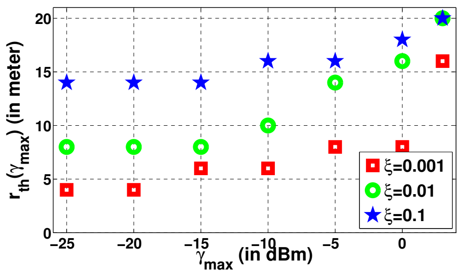

Fig. 3. Relaying only via last placed relay: $r_{th}(\gamma_{max})$ vs. γ_{max}, for various relay costs, ξ, for the numerical example described in Section III-B6.

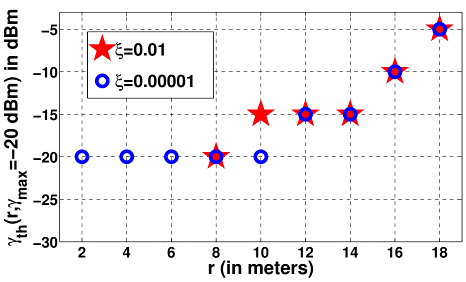

Fig. 4. Relaying only via last placed relay: $\gamma_{th}(r, \gamma_{max})$ vs. r, for $\gamma_{max} = -20$ dBm and various ξ, for the numerical example in Section III-B6.

required to establish a link to the last node will be below γ_{max}, and hence we don't need to place a relay. But if r is large, then it is more likely that the power will cross γ_{max} in the next step, and hence we will have a threshold $r_{th}(\gamma_{max})$ beyond which we have to place the relay. As γ_{max} increases, the probability that the power required to establish a link to the last node exceeding γ_{max} decreases for each r, thereby increasing $r_{th}(\gamma_{max})$. Also, $r_{th}(\gamma_{max})$ increases with ξ because if ξ increases, we will place relays less frequently.

Figure 4 shows the variation of $\gamma_{th}(r, \gamma_{max})$ with r for $\gamma_{max} = -20$ dBm and two different values of ξ. For very small ξ (e.g., $\xi = 0.00001$), $\gamma_{th}(r, \gamma_{max})$ can be more than γ_{max} for all r. For moderate values of ξ (e.g., $\xi = 0.01$ in Figure 3), $\gamma_{th}(r, \gamma_{max})$ vs. r curve crosses γ_{max} at $r = r_{th}(\gamma_{max})$. Also, we have seen numerically that for ξ very large, $\gamma_{th}(r, \gamma_{max})$ is 0 mW for $r \leq 9$ steps; for large ξ we will place a relay only when $r = 10$ steps (20 meters).

Table II shows that as ξ increases, the mean number of relays decrease, and the power cost and $J_\xi(\mathbf{0})$ increase. Note that, for any given deployment and any given relay cost ξ, the sum power is always greater than the max power in the network. Hence, for a given ξ, the optimal mean power cost and $J_\xi(\mathbf{0})$ for the sum power objective will be greater than the corresponding values for the max-power objective, as seen in Table I and Table II.

The estimated probability of deployment failure obtained from simulation for $\xi = 0.001, 0.01, 0.1$ and 1 are $0.01\%, 0.145\%, 0.73\%$ and 5.39% respectively.

IV. Relaying via Any Previous Node

In Section III, we considered the case where, after the deployment is over, only the links between adjacent nodes are permitted, i.e., only the links represented by the solid lines

	$\xi = 0.001$	$\xi = 0.01$	$\xi = 0.1$
$\mathbb{E}(N)$	18.1178	8.6875	4.6615
Relay Cost $\xi\mathbb{E}(N)$	0.01812	0.08688	0.46615
Power Cost	0.01524	0.04436	0.15079
$J_\xi(\mathbf{0})$	0.03336	0.13124	0.61693

TABLE II
Relaying via the last placed relay: break-up of the optimal cost for the example in Section III-B6, for three values of ξ.

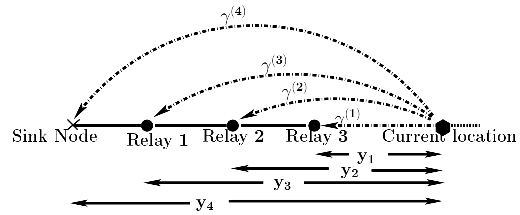

Fig. 5. Measurement-based sequential relay placement. When standing at the "current location," the deployment operative, having already deployed Relays 1, 2, 3, makes the power measurements $\gamma^{(k)}$, and knows the distances y_k.

in Figure 1 can be used. However, as discussed in Section II, while formulating the problem we need to take into account the fact that some relays might be skipped after deployment, i.e., some of the links represented by the dotted lines in Figure 1 can be used. This section is dedicated to such formulations.

A. Sum-Power Objective

1) Problem Definition: Given a deployment of N relays, indexed $1, 2, \cdots, N$, consider the directed acyclic graph on these relays along with the sink (Node 0) and the source (Node $N+1$), whose links are all directed edges from each node to every node with smaller index. Hence, if i and j are two nodes with $i > j$, there is only one link (i, j) between them. Consider all directed acyclic paths from the source to sink, on this graph. Let us denote by $\mathbf{p}$ any arbitrary directed acyclic path from the source to the sink, and by $\mathbf{E}(\mathbf{p})$ the set of (directed) links of the path $\mathbf{p}$. We also define $\mathcal{P}_n := \{\mathbf{p} : (i, j) \in \mathbf{E}(\mathbf{p}) \implies i > j, |i - j| \leq n\}$ a subcollection of paths between the source and the sink on the directed acyclic graph, such that no path in $\mathcal{P}_n$ contains a link between two nodes whose indices differ by a number larger that n. We call n the "memory" of the class of policies we are considering.

Here we consider the following problem:

$$\min_{\pi \in \Pi} \mathbb{E}_\pi \left(\min_{\mathbf{p} \in \mathcal{P}_n} \sum_{e \in \mathbf{E}(\mathbf{p})} \Gamma^{(e)} + \xi N \right) \tag{13}$$

where $\Gamma^{(e)}$ is the power used on the link e. We call $\sum_{e \in \mathbf{E}(\mathbf{p})} \Gamma^{(e)}$ the "length" of the path $\mathbf{p}$, and $\min_{\mathbf{p} \in \mathcal{P}_n} \sum_{e \in \mathbf{E}(\mathbf{p})} \Gamma^{(e)}$ the length of the "shortest path" from the source to the sink (over the relays deployed by policy π in a given realization of the decision process).

2) MDP Formulation: Consider the evolution of the network as the relays are deployed. Suppose that at some point in the deployment process there are m preceding nodes, *including the sink*; see Figure 5 where $m = 4$. The transmit power required to establish a link from the current location to the k-th previous node is denoted by $\gamma^{(k)}$, and the distance of the current location from the k-th previous node is denoted by y_k. Let $P_n^{(k)}$ denote the length of the "shortest path" from the k-th previous node to the sink. We define $P_n^{(m)} := 0$ if $m \leq n$, i.e.,

$$
\begin{aligned}
& J_\xi\Big(\{y_k\}_{k=1}^{n};\{P^{(k)}\}_{k=1}^{n};\{\gamma^{(k)}\}_{k=1}^{n}\Big) = \min\Big\{\xi + \theta\mathbb{E}\min\Big\{\min_{k\in\{1,2,\cdots,n-1\}}(\Gamma_{y_k+1}+P^{(k)}),\Gamma_1+\min_{k\in\{1,2,\cdots,n\}}(\gamma^{(k)}+P^{(k)})\Big\} \\
+ \quad & (1-\theta)\mathbb{E}J_\xi\Big(1,y_1+1,\cdots,y_{n-1}+1;\min_{k\in\{1,\cdots,n\}}(\gamma^{(k)}+P^{(k)}),P^{(1)},\cdots,P^{(n-1)};\Gamma_1,\Gamma_{y_1+1},\cdots,\Gamma_{y_{n-1}+1}\Big), \\
& \theta\mathbb{E}\min_{k\in\{1,\cdots,n\}}\Big(\Gamma_{y_k+1}+P^{(k)}\Big)+(1-\theta)\mathbb{E}J_\xi\Big(\{y_k+1\}_{k=1}^{n};\{P^{(k)}\}_{k=1}^{n};\{\Gamma_{y_k+1}\}_{k=1}^{n}\Big)\Big\} \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
& J_\xi\Big(\{y_k\}_{k=1}^{m};\{P^{(k)}\}_{k=1}^{m};\{\gamma^{(k)}\}_{k=1}^{m}\Big) = \min\Big\{\xi + \theta\mathbb{E}\min\Big\{\min_{k\in\{1,2,\cdots,m\}}(\Gamma_{y_k+1}+P^{(k)}),\Gamma_1+\min_{k\in\{1,2,\cdots,m\}}(\gamma^{(k)}+P^{(k)})\Big\} \\
+ \quad & (1-\theta)\mathbb{E}J_\xi\Big(1,y_1+1,\cdots,y_m+1;\min_{k\in\{1,\cdots,m\}}(\gamma^{(k)}+P^{(k)}),P^{(1)},\cdots,P^{(m)};\Gamma_1,\Gamma_{y_1+1},\cdots,\Gamma_{y_m+1}\Big), \\
& \theta\mathbb{E}\min_{k\in\{1,\cdots,m\}}\Big(\Gamma_{y_k+1}+P^{(k)}\Big)+(1-\theta)\mathbb{E}J_\xi\Big(\{y_k+1\}_{k=1}^{m};\{P^{(k)}\}_{k=1}^{m};\{\Gamma_{y_k+1}\}_{k=1}^{m}\Big)\Big\} \qquad (12)
\end{aligned}
$$

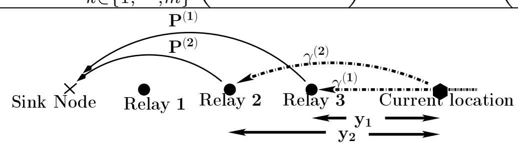

Fig. 6. Sequential deployment with $n = 2$. When standing at the "current location," the deployment operative, having already deployed Relays 1, 2, and 3, measures the powers $\gamma^{(1)}$, $\gamma^{(2)}$, and knows the distances y_1, y_2 and the lengths $P^{(1)}$, $P^{(2)}$ of the shortest paths from relay 3 and relay 2 to the sink.

the length of the shortest path from the sink to itself is 0 (when $m \leq n$, the m-th previous node is the sink). For notational simplicity, we drop the subscript and denote $P_n^{(k)}$ by $P^{(k)}$. The deployment operative decides whether to place a node at his current position based on (i) the powers $\gamma^{(1)}, \gamma^{(2)}, \cdots, \gamma^{(n)}$, (ii) the distances $y_1, y_2, \cdots, y_n$, and (iii) the length of the shortest paths $P^{(1)}, P^{(2)}, \cdots, P^{(n)}$. If $n = 2$, at the "current location" shown in Figure 6, the decision will be based on the powers $\gamma^{(1)}$, $\gamma^{(2)}$, the distances y_1, y_2, and the shortest paths $P^{(1)}$ and $P^{(2)}$ at nodes 3 and 2 respectively. However, in case $m < n$, we do not have measurements for n previous nodes. Hence, let us define $l_m := \min\{m, n\}$. At each step, the deployment operative knows the distance $\{y_k\}_{k=1}^{l_m}$, the power $\{\gamma^{(k)}\}_{k=1}^{l_m}$ and the lengths of the shortest paths $\{P^{(k)}\}_{k=1}^{l_m}$. He decides based on this information whether to place a relay at the current position or not. We formulate this problem as an MDP with state $(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$, and the action space $\{$*place*, *do not place*$\}$. The state at the sink is denoted by $\mathbf{0}$. Since the set $\mathcal{S}$ of transmit power levels is countable, $\{P^{(k)}\}_{k=1}^{l_m}$ also take values from a countable set. Hence, the state space is countable in our problem.

If the state is $(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$ and a relay is placed, the relay cost ξ is incurred. The power cost is incurred only after the source is placed, and that cost will be the length of the shortest path in $\mathcal{P}_n$ from the source to the sink. Let us define $J_\xi(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$ and $J_\xi(\mathbf{0})$ to be the optimal expected cost-to-go starting from state $(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$ and state $\mathbf{0}$ respectively.

3) Bellman Equation: $J_\xi(\cdot)$ satisfies the Bellman equations (11) for $m \geq n$ and (12) for $m < n$, for the optimal cost function. The first term in the $\min\{\cdot,\cdot\}$ is the cost if we place a relay at the state $(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$, and the second term is the cost if we do not place a relay.

Observe that it is never optimal to place a relay at state $\mathbf{0}$ because, in doing so, a cost ξ will unnecessarily be incurred. Hence, $J_\xi(\mathbf{0}) = \theta\mathbb{E}(\Gamma_1) + (1-\theta)\mathbb{E}J_\xi(1; 0; \Gamma_1)$.

When $m \geq n$, if we place a relay at the current location and if the line ends in the next step, the length of the shortest path from the source to the sink will be seen as a terminal cost, and is equal to $\mathbb{E}\min\{\min_{k\in\{1,\cdots,n-1\}}(\Gamma_{y_k+1} + P^{(k)}), \Gamma_1 + \min_{k\in\{1,\cdots,n\}}(\gamma^{(k)} + P^{(k)})\}$. Note that in this case the shortest path from the source to the sink can pass via the relay placed at the "current location", or via one of the $(n-1)$ previous relays. For example, in the scenario shown in Figure 6 (with $n = 2$), if we place a relay at the "current location" and the line ends at the next step, then the neighbouring node of the source along the shortest path can be the relay placed at the "current location" or relay 3 (source is not allowed to transmit directly to relay 2 because $n = 2$). Keeping this in mind, $\Gamma_{y_k+1} + P^{(k)}$ is the sum of two costs: the (random) power Γ_{y_k+1} from the source to the $(k+1)$-st previous node w.r.t the source (after placing the relay at the current location, the current k-th previous node will become the $(k+1)$-st previous node in the next step, where the source will be placed) and the length of the shortest path $P^{(k)}$ from that node to the sink. $\Gamma_1 + \min_{k\in\{1,\cdots,n\}}(\gamma^{(k)} + P^{(k)})$ is the sum of the random power Γ_1 required to establish a link from the source to the relay deployed at the current location, and the length of the shortest path from this relay to the sink.

When $m \geq n$, if we place a relay and the line does not end in the next step, the terms y_n, $P^{(n)}$ and $\gamma^{(n)}$ disappear from the state (because a new relay has been placed, which must be taken into account in the state) and the distance 1 of the next location from the newly placed relay at the current location is absorbed into the state. Other distances in the state increase by 1 each. The length of the shortest path from the newly placed relay to the sink, i.e., $\min_{k\in\{1,\cdots,n\}}(\gamma^{(k)}+P^{(k)})$ enters the state, and the power required at the next location to connect to the n previous relays (w.r.t the next location) are independently sampled again. Hence, the new state becomes:

$$
\begin{aligned}
&(1, y_1+1, \cdots, y_{n-1}+1; \min_{k\in\{1,\cdots,n\}}(\gamma^{(k)} + P^{(k)}), \\
&P^{(1)}, \cdots, P^{(n-1)}; \Gamma_1, \Gamma_{y_1+1}, \cdots, \Gamma_{y_{n-1}+1})
\end{aligned}
$$

Similarly, if $m \geq n$ and we do not place a relay at the current location, in the next step the line may end with probability θ and may not end with probability $(1-\theta)$. If the line ends, a cost of the shortest path $\mathbb{E}\min_{k\in\{1,\cdots,n\}}(\Gamma_{y_k+1}+P^{(k)})$ will be incurred. If the line does not end, the next state will be $(\{y_k+1\}_{k=1}^{n}; \{P^{(k)}\}_{k=1}^{n}; \{\gamma_k\}_{k=1}^{n})$, where for each k, γ_k will be drawn independently from $G_{y_k+1}(\cdot)$.

Similar arguments can be used to explain (12) in case $m < n$. The difference is that if we place a relay at the current location and the line does not end in the next step, the next state will have three more terms, since the information for the newly placed relay can be accomodated into the state. On the other hand, if the line ends in the next step, the source will be able to communicate to the sink via one of the m relays (there will be $(m+1)$ preceding nodes, including the sink).

4) Results and Discussion:

Theorem 3: Policy Structure: For the state $(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$, the optimal relay placement policy is the following:

Place a relay if and only if $\min_{k\in\{1,\cdots,l_m\}}(\gamma^{(k)}+P^{(k)}) \leq c(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m})$ where $c(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m})$ is a threshold value. □

Discussion of the Policy Structure: The structure of the optimal policy as stated in Theorem 3 is intuitive because here we need to check whether the quantity $\min_{k\in\{1,\cdots,l_m\}}(\gamma^{(k)}+P^{(k)})$ which is the length of the shortest path from the current location to the sink, is below a certain threshold.

Evidently, the optimal cost $J_\xi(\mathbf{0})$ of (13) is always less than or equal to that of (1), for the same ξ. This is because each policy for $n=1$ will be a policy for $n=2$ as well. Also, $n=\infty$ provides the best policy since there we consider information from all previous nodes.

B. Max-Power Objective

Here we are going to address the following problem:

$$\min_{\pi\in\Pi} \mathbb{E}_\pi \left(\min_{\mathbf{p}\in\mathcal{P}_n} \max_{e\in\mathbf{E}(\mathbf{p})} \Gamma^{(e)} + \xi N \right) \tag{14}$$

We call $\max_{e\in\mathbf{E}(\mathbf{p})} \Gamma^{(e)}$ the "length" of the path $\mathbf{p}$, and $\min_{\mathbf{p}\in\mathcal{P}_n} \max_{e\in\mathbf{E}(\mathbf{p})} \Gamma^{(e)}$ the length of the "shortest path" from the source to the sink.

The Bellman equations will be similar to (11) and (12), except that the big $+$ operations will be replaced by $\max$ operations (for exact Bellman equation, see [4]).

Theorem 4: Policy Structure: For the state $(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m}; \{\gamma^{(k)}\}_{k=1}^{l_m})$, the optimal relay placement policy is the following:

Place a relay if and only if $\min_{k\in\{1,\cdots,l_m\}} \max\{\gamma^{(k)}, P^{(k)}\} \leq c(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m})$ where $c(\{y_k\}_{k=1}^{l_m}; \{P^{(k)}\}_{k=1}^{l_m})$ is a threshold value. □

Note that (3) and (5) can be obtained from the corresponding Bellman equations for arbitrary n, by putting $n=1$; see [4], Section IV.

C. Performance comparison between $n=1$ and $n=2$

A comparative study of the optimal costs with $n=1$ and $n=2$ is summarized in Table III. Here we have used the same model as used in Section III in the max-power case, *but we have considered $\mathcal{S} = \{0.1, 0.2, \cdots, 2\}$ mW in the sum-power case in order to avoid huge computational requirement* (see [4]). The study suggests that, for small relay cost, $n=2$ can provide a significant percentage gain over the optimal cost for $n=1$. Since at small ξ we tend to place more relays (but the relay cost is small compared to $J_\xi(\mathbf{0})$, see Table I and Table II), skipping relays could be useful. For large ξ, we place fewer relays, but the relay cost will dominate. As ξ becomes very high, we will always place the relays at every 10 steps, and nowhere else; hence the relay cost becomes independent of n and the little variation in power cost will be insignificant.

	$\xi=0.001$	$\xi=0.01$	$\xi=0.1$	$\xi=1$
Sum-power ($n=1$)	0.61946	0.65759	1.0414	4.49396
Sum-power ($n=2$)	0.50723	0.56834	1.0233	4.43836
Max-power ($n=1$)	0.03336	0.13124	0.61693	4.10798
Max-power ($n=2$)	0.02119	0.10686	0.60548	4.09718

TABLE III
COMPARISON OF $J_\xi(\mathbf{0})$ FOR $n=1$ AND $n=2$ FOR VARIOUS ξ: SUM-POWER OBJECTIVE AND MAX-POWER OBJECTIVE. TWO DIFFERENT TRANSMIT POWER SETS ARE USED FOR THE TWO OBJECTIVES.

D. Computational Issues

The dimension of the state space is $3n$ in the value iteration, and hence the computational complexity and memory requirement increases with n. However, we can reduce the value iteration to a function iteration as in (7) and (10), and reduce the dimension of the domain of the function to $2n$.

V. CONCLUSION

In this paper, we explored several sequential relay placement problems for as-you-go deployment of wireless relay networks, assuming very light traffic. The problems were formulated as MDPs, optimal policies were derived, and the procedure illustrated via numerical examples. There are numerous issues to improve upon: (i) the light traffic ("lone packet model") assumption, (ii) the assumption of independent shadow fading from link to link, and (iii) the deployment failure issue. Extension to positive traffic might require a different approach: perhaps one that requires a performance analysis model working in conjunction with an optimal sequential decision technique. We are addressing these issues in our ongoing work.

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