Robust Beamformer Design for Underlay Cognitive Radio Network Using Worst Case Optimization

Uditha L. Wijewardhana, Marian Codreanu, and Matti Latva-aho
Centre for Wireless Communications, University of Oulu, Finland
{uditha, codreanu, matti.latva-aho}@ee.oulu.fi

Abstract—**We propose a robust beamforming design for underlay cognitive radio networks where multiple secondary transmitters communicate with corresponding secondary receivers and coexist with a primary network. The main focus is to design the optimal transmit beamforming vectors for secondary transmitters that maximize the minimum of the received signal-to-interference-plus-noise ratios of the cognitive users. We consider a scenario where all transmitters have multiple antennas and all primary and secondary receivers are equipped with a single antenna. Individual transmit powers of the transmitters are limited and interference power constraints to the primary receivers guarantee the performance of the primary network. Imperfect channel state information (CSI) in all relevant channels are considered and bounded ellipsoidal uncertainty model is used to model the CSI errors. We recast the problem in the form of semi-definite program and an iterative algorithm is proposed to achieve the optimal solution. Numerical simulation are conducted to show the effectiveness of the proposed method against the non-robust design.**

Index Terms—**cognitive radio network, robust beamforming, imperfect channel state information, convex optimization, second order cone programming (SOCP).**

I. INTRODUCTION

Cognitive radio networks (CRNs) [1]–[3] operate on the idea of the secondary usage of spectrum. Here the secondary network is allowed to opportunistically accesses the spectrum owned by the primary network provided that it does not degrade the performance of the primary network. Hence, there are two major challenges that should be addressed by a CRN. The first is to guarantee the performance or the quality of service (QoS) of the primary network. The second is to meet the QoS requirements of the users in the CRN as much as possible. Specifically, in *underlay* CRNs the maximum interference power to the primary users (PUs) should remain below a threshold [3] while maintaining QoS for the secondary (cognitive) users.

Resource allocation problems for underlay CRNs has been studied recently in [4]–[6], assuming that perfect channel state information (CSI) knowledge for all relevant links are available for the design. Beamformer design for multiple-input-multiple-output (MIMO) ad hoc CRN is presented in [4]. The weighted sum-rate of the CRN is maximized in [4] subject to the individual power constraints and interference constraints to the primary network. A semi-distributed algorithms is proposed to achieve local optimum solution for multi user CRN co-exists with a single PU. A game theoretic approach for the same problem with multiple PUs is presented in [5].

However, the channel vectors are estimated with error from training sequences in practice. These imperfect estimation and errors in the channels can greatly affect the performance of the network, resulting in degradation in users' QoS. Such channel errors can be modeled either by a bounded uncertainty model such as D-norm, polyhedron, ellipsoidal [7] or a stochastic error model [8].

Robust beamformer design over imperfect CSI has received considerable attention recently. Usually, this problem is tackled by either worst-case optimization [9]–[18] or stochastic optimization [17], [19]. In worst-case optimization (or maximin optimization), the uncertain parameters can take some given set of possible values, but without any known distribution. Then the optimization variables are designed such a way that an objective value is maximized while guaranteeing the feasibility of the constraints over the given set of possible values of the parameters. Hence, for bounded CSI error model, the worst-case optimization method is more suitable. This method has been applied to design the robust beamforming vectors for underlay CRNs in [13]–[18], where the channel errors are either norm bound or bounded by ellipsoids. With the exception of [17] and [18], most of the above mentioned work consider a CRN where a single secondary transmitter (TX) co-exist in a primary network. Various different problems has been studied and the problem of maximizing the minimum SINR in an underlay CRN, where the transmitter communicates with multiple secondary receivers (RXs) is studied in [14]. An iterative solution has been proposed based on semi-definite relaxation, and if the solution is not rank-one, rank-one approximations have to be used to achieve the beamforming vectors.

The sum mean square error is minimized in [17] for an underlay MIMO ad hoc CRN constrained to individual power budgets of secondary TXs, where the channel errors are bounded by Euclidean balls. There the authors have cast the problem as a semi-definite program (SDP) and solved iteratively via standard interior point methods. In [18], the same problem has been considered and a distributed solution is proposed under the assumption that secondary TXs have perfect CSI knowledge of the channels to secondary RXs.

The focus of this paper is to design the optimal beamformer vectors for secondary TXs in a CRN that maximize the worst

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SINR of any secondary user subject to interference constraints to primary network and individual power constraints. Further, we assume that the network controller has imperfect CSI knowledge of all relevant links and the possible channel errors vary in bounded uncertainty ellipsoids [7], [10], [11]. An equivalent reformulation of the problem is obtained and then the S-procedure [20], [21] is used to handle the non-convex quadratic constraints due to channel uncertainties. In particular, each quadratic constraint pair due to channel errors can be reformulated as a linear matrix inequality (LMI) for a fixed objective value by means of the S-lemma [12], [14], which leads to a SDP that can be solved iteratively. Finally, by introducing an objective function to the feasibility check step of the iterative algorithm, we show that the optimal solution for the original problem can be achieved since this objective function guarantees that the solution of the SDP is always rank-one.

Organization: Section II describe the network model and the channel uncertainty model use in this paper. The problem formulation for the worst-case scenario and a suitable equivalent reformulation is presented in Section III. In Section IV, the SDP based solution is presented and an iterative algorithm is proposed to obtain the optimal beamforming vectors. Simulation results are presented in Section V. Finally, we concludes the paper with Section VI.

Notations: Throughout this paper, $\mathbb{C}^n$, $\mathbb{H}^n$ and $\mathbb{R}$ denotes the set of n-dimensional complex vectors, the set of n-dimensional complex hermitian matrices and the set of real values respectively. Further, the complex column vectors and matrices are represented by the boldfaced lowercase and uppercase letters respectively, e.g. $\mathbf{w}$ and $\mathbf{W}$, and a real scalar by a lower case letter. The superscript $(\cdot)^H$ denotes the hermitian (conjugate transpose) operation for a vector or a matrix. $\mathbf{W} \succeq 0$ and $\mathbf{W} \succ 0$ means that $\mathbf{W}$ is positive semi-definite and positive definite, respectively. Rank and the trace of a matrix are represented by $\mathrm{Rank}(\mathbf{W})$ and $\mathrm{Trace}(\mathbf{W})$ respectively. Modulus of a scalar is denoted by $|\cdot|$ and $\|\cdot\|$ denotes the Euclidean norm of a vector. The expectation of a random variable is denoted by $E\{\cdot\}$. In addition, I_n denotes the n-dimensional identity matrix while $\mathbf{0}$ denotes an all-zero vector or matrix with appropriate dimensions. Finally, in an optimization problem if w is a variable then w^* denotes the optimal value or the optimal solution of the problem for the corresponding variable.

II. System Model

In this section we provide the detailed description of the considered network and the ellipsoidal channel uncertainty model.

A. Network Model

The system model is shown in Fig. 1, where an underlay CRN made up of multiple transmitter receiver pairs coexists with a primary network. The set of secondary TXs is denoted by $\mathcal{N}$ and we label them as $i = 1, \ldots, N$. We consider that the secondary TX nodes are equipped with n_t antennas to communicate with their RXs. We represent the set of all PUs by $\mathcal{K}$ and label them as $k = 1, \ldots, K$. Further, we assume that all the primary and secondary RXs are equipped with a single antenna.

We assume that the secondary network operates in a centralized manner and a network controller decides the resource allocation for each secondary TX. All secondary TXs operate in the same frequency band as the primary network and use transmit beamforming to communicate with their corresponding receivers.

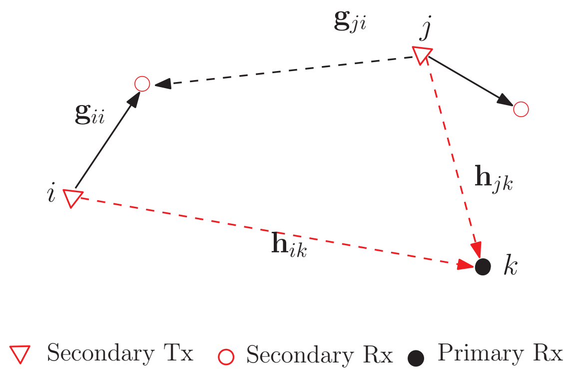

Fig. 1. System model **(Straight arrows show the desired channels and the dashed arrows show the interfering channels)**

The signal transmitted by ith secondary TX is given by,

$$\mathbf{x}_i = \mathbf{w}_i d_i, \tag{1}$$

where d_i denotes the transmitted (complex) information symbol from ith secondary TX and $\mathbf{w}_i$ is its associated beamforming vector. We assume that the information symbols are independent, i.e. $E\{d_i d_j^H\} = 0$ for all $j \neq i, \ i, j \in \mathcal{N}$, and normalized such that $E\{|d_i|^2\} = 1$ for all $i \in \mathcal{N}$. Therefore the transmit power of secondary TX i is given by $\|\mathbf{w}_i\|^2$ and is limited by $P_{\max}$, the maximum available transmit power, for all $i \in \mathcal{N}$.

We denote the channel vector from jth secondary TX to ith secondary RX by $\mathbf{g}_{ji} \in \mathbb{C}^{n_t}$. The interfering channel from ith secondary TX to kth primary RX is denoted by $\mathbf{h}_{ik} \in \mathbb{C}^{n_t}$. Then the received signal, y_i, at ith secondary RX can be written as,

$$y_i = \mathbf{g}_{ii}^H \mathbf{w}_i d_i + \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathbf{g}_{ji}^H \mathbf{w}_j d_j + z_i, \tag{2}$$

where the first term is the signal of interest, the second term represents the interference from the secondary network and $z_i \in \mathbb{C}$ is the experienced additive noise at ith secondary RX. The term z_i includes the receiver's thermal noise and the interference from the primary network with power σ_i^2. Therefore, the instantaneous SINR at the ith secondary RX can be expressed as,

$$\mathrm{SINR}_i = \frac{|\mathbf{g}_{ii}^H \mathbf{w}_i|^2}{\sum_{\substack{j=1 \\ j \neq i}}^{N} |\mathbf{g}_{ji}^H \mathbf{w}_j|^2 + \sigma_i^2}. \tag{3}$$

The total interference power, I_k, cause by the secondary network on kth primary RX can be written as,

$$I_k = \sum_{i=1}^{N} |\mathbf{h}_{ik}^H \mathbf{w}_i|^2, \tag{4}$$

which should be limited by the interference threshold I_{th} in order to guarantee the QoS of the primary network.

B. Channel Uncertainty Model

We assume that the channels are uncertain at the network controller, but they belong to a known compact sets of possible values. Specifically, we assume that the channel vectors, $\mathbf{g}_{ji}$ and $\mathbf{h}_{ik}$ for all $i, j \in \mathcal{N}$ and $k \in \mathcal{K}$, belong to known ellipsoidal uncertainty sets.

We model the channel vector, $\mathbf{g}_{ji}$, from jth secondary TX to ith secondary RX as the sum of two components, i.e.,

$$\mathbf{g}_{ji} = \hat{\mathbf{g}}_{ji} + \mathbf{e}_{ji}, \tag{5}$$

where $\hat{\mathbf{g}}_{ji} \in \mathbb{C}^{n_t}$ denotes the estimated value at the network controller and $\mathbf{e}_{ji}$ represents the corresponding channel estimation error. It is assumed that $\mathbf{e}_{ji}$ can take any value inside a n_t-dimensional complex ellipsoid described by,

$$\mathbf{e}_{ji}^H \mathbf{Q}_{ji} \mathbf{e}_{ji} \le 1, \tag{6}$$

where $\mathbf{Q}_{ji}$ is a complex Hermitian positive definite matrix ($\mathbf{Q}_{ji} \succ 0$), assumed to be known, which specifies the size and shape of the ellipsoid. For example, when $\mathbf{Q}_{ji} = (1/\xi_{ji}^2)\mathbf{I}$, the ellipsoidal channel error model (6) reduces to $\|\mathbf{e}_{ji}\|^2 \le \xi_{ji}^2$ the popular Frobenious norm bound (ball) error model [22] with uncertainty ball radius ξ_{ji}. The CSI knowledge change from perfect to worst can be model using this model as ξ varies from zero to infinity [14].

We use the same uncertainty model for the channel vector, $\mathbf{h}_{ik}$, from ith secondary TX to kth primary RX, i.e.,

$$\mathbf{h}_{ik} = \hat{\mathbf{h}}_{ik} + \tilde{\mathbf{e}}_{ik}, \tag{7}$$

where $\hat{\mathbf{h}}_{ik} \in \mathbb{C}^{n_t}$ is the estimated value at the network controller and $\tilde{\mathbf{e}}_{ik}$ denotes the corresponding channel estimation error. The n_t-dimensional complex ellipsoid that the error $\tilde{\mathbf{e}}_{ik}$ can vary is defined as,

$$\tilde{\mathbf{e}}_{ik}^H \tilde{\mathbf{Q}}_{ik} \tilde{\mathbf{e}}_{ik} \le 1, \tag{8}$$

where ($\tilde{\mathbf{Q}}_{ik} \succ 0$) specifies the size and shape of the ellipsoid.

III. Problem Formulation

Our objective is to maximize the performance of the CRN while satisfying the QoS requirements of the primary network. We consider the minimum SINR among all secondary receivers as the performance indicator of the CRN and interference received from CRN as the QoS measurement for primary network. Then the solution of the problem will guarantee a certain SINR for all secondary RXs while the interference to all the users in the primary network will be lower than a predefined threshold I_{th}. Mathematical formulation of the mentioned problem and a suitable reformulation is presented in this section.

A. Worst-case Problem Formulation

Now, with the channel uncertainty model, we can re-write the instantaneous SINR at ith secondary RX as,

$$\text{SINR}_i = \frac{|(\hat{\mathbf{g}}_{ii} + \mathbf{e}_{ii})^H \mathbf{w}_i|^2}{\sum_{\substack{j=1 \\ j \ne i}}^{N} |(\hat{\mathbf{g}}_{ji} + \mathbf{e}_{ji})^H \mathbf{w}_j|^2 + \sigma_i^2} \tag{9}$$

and the interference power, I_k, cause by the secondary network on kth primary RX as,

$$I_k = \sum_{i=1}^{N} |(\hat{\mathbf{h}}_{ik} + \tilde{\mathbf{e}}_{ik})^H \mathbf{w}_i|^2. \tag{10}$$

Specifically the resource allocation problem we address in this work is to optimize the transmit beamforming vectors in CRN, $\{\mathbf{w}_i\}_{i=1}^N$, to maximize the minimum SINR of the secondary RXs for given parameters $P_{\max}$ and I_{th}. Due to uncertain knowledge in CSIs, the beamfomer design should guarantee a certain SINR for all secondary RXs for any channel error that is inside the given uncertainty region. Further, the design should keep the interference to the primary RXs below the threshold for all channel errors inside the uncertainty region to guarantee the performance of the primary network. This problem can be mathematically expressed as,

$$\begin{array}{ll} \text{maximize} & \min_{i=1,\ldots,N} \inf_{\mathbf{e}_{ji} | \mathbf{e}_{ji}^H \mathbf{Q}_{ji} \mathbf{e}_{ji} \le 1, j \in \mathcal{N}} \text{SINR}_i \\ \text{subject to} & \sup_{\tilde{\mathbf{e}}_{ik} | \tilde{\mathbf{e}}_{ik}^H \tilde{\mathbf{Q}}_{ik} \tilde{\mathbf{e}}_{ik} \le 1, i \in \mathcal{N}} I_k \le I_{\text{th}}, \quad k \in \mathcal{K} \\ & \|\mathbf{w}_i\|_2^2 \le P_{\max} \quad i \in \mathcal{N}, \end{array} \tag{11}$$

where the optimization variables are $\mathbf{w}_i, \mathbf{e}_{ji}, \tilde{\mathbf{e}}_{ik}$ for $i, j \in \mathcal{N}, k \in \mathcal{K}$. Note that SINR_i depends on $\mathbf{e}_{ji}$ for all $i, j \in \mathcal{N}$ (see (9)) and I_k depends on $\tilde{\mathbf{e}}_{ik}$ for all $i \in \mathcal{N}$ (see (10)). In problem (11) the infimum in the objective function and supremum in the first constraint are taken over all possible channel errors contained in the given uncertainty region. Hence the solution should satisfy the constraints $\mathbf{e}_{ji}^H \mathbf{Q}_{ji} \mathbf{e}_{ji} \le 1, i, j \in \mathcal{N}$ and $\tilde{\mathbf{e}}_{ik}^H \tilde{\mathbf{Q}}_{ik} \tilde{\mathbf{e}}_{ik} \le 1, i \in \mathcal{N}, k \in \mathcal{K}$.

B. An Equivalent reformulation

Since the minimization in the objective function of problem (11) is over all secondary RXs, the optimal value should be less than or equal to any SINR value that can be achieved by a secondary RX for the optimal beamformer design.

Therefore, the optimization problem (11), using the epigraph form (strictly speaking this is a hypograph form, as problem (11) is a maximization), can be equivalently written as,

$$\begin{array}{ll} \text{maximize} & \gamma \\ \text{subject to} & \gamma \le \dfrac{|(\hat{\mathbf{g}}_{ii} + \mathbf{e}_{ii})^H \mathbf{w}_i|^2}{\sum_{\substack{j=1 \\ j \ne i}}^{N} |(\hat{\mathbf{g}}_{ji} + \mathbf{e}_{ji})^H \mathbf{w}_j|^2 + \sigma_i^2}, i \in \mathcal{N} \\ & \mathbf{e}_{ji}^H \mathbf{Q}_{ji} \mathbf{e}_{ji} \le 1, \quad i, j \in \mathcal{N} \\ & \sum_{i=1}^{N} |(\hat{\mathbf{h}}_{ik} + \tilde{\mathbf{e}}_{ik})^H \mathbf{w}_i|^2 \le I_{\text{th}}, \quad k \in \mathcal{K} \\ & \tilde{\mathbf{e}}_{ik}^H \tilde{\mathbf{Q}}_{ik} \tilde{\mathbf{e}}_{ik} \le 1, \quad i \in \mathcal{N}, k \in \mathcal{K} \\ & \|\mathbf{w}_i\|_2^2 \le P_{\max}, \quad i \in \mathcal{N}, \end{array} \tag{12}$$

where the optimization variables are $\mathbf{w}_i, \mathbf{e}_{ji}, \tilde{\mathbf{e}}_{ik}$ for $i, j \in \mathcal{N}, k \in \mathcal{K}$ and γ.

We introduce the variables,

$$s_{ji} = |(\hat{\mathbf{g}}_{ji} + \mathbf{e}_{ji})^H \mathbf{w}_j|^2, \quad j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} \tag{13}$$

$$I_{ik} = |(\hat{\mathbf{h}}_{ik} + \tilde{\mathbf{e}}_{ik})^H \mathbf{w}_i|^2, \quad i \in \mathcal{N}, k \in \mathcal{K}. \tag{14}$$

Then, problem (12) can be recast as follows,

$$\begin{aligned}
&\text{maximize} \quad \gamma && \text{(15a)}\\
&\text{subject to} \quad (\hat{\mathbf{g}}_{ii} + \mathbf{e}_{ii})^H \mathbf{w}_i \mathbf{w}_i^H (\hat{\mathbf{g}}_{ii} + \mathbf{e}_{ii}) \geq \\
&\qquad \gamma \left(\sum_{\substack{j=1\\ j \neq i}}^{N} s_{ji} + \sigma_i^2 \right), i \in \mathcal{N} && \text{(15b)}\\
&\qquad \mathbf{e}_{ii}^H \mathbf{Q}_{ii} \mathbf{e}_{ii} \leq 1, \quad i \in \mathcal{N} && \text{(15c)}\\
&\qquad (\hat{\mathbf{g}}_{ji} + \mathbf{e}_{ji})^H \mathbf{w}_j \mathbf{w}_j^H (\hat{\mathbf{g}}_{ji} + \mathbf{e}_{ji}) \leq s_{ji}, \\
&\qquad\qquad j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} && \text{(15d)}\\
&\qquad \mathbf{e}_{ji}^H \mathbf{Q}_{ji} \mathbf{e}_{ji} \leq 1, \quad j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} && \text{(15e)}\\
&\qquad \sum_{i=1}^{N} I_{ik} \leq I_{\text{th}}, \quad k \in \mathcal{K} && \text{(15f)}\\
&\qquad (\hat{\mathbf{h}}_{ik} + \tilde{\mathbf{e}}_{ik})^H \mathbf{w}_i \mathbf{w}_i^H (\hat{\mathbf{h}}_{ik} + \tilde{\mathbf{e}}_{ik}) \leq I_{ik}, \\
&\qquad\qquad i \in \mathcal{N}, k \in \mathcal{K} && \text{(15g)}\\
&\qquad \tilde{\mathbf{e}}_{ji}^H \tilde{\mathbf{Q}}_{ik} \tilde{\mathbf{e}}_{ik} \leq 1, \quad i \in \mathcal{N}, k \in \mathcal{K} && \text{(15h)}\\
&\qquad \mathbf{w}_i^H \mathbf{w}_i \leq P_{\max}, \quad i \in \mathcal{N}, && \text{(15i)}
\end{aligned}$$

where the variables are $\mathbf{w}_i, \mathbf{e}_{ji}, \tilde{\mathbf{e}}_{ik}, s_{ji}, I_{ik}$ for all $i, j \in \mathcal{N}, k \in \mathcal{K}$ and γ. Note that we have re-written 2nd constraint in problem (12) as two separate ones, i.e., (15c) and (15e). Furthermore, it easy to show (e.g., by contradiction) that constraints (15d) and (15g) are tight (i.e., they hold with equality at optimality). Hence, problem (12) and (15) are equivalent.

IV. Optimal Beamformer Design

The outer product $\mathbf{w}_i \mathbf{w}_i^H$ in problem (15) is a rank one positive semidefinite matrix. We introduce a new set of variables $\mathbf{W}_i = \mathbf{w}_i \mathbf{w}_i^H$ for all $i \in \mathcal{N}$. Then the constraints (15b),(15d),(15g) and (15i) become linear in $\mathbf{W}_i$ and $\mathbf{W}_i$ should be rank one. Furthermore, we can see from problem (15) that the constraints (15b), (15c) are quadratic in $\mathbf{e}_{ii}$, constraints (15d), (15e) are quadratic in $\mathbf{e}_{ji}$ and (15g),(15h) are quadratic in $\tilde{\mathbf{e}}_{ik}$. This suggests that we can use the following lemma to recast these constraints as linear matrix inequalities (LMIs).

S-lemma [12], [20], [21] : Let Φ_i be a real valued function of an m-dimensional complex vector, $\mathbf{y}$, defined as,

$$\Phi_i(\mathbf{y}) = \mathbf{y}^H \mathbf{A}_i \mathbf{y} + 2\text{Re}(\mathbf{b}_i^H \mathbf{y}) + c_i,$$

where $\mathbf{A}_i \in \mathbb{H}^m$, $\mathbf{b}_i \in \mathbb{C}^m$, $c_i \in \mathbb{R}$ and $i = 0, 1$. Assume that there exists a vector $\hat{\mathbf{y}} \in \mathbb{C}^m$ such that $\Phi_1(\hat{y}) < 0$. Then the following conditions are equivalent:

S1 : $\Phi_0(\mathbf{y}) \geq 0$ for all $\mathbf{y} \in \mathbb{C}^m$ such that $\Phi_1(\mathbf{y}) \leq 0$.

S2 : There exists $\lambda \geq 0$ such that the following LMI is feasible:

$$\begin{bmatrix} \mathbf{A}_0 & \mathbf{b}_0 \\ \mathbf{b}_0^H & c_0 \end{bmatrix} + \lambda \begin{bmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{bmatrix} \succeq 0.$$

■

First two constraints in problem (15) can be re-written as,

$$\begin{aligned}
\mathbf{e}_{ii}^H \mathbf{W}_i \mathbf{e}_{ii} + 2\text{Re}((\mathbf{W}_i \hat{\mathbf{g}}_{ii})^H \mathbf{e}_{ii}) + \hat{\mathbf{g}}_{ii}^H \mathbf{W}_i \hat{\mathbf{g}}_{ii} \\
-\gamma \left(\textstyle\sum_{\substack{j=1\\ j \neq i}}^{N} s_{ji} + \sigma_i^2 \right) \geq 0, \quad i \in \mathcal{N}
\end{aligned} \tag{16}$$

$$\mathbf{e}_{ii}^H \mathbf{Q}_{ii} \mathbf{e}_{ii} - 1 \leq 0, \quad i \in \mathcal{N}. \tag{17}$$

The existence of an $\mathbf{e}_{ii}$ for which (17) holds strictly is obvious (e.g., $\mathbf{e}_{ii} = 0$). Hence, we can regard the left hand sides of (16) and (17) as $\Phi_0(\mathbf{e}_{ii})$ and $\Phi_1(\mathbf{e}_{ii})$ in S-Lemma. Then, according to S-Lemma the inequality (16) is satisfied for all channel errors $\mathbf{e}_{ii}$ that satisfy (17) if there exists $\mu_{ii} \geq 0$ such that (18) (defined at the top of the next page) is satisfied.

Following the same procedure, we can say that the inequality (15d) is satisfied for all channel errors $\mathbf{e}_{ji}$ that satisfy (15e) if there exists $\mu_{ji} \geq 0$ such that (19) is satisfied. Similarly, the inequality (15g) is satisfied for all channel errors $\tilde{\mathbf{e}}_{ik}$ that satisfy (15h) if there exists $\nu_{ik} \geq 0$ such that (19) is satisfied.

Thus we can rewrite problem (15) equivalently as follows,

$$\begin{aligned}
&\text{maximize} && \gamma \\
&\text{subject to} && \mathbf{\Delta}_{ii} \succeq 0, \quad i \in \mathcal{N} \\
&&& \mathbf{\Phi}_{ji} \succeq 0, \quad j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} \\
&&& \mathbf{\Theta}_{ik} \succeq 0, \quad i \in \mathcal{N}, k \in \mathcal{K} \\
&&& \textstyle\sum_{i=1}^{N} I_{ik} \leq I_{\text{th}}, \quad k \in \mathcal{K} \\
&&& \mu_{ji} \geq 0, \quad i, j \in \mathcal{N} \\
&&& \nu_{ik} \geq 0, \quad i \in \mathcal{N}, k \in \mathcal{K} \\
&&& \mathbf{W}_i \succeq 0, \quad i \in \mathcal{N} \\
&&& \text{Trace}(\mathbf{W}_i) \leq P_{\max}, \quad i \in \mathcal{N} \\
&&& \text{Rank}(\mathbf{W}_i) = 1, \quad i \in \mathcal{N},
\end{aligned} \tag{21}$$

where γ and $\mathbf{W}_i, \mu_{ji}, \nu_{ik}, s_{ji}, I_{ik}$ for $i, j \in \mathcal{N}, k \in \mathcal{K}$ are the optimization variables.

Without the rank constraints, problem (21) can be solved using bisection. For a fixed γ the feasibility can be checked by solving the problem,

$$\begin{aligned}
P_0(\gamma): \quad &\text{maximize} && 0 \\
&\text{subject to} && \mathbf{\Delta}_{ii}(\gamma) \succeq 0, \quad i \in \mathcal{N} \\
&&& \mathbf{\Phi}_{ji} \succeq 0, \quad j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} \\
&&& \mathbf{\Theta}_{ik} \succeq 0, \quad i \in \mathcal{N}, k \in \mathcal{K} \\
&&& \textstyle\sum_{i=1}^{N} I_{ik} \leq I_{\text{th}}, \quad k \in \mathcal{K} \\
&&& \mu_{ji} \geq 0, \quad i, j \in \mathcal{N} \\
&&& \nu_{ik} \geq 0, \quad i \in \mathcal{N}, k \in \mathcal{K} \\
&&& \mathbf{W}_i \succeq 0, \quad i \in \mathcal{N} \\
&&& \text{Trace}(\mathbf{W}_i) \leq P_{\max}, \quad i \in \mathcal{N},
\end{aligned} \tag{22}$$

using a standard SDP solver. The optimal beamforming matrices can be obtained when the maximum feasible γ is achieved.

In order to obtain a rank one solution, we replace the dummy objective function in (22) with sum power minimization of the secondary network. Then the problem use to check

$$\mathbf{\Delta}_{ii} \triangleq \begin{bmatrix} \mathbf{W}_i & \mathbf{W}_i\hat{\mathbf{g}}_{ii} \\ \hat{\mathbf{g}}_{ii}^H\mathbf{W}_i & \hat{\mathbf{g}}_{ii}^H\mathbf{W}_i\hat{\mathbf{g}}_{ii} - \gamma\left(\sum_{\substack{j=1\\j\neq i}}^N s_{ji} + \sigma_i^2\right) \end{bmatrix} + \mu_{ii}\begin{bmatrix} \mathbf{Q}_{ii} & 0 \\ 0 & -1 \end{bmatrix} \succeq 0, \quad i \in \mathcal{N} \tag{18}$$

$$\mathbf{\Phi}_{ji} \triangleq \begin{bmatrix} -\mathbf{W}_j & -\mathbf{W}_j\hat{\mathbf{g}}_{ji} \\ -\hat{\mathbf{g}}_{ji}^H\mathbf{W}_j & s_{ji} - \hat{\mathbf{g}}_{ji}^H\mathbf{W}_j\hat{\mathbf{g}}_{ji} \end{bmatrix} + \mu_{ji}\begin{bmatrix} \mathbf{Q}_{ji} & 0 \\ 0 & -1 \end{bmatrix} \succeq 0, \quad j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} \tag{19}$$

$$\mathbf{\Theta}_{ik} \triangleq \begin{bmatrix} -\mathbf{W}_i & -\mathbf{W}_i\hat{\mathbf{h}}_{ik} \\ -\hat{\mathbf{h}}_{ik}^H\mathbf{W}_i & I_{ik} - \hat{\mathbf{h}}_{ik}^H\mathbf{W}_i\hat{\mathbf{h}}_{ik} \end{bmatrix} + \nu_{ik}\begin{bmatrix} \tilde{\mathbf{Q}}_{ik} & 0 \\ 0 & -1 \end{bmatrix} \succeq 0, \quad i \in \mathcal{N}, k \in \mathcal{K} \tag{20}$$

the feasibility can be rewrite as,

$$P_1(\gamma): \quad \begin{array}{ll} \text{minimize} & \sum_{i=1}^N \text{Trace}(\mathbf{W}_i) \\ \text{subject to} & \text{constraints of } P_0(\gamma) \end{array} \tag{23}$$

with the optimization variables $\mathbf{W}_i, \mu_{ji}, \nu_{ik}, s_{ji}, I_{ik}$ for all $i, j \in \mathcal{N}, k \in \mathcal{K}$. Clearly, problem (22) is feasible if and only problem (23) is feasible. Furthermore, the following proposition ensure that problem (23) returns always a set of rank one matrices $\mathbf{W}_i$.

Proposition 1. *If there exists any γ value such that the problem* (23) *is feasible, then the corresponding feasible beamforming matrices are always rank-one, that is,* $\mathbf{W}_i = \mathbf{w}_i(\mathbf{w}_i)^H$ *for all* $i \in \mathcal{N}$.

Proof: The proof is presented in the Appendix A. ■

Hence, the optimal beamforming vectors that maximize the minimum SINR can be found by eigen-decomposition, when the maximum feasible γ is achieved. This implies that, we have solved the original problem (11) without any relaxation and hence the global optimal solution is achieved.

Following iterative algorithm can be used to design the optimal robust beamforming vectors for an underlay ad hoc CRN that maximize the minimum SINR of the cognitive users.

Algorithm 1: Robust Cognitive Beamforming

1) Inputs: network parameters
   $\{\hat{\mathbf{g}}_{ji}\}$, $\{\hat{\mathbf{h}}_{ik}\}$, $\{\mathbf{Q}_{ji}\}$, $\{\tilde{\mathbf{Q}}_{ik}\}$, $\{\sigma_i\}$, I_{th}, P_{max}.
2) Initialization: given tolerance $\epsilon > 0$. The initial lower and upper bounds of optimal value γ_{low} and γ_{upp}.
3) Set $\gamma = (\gamma_{\text{low}} + \gamma_{\text{upp}})/2$.
4) If $P_1(\gamma)$ is feasible, set $\gamma_{\text{low}} = \gamma$ and denote the solution by $\{\mathbf{W}_i^*\}$. Else set $\gamma_{\text{upp}} = \gamma$.
5) Stopping criterion: if $\gamma_{\text{upp}} - \gamma_{\text{low}} \leq \epsilon$ STOP; otherwise go to step 3.
6) Outputs: γ and $\{\mathbf{w}_i^*(\mathbf{w}_i^*)^H = \mathbf{W}_i^*\}$.

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the initial lower and upper bounds for γ can be efficiently initialized by solving modified (i.e., relaxed and restricted) versions of the original problem.

V. Results and Discussion

Numerical simulations are performed to validate and assess the performance of the proposed beamforming scheme. We consider a simple network model where two secondary transmitter-receiver pairs ($N = 2$) coexist in a primary network with two primary RXs ($K = 2$). We assume that each secondary TX is equipped with four antennas ($n_t = 4$). Further, we use an independent and identically distributed $\mathcal{CN}(0, 1)$ to generate all the estimated channel vectors. The CSI uncertainties are assumed to be within the bounded uncertainty balls $\|\mathbf{e}_{ii}\|^2 = \|\mathbf{e}_{ji}\|^2 = \|\tilde{\mathbf{e}}_{ik}\|^2 \leq \xi^2$ for all $i, j \in \mathcal{N}, k \in \mathcal{K}$. Each entry in the error vectors follows a truncated Gaussian distribution of $\mathcal{CN}(0, \xi^2/9n_t)$ truncated at $-\xi/\sqrt{2n_t}$ and $+\xi/\sqrt{2n_t}$, where ξ is radius of the uncertainty ball. We assume that $\sigma_i^2 = \sigma^2 = 1(0 \text{ dB})$ for all $i \in \mathcal{N}$ and the maximum available transmit power of a secondary TX is $P_{\text{max}}/\sigma^2 = 10$ dB. To maintain the QoS requirements for the primary network the maximum allowable interference power from the secondary network is limited to 0 dB, so that the primary network can operate neglecting the interference from the secondary network.

We present results for both robust and non-robust beamforming designs for comparison. The robust beamformer design is acquired directly following algorithm 1 and since all the obtained beamforming matrices are with unit rank, those are directly used in the calculation of interference powers to primary RXs and SINRs for secondary RXs to reduce the complexity of the simulations. For the non-robust case, the beamforming vectors are obtained based on the estimated channels by ignoring the uncertainty. There, the algorithm 1 is followed and instead of problem (23) the following problem,

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^N \text{Trace}(\mathbf{W}_i) \\ \text{subject to} & \hat{\mathbf{g}}_{ii}^H\mathbf{W}_i\hat{\mathbf{g}}_{ii} \geq \gamma\left(\sum_{\substack{j=1\\j\neq i}}^N \hat{\mathbf{g}}_{ji}^H\mathbf{W}_j\hat{\mathbf{g}}_{ji} + \sigma_i^2\right), \\ & \qquad i \in \mathcal{N} \\ & \sum_{i=1}^N \hat{\mathbf{h}}_{ik}^H\mathbf{W}_i\hat{\mathbf{h}}_{ik} - I_{\text{th}} \leq 0, k \in \mathcal{K} \\ & \text{Trace}(\mathbf{W}_i) - P_{\text{max}} \leq 0, i \in \mathcal{N} \\ & \mathbf{W}_i \succeq 0, i \in \mathcal{N}, \end{array} \tag{24}$$

is used, where the optimization variable is $\mathbf{W}_i$ for all $i \in \mathcal{N}$. The MATLAB toolbox CVX is used to solve all the optimization problems and there the SDPs are solved using the solver SeDuMi.

Fig. 2 and Fig. 3 displays the cumulative density function (CDF) of received secondary interference at the first primary RX for different uncertainty balls. In Fig. 2, the CDF is taken over the distribution of 10000 different possible channels lie in the corresponding uncertainty balls for a given channel estimate. Fig. 3 is drawn by averaging over 1000 such estimated

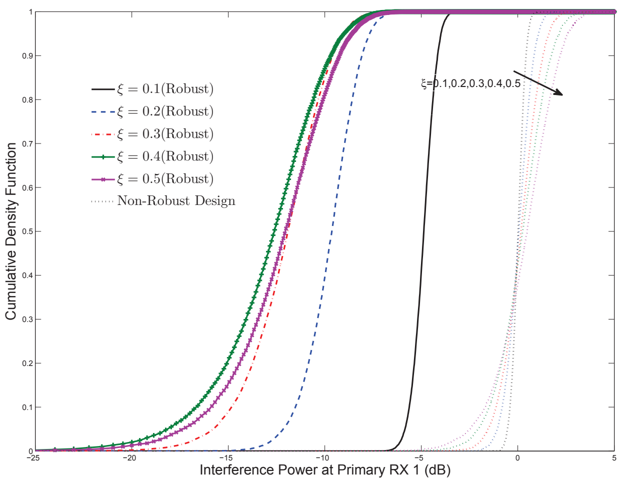

Fig. 2. CDF of the interference due to the secondary network at 1st primary RX for different uncertainty spheres for a network with $n_t = 4, N = 2$ and $K = 2$. The interference threshold for the primary network is 0 dB. CDF is taken over different possible channel errors for a single channel estimate.

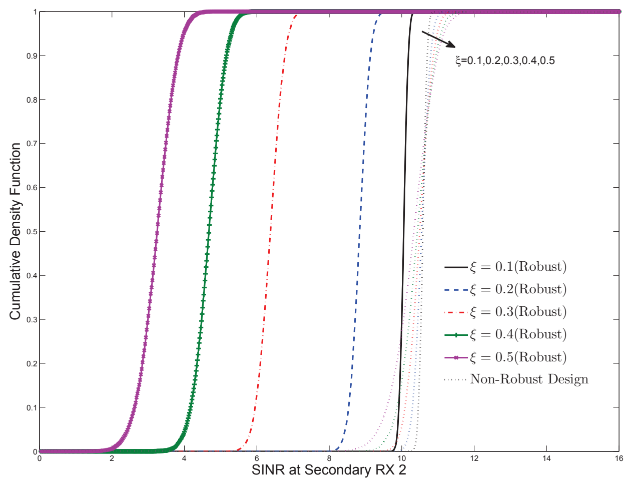

Fig. 4. CDF of the received SINR at secondary RX 2 for different uncertainty spheres for a network with $n_t = 4, N = 2$ and $K = 2$. CDF is taken over different possible channel errors for a single channel estimate.

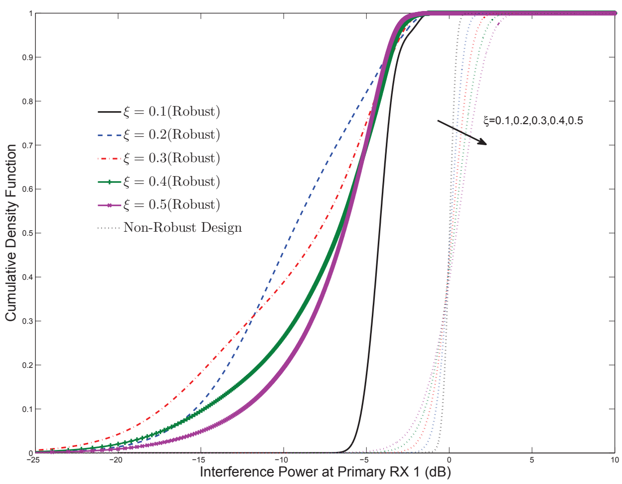

Fig. 3. CDF of the interference due to the secondary network at 1st primary RX for different uncertainty spheres for a network with $n_t = 4, N = 2$ and $K = 2$. The interference threshold for the primary network is 0 dB. CDF is taken over different possible channel errors and averaged over multiple channel estimates.

channel realizations. As we expected, it is clear from Fig. 2 that the non-robust design gives the worst performance. In fact, according to Fig. 2, around $52\%, 55\%$, 57%, 59% and 61% of the simulated channel errors the non-robust design exceeds the interference threshold of the primary network (0 dB) for uncertainty ball radius $0.1, 0.2, 0.3, 0.4$ and 0.5 respectively. These values in average are around $52\%, 54\%, 56\%, 59\%$ and 61% for uncertainty ball radius $0.1, 0.2, 0.3, 0.4$ and 0.5 according to Fig. 3. On the other hand, for the proposed robust algorithm, the interference to the primary RX is always less than the threshold level which guarantees the required QoS of the primary network. Moreover, it can be observed from the figure 2 that the CDF spread out wide with the uncertainty radius varies from 0.1 to 0.4. When the uncertainty ball is small, since the range of the possible channels are less, the beamformer design can direct sharp nulls towards primary RXs as the channel is precisely known. As radii become larger, the range of possible channels in the uncertainty ball get increased and hence the nulls become less focused. But most of those channels lie in deep null and hence the CDFs spread out wide. Further increase in the radii force the beam pattern to flattens in the direction of the uncertainty region and therefore the interference to the primary RXs get less variable as radius varies from 0.4 to 0.5 in Fig. 2. This phenomenon can also be observed in Fig. 3.

The CDF of the received SINR at 2nd secondary RX for different uncertainty radius values is plotted in Fig. 4 for channel estimate used in Fig. 2, since secondary RX 2 result in the minimum SINR for that channel estimate. As same as in Fig. 2, the CDF is taken over 10000 possible channels inside the uncertainty region. Further, the same figure averaged over 1000 different channel estimates is plotted in Fig. 5. The ability of the secondary TXs to focus sharp beams towards the corresponding secondary RXs while simultaneously directing nulls towards other secondary RXs and primary RXs reduces as the channel knowledge deceases. Hence, it can be seen from the Fig. 4 and Fig. 5 that, as expected, the mean SINR reduces with increase in the radius of uncertainty balls. Furthermore, for small uncertainty regions the beamformers are designed as the channel is precisely known and hence in Figure 4 the CDF is narrowed for small uncertainty radius values.

Finally, the variation in the optimal objective value with the radius of uncertainty balls is illustrated in Fig. 6. The figure is averaged over 1000 channel realizations. As mentioned earlier, since the ability of the transmitters to focus the beams towards secondary RXs and nulls towards primary RXs increase rapidly with smaller uncertainty regions, the optimal value rapidly increase at smaller values of uncertainty ball

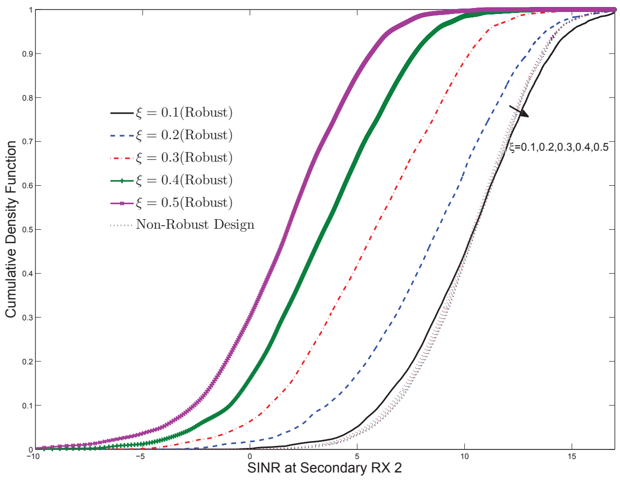

Fig. 5. CDF of the received SINR at secondary RX 2 for different uncertainty spheres for a network with $n_t = 4, N = 2$ and $K = 2$. CDF is taken over different possible channel errors and averaged over multiple channel estimates.

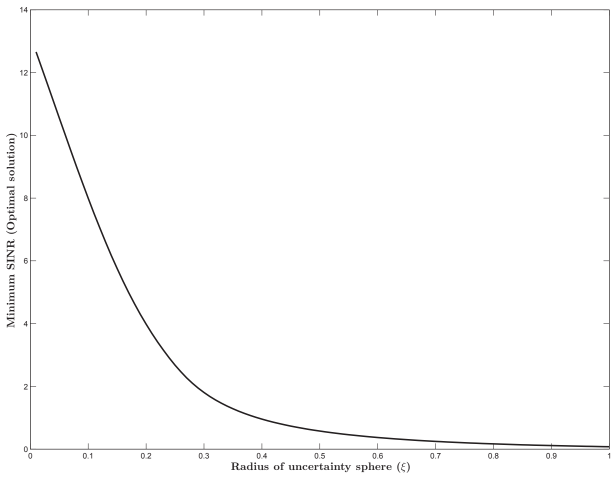

Fig. 6. Variation of the objective value with respect to the radius of the uncertainty balls for a network with $n_t = 4, N = 2$ and $K = 2$. The curve is averaged over multiple channel estimates.

radius in Fig. 6. At higher values of $\xi_{ii}, \xi_{ji}, \delta_{ik}$ the beams become flat in all directions and the transmit powers become reasonably small, hence the optimal objective value and also the variation become small.

VI. Conclusion

We have presented a centralized method to design optimal beamforming vectors for an underlay cognitive ad hoc network to maximize the minimum of the received SINRs of the cognitive users subject to primary interference constraints. The developed robust design based on SDP can handle the channel uncertainties in all relevant channels where the uncertainties are modeled by complex ellipsoids. Further, we have shown that for this problem no relaxation is required, and the optimal solution can be achieved guarantying the global optimality of the original problem. The simulation results show that our robust design is more effective than the non-robust design.

Appendix A
Proof of Proposition 1

Let us rewrite the problem (23) as follows,

$$\begin{array}{ll} \text{minimize} & \sum_{i=1}^{N} \text{Trace}(\mathbf{W}_i) \\ \text{subject to} & \mathbf{\Delta}_{ii}\left(\mathbf{W}_i, \gamma, \mu_{ii}, \{s_{ji}\}_{j\neq i}\right) \succeq 0, \quad i \in \mathcal{N} \\ & \mathbf{\Phi}_{ji}\left(\mathbf{W}_j, \mu_{ji}, s_{ji}\right) \succeq 0, j \in \mathcal{N}\backslash\{i\}, i \in \mathcal{N} \\ & \mathbf{\Theta}_{ik}\left(\mathbf{W}_i, \nu_{ik}, I_{ik}\right) \succeq 0, \quad i \in \mathcal{N}, k \in \mathcal{K} \\ & \mathbf{W}_i \succeq 0, \quad i \in \mathcal{N} \\ & \text{Trace}(\mathbf{W}_i) \leq P_{\max}, \quad i \in \mathcal{N} \\ & \sum_{i=1}^{N} I_{ik} \leq I_{\text{th}}, \quad k \in \mathcal{K} \\ & \mu_{ji} \geq 0, \quad i, j \in \mathcal{N} \\ & \nu_{ik} \geq 0, \quad i \in \mathcal{N}, k \in \mathcal{K}, \end{array} \tag{A.1}$$

where the optimization variables are $\mathbf{W}_i, \mu_{ji}, \nu_{ik}, s_{ji}, I_{ik}$ for all $i, j \in \mathcal{N}, k \in \mathcal{K}$ for a feasible γ value. Proposition 1 can be proved following the method used to prove proposition 1 in [12], by investigating the KKT conditions of the problem (A.1).

Let the optimal dual variable associated with the first five constraints be $\{\mathbf{\Psi}_{ii}^* \succeq 0\}$, $\{\mathbf{\Psi}_{ji}^* \succeq 0\}$,$\{\mathbf{\Lambda}_{ik}^* \succeq 0\}$,$\{\mathbf{Z}_i^* \succeq 0\}$ and $\{\beta_i^* \geq 0\}$ respectively. Further, let

$$\mathbf{\Psi}_{ii}^* = \begin{bmatrix} \mathbf{A}_{ii} & \mathbf{b}_{ii} \\ \mathbf{b}_{ii}^H & c_{ii} \end{bmatrix} \succeq 0 \quad i \in \mathcal{N}, \tag{A.2}$$

where the matrix $\mathbf{A}_{ii} \in \mathbb{H}^{n_t}$ is PSD, $\mathbf{b}_{ii} \in \mathbb{C}^{n_t}$ and $c_{ii} \in \mathbb{R}$ is non-negative according to the properties of PSD matrices. We can verify that according to the KKT conditions of (A.1) the optimal $\mathbf{W}_i^*, \mu_{ji}^*, \nu_{ik}^*, s_{ji}^*, I_{ik}^*$ for all $i, j \in \mathcal{N}, k \in \mathcal{K}$ should satisfy the following conditions,

$$\begin{aligned} \mathbf{Z}_i^* = \beta_i^* \mathbf{I}_{n_t} + \sum_{\substack{j=1 \\ j\neq i}}^{N} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{g}}_{ij} \end{bmatrix} \mathbf{\Psi}_{ij}^* \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{g}}_{ij}^H \end{bmatrix} \\ + \sum_{k=1}^{K} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{h}}_{ik} \end{bmatrix} \mathbf{\Lambda}_{ik}^* \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{h}}_{ik}^H \end{bmatrix} \\ -\frac{1}{\gamma} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{g}}_{ii} \end{bmatrix} \mathbf{\Psi}_{ii}^* \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{g}}_{ii}^H \end{bmatrix} \succeq 0, \end{aligned} \tag{A.3}$$

$$\mathbf{Z}_i^* \mathbf{W}_i^* = \mathbf{0}, \mathbf{W}_i^* \neq \mathbf{0}, \tag{A.4}$$

$$\mathbf{\Delta}_{ii}\left(\mathbf{W}_i, \gamma, \mu_{ii}, \{s_{ji}\}_{j\neq i}\right) \mathbf{\Psi}_{ii}^* = \mathbf{0}, \mathbf{\Psi}_{ii}^* \neq \mathbf{0}, \tag{A.5}$$

$$\text{Trace}\left(\mathbf{Q}_{ii} \mathbf{A}_{ii}\right) \leq c_{ii}, \tag{A.6}$$

$$s_{ji}^* > 0 \quad \forall j \in \mathcal{N}/\{i\}, \tag{A.7}$$

$$\mu_{ii}^* > 0, \tag{A.8}$$

for all $i \in \mathcal{N}$. Equality in (A.3) is obtained using the stationary property of $\mathbf{W}_i^*$ and the inequality is directly given by dual feasibility of $\mathbf{Z}_i^*$. In (A.4) and (A.5) the equalities are obtained from complementary slackness of constraints four and one in (A.1) respectively. Since the target SINR should satisfy $\gamma > 0$, the beamforming matrix must satisfy the condition $\mathbf{W}_i^* \neq \mathbf{0}$. If $\mathbf{\Psi}_{ii}^* = \mathbf{0}$, (A.3) will lead to $\mathbf{Z}_i^* \succ 0$ and as a result contradicts with (A.4) since the equality condition of (A.4) forced to $\mathbf{W}_i^* = \mathbf{0}$. Further, the stationary property of μ_{ii} and dual feasibility of constraint $\mu_{ii} \geq 0$ leads to condition (A.6).

Condition (A.7) is achieved investigating the primal feasibility of constraint $\mathbf{\Phi}_{ji} \succeq 0$, and finally to show (A.8), we can see from (18) that when $\mu_{ii} = 0$,

$$\begin{aligned} \begin{bmatrix} -\hat{\mathbf{g}}_{ii}^H & 1 \end{bmatrix} \mathbf{\Delta}_{ii}(\mathbf{W}_i^*, \gamma, \mu_{ii}^*, \{s_{ji}^*\}_{j\neq i}) \begin{bmatrix} -\hat{\mathbf{g}}_{ij} \\ 1 \end{bmatrix} \\ = -\gamma \left(\sum_{\substack{j=1 \\ j\neq i}}^{N} s_{ji} + \sigma_i^2 \right) < 0, \end{aligned} \tag{A.9}$$

which contradicts with constraint $\mathbf{\Delta}_{ii} \succeq 0$. Rest of the proof directly follows the proof of proposition 1 in [12].

First assume that $\mathbf{\Psi}_{ii}^*$ is rank one and therefore it can be written as $\mathbf{\Psi}_{ii}^* = \mathbf{v}\mathbf{v}^H$, where $\mathbf{v} \in \mathbb{C}^{n_t+1}$. Then, $\bar{\mathbf{Z}}_i^* = \beta_i^* \mathbf{I}_{n_t} + \sum_{\substack{j=1 \\ j\neq i}}^{N} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{g}}_{ij} \end{bmatrix} \mathbf{\Psi}_{ij}^* \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{g}}_{ij}^H \end{bmatrix} + \sum_{k=1}^{K} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{h}}_{ik} \end{bmatrix} \mathbf{\Lambda}_{ik}^* \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{h}}_{ik}^H \end{bmatrix} \succ 0$, become full ranked and $\frac{1}{\gamma} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{g}}_{ii} \end{bmatrix} \mathbf{v}\mathbf{v}^H \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{g}}_{ii}^H \end{bmatrix}$ becomes rank one matrices. Hence from (A.5),

$$\begin{aligned} \text{Rank}(\mathbf{Z}_i^*) = \\ \text{Rank}\left(\bar{\mathbf{Z}}_i^* - \frac{1}{\gamma} \begin{bmatrix} \mathbf{I}_{n_t} & \hat{\mathbf{g}}_{ii} \end{bmatrix} \mathbf{v}\mathbf{v}^H \begin{bmatrix} \mathbf{I}_{n_t} \\ \hat{\mathbf{g}}_{ii}^H \end{bmatrix} \right) \geq n_t - 1. \end{aligned} \tag{A.10}$$

Using Sylvester's rank inequality we can write following for rank of $\mathbf{W}_i^*$,

$$\text{Rank}(\mathbf{W}_i^*) \leq n_t + \text{Rank}(\mathbf{Z}_i^* \mathbf{W}_i^*) - \text{Rank}(\mathbf{Z}_i^*) \tag{A.11}$$

It follows from (A.4), (A.10) and (A.11) that,

$$0 \leq \text{Rank}(\mathbf{W}_i^*) \leq 1 \tag{A.12}$$

implying $\mathbf{W}_i^*$ must be rank one. What remains is to prove that $\mathbf{\Psi}_{ii}^*$ is indeed rank one. By substituting (18) and (A.2) in to condition (A.5), the following two equalities can be easily obtained,

$$(\mathbf{W}_i^* + \mu_{ii}^* \mathbf{Q}_{ii})\mathbf{A}_{ii} + \mathbf{W}_i^* \hat{\mathbf{g}}_{ii} \mathbf{b}_{ii}^H = \mathbf{0}, \tag{A.13}$$

$$(\mathbf{W}_i^* + \mu_{ii}^* \mathbf{Q}_{ii})\mathbf{b}_{ii} + \mathbf{W}_i^* \hat{\mathbf{g}}_{ii} c_{ii} = \mathbf{0}. \tag{A.14}$$

Moreover, $c_{ii} > 0$ must be satisfied, if not we must have $\mathbf{\Psi}_{ii}^* = \mathbf{0}$ by (A.2) and (A.6) since $\mathbf{Q}_{ii} \succ 0$ which contradicts with (A.5). Post-multiply (A.14) with $-\mathbf{b}_{ii}^H / c_{ii}$ and add the resultant equality to A.13 result in,

$$(\mathbf{W}_i^* + \mu_{ii}^* \mathbf{Q}_{ii})(\mathbf{A}_{ii} - \mathbf{b}_{ii}\mathbf{b}_{ii}^H / c_{ii}) = \mathbf{0}. \tag{A.15}$$

Since $\mu_{ii}^* > 0$ and $\mathbf{Q}_{ii} \succ \mathbf{0}$ the term $(\mathbf{W}_i^* + \mu_{ii}^* \mathbf{Q}_{ii}) \succ \mathbf{0}$, (A.15) implies that $\mathbf{A}_{ii} = \mathbf{b}_{ii}\mathbf{b}_{ii}^H / c_{ii}$. Hence (A.2) transforms to

$$\begin{aligned} \mathbf{\Psi}_{ii}^* = \begin{bmatrix} \mathbf{b}_{ii}\mathbf{b}_{ii}^H / c_{ii} & \mathbf{b}_{ii} \\ \mathbf{b}_{ii}^H & c_{ii} \end{bmatrix} \\ = \begin{bmatrix} \mathbf{b}_{ii} / \sqrt{c_{ii}} \\ \sqrt{c_{ii}} \end{bmatrix} \begin{bmatrix} \mathbf{b}_{ii}^H / \sqrt{c_{ii}} & \sqrt{c_{ii}} \end{bmatrix} \succeq 0, \end{aligned} \tag{A.16}$$

which is indeed a rank-one matrix. This concludes the proof.

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