A maximum dispersion approach for rate feasibility problems in SINR model

Shashi Prabh
School of Engineering
Shiv Nadar University
Dadri, Gautam Buddh Nagar, UP 201314. India
Email: shashi.prabh@snu.edu.in

***Abstract*—In this paper, we propose a maximum dispersion based approach for the problem of determining the feasibility of a set of single-hop rates in SINR model. Recent algorithmic work on capacity maximization has focused on maximizing the number of simultaneously scheduled links. We show that such an approach is not suitable for the problem. We present a polynomial time algorithm to determine the feasibility. We present several simulation results to evaluate the proposed approach. The results confirm that maximum dispersion-based approach is well suited for the problem and that such an approach performs significantly better than the existing work.**

I. Introduction

In this paper, we consider the problem of feasibility of a set of single-hop rates in SINR model. The problem that we address here is described as follows: we are given M directed links in a wireless network of N nodes, and associated with these links is also given a vector of M target data rates $\boldsymbol{\rho} = \{\rho_1, \ldots, \rho_M\}$. Assuming that the senders remain backlogged, the problem asks whether the given link rate vector is feasible, where, by feasibility (formally defined in Sec. II) we mean that the rate vector is achievable over a sufficiently long time horizon provided the SINR at any receiver never falls below the SINR threshold. The SINR model states that transmission from node i to j can be decoded provided the following condition remains true at the receiver j:

$$\mathrm{SINR}_{ij} \triangleq \frac{\mathrm{Signal}}{\mathrm{Interference} + \mathrm{Noise}} \geq \beta \tag{1}$$

The threshold β depends on hardware and is usually modeled as a constant in the literature. Although we adopt this convention here, we note that this work can be easily extended to the case where β varies from receiver to receiver.

Since the set of simultaneously active links will change with time, SINR at a receiver is time-dependent. Let P_i denote the transmit power of node i, and G_{ij} denote the gain at node j. A commonly used channel gain model is a polynomially decaying function of the Euclidean distance, that is, $G_{ij} = d(i,j)^{-\alpha}$, where $d(i,j)$ denotes the Euclidean distance between nodes i and j and α is the path-loss exponent. The value of α depends on the physical environment and usually lies between 2 (free space) and 6. Therefore, the SINR constraint (1) becomes

$$\mathrm{SINR}_{ij}(t) = \frac{P_i G_{ij}}{\sum_{k \in S(t) \setminus \{i\}} P_k G_{kj} + \mathrm{Noise}} \geq \beta, \ \forall t \tag{2}$$

where $S(t)$ denotes the set of nodes transmitting at time t. Shannon's formula specifies the upper bound on the data rate on link (i,j) as:

$$r_{ij}(t) = W \lg(1 + \mathrm{SINR}_{ij}(t)), \tag{3}$$

where W denotes the bandwidth.

Exhaustive search is intractable since there are 2^M subsets of links that can be scheduled simultaneously. As we discuss later in the paper, this problem is NP-hard. Developing tractable algorithms for providing certain rate guarantees had been a problem of great research interest for several decades. This problem has gained added relevance due to the upcoming 5G networks where link density can be very high.

A closely related work is the so-called "emptying the network" problem studied in [1]. Here, each transmitter is assumed to have a finite amount of traffic and the problem is to empty the packet queues as quickly as possible. The main limitation of [1] is that the problem does not allow new arrivals until all queued packets have been transmitted. The problem that we are addressing in this paper does not have this limitation. There exist several studies in the literature where sum-rate maximization is achieved by controlling transmit power [2], [3]. In our problem formulation, we treat transmit power vector as

978-3-901882-90-6/17

a given input. Although sum-rate maximization is one of the steps we take to solve the feasibility problem, maximizing the sum-rate alone leads to starvation, since links on which the data rate is low may never get scheduled (see [4], for example).

There has been a large number of published results that maximize the number of simultaneously scheduled transmissions [5]–[13]. One line of approach used in these papers is to greedily select links based on criteria, such as, link length or certain approximations of interference caused by different links etc. The greedy algorithm presented by Borbash and Ephremides in [5] seems to be the first such work. Another line of approach imposes certain structure on links, e.g., [6], [7]. Yet another approach is taken in [14] which starts with conflict graph and constructs SINR schedule in exponential time. Distributed, though suboptimal, scheduling algorithms using SINR model can be found in [15], [16]. Although some of the above mentioned papers and others in the literature imply that maximizing the number of simultaneously scheduled links maximizes the sum of rates, unfortunately this is not the case.

The following counterexample shows that maximizing the number of links does not necessarily maximize the sum-rate. Consider the two sender-receiver pairs, shown in Figure 1, where $d(u,v) = 1 = d(w,x)$, and the distance between either one of the interfering senders to a receiver $d(u,x) = 3/4 = d(w,v)$. Further, assume that the transmit power to noise ratio is 3, the path loss constant is 3 and SINR threshold is 1. The data rate when any one of the pairs is active is $2W$ bps, whereas the sum of rates when both are active falls to $0.908W$ bps, less than half of the single pair rate even though the SINR values remain above the threshold.

In addition to SINR-based models, there exists vast amount of literature on scheduling in wireless networks where some (non-geometric) graph-based model is used. A survey of these is presented in [17].

Given N locations, the objective of the *max-sum p-dispersion problem* is to find a p-cardinality set $S \subset N$ such that $\sum_{i,j \in S} d(i,j)$ is maximized. This problem has been shown to be NP-hard [18]. There exist greedy algorithms that achieve 2-approximation [19], [20]. In this paper we present a sum-rate maximization algorithm that selects subset of links based on the idea of maximum dispersion and show that the algorithm gives significant performance improvement over the state-of-the-art. We note that for our problem p is no longer a fixed given quantity but becomes a variable instead.

The rest of this paper is organized as follows: in Sec. II we present problem formulation. In Sec. III, we first present our max-sum p-dispersion based algorithm that maximizes the sum of rates, and then an algorithm that determines the feasibility of a target rate vector in polynomial time. We present simulation results in Sec. IV that compare the performance of the proposed algorithm with an optimal algorithm (based on exhaustive search), the algorithm presented in [13] that maximizes the number of simultaneously scheduled links, and a modified version of the max-weight algorithm [21]. We conclude the paper in Sec. V.

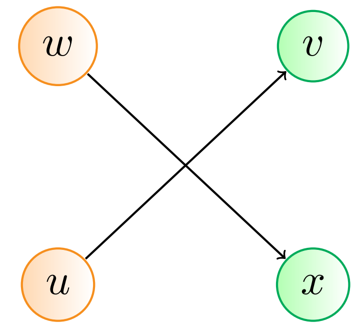


Fig. 1: The sum of rates achieved when both the transmissions $u \to v$ and $w \to x$ are active is less than the rate when only either one is active.

II. Problem formulation

Consider M given directed links in a wireless network. Associated with these links is given a vector of M target data rates $\boldsymbol{\rho} = \{\rho_1, \ldots, \rho_M\}$. We want to determine whether the given target rates can be achieved over a sufficiently large time interval I. We consider the SINR model (2) where the transmit power of senders are given.

Consider unit time interval. We discretize it into $\mathcal{T}$ equal size sub-intervals $T_i, i = 1, 2, \ldots, \mathcal{T}$. Therefore, $\sum_{i=1,\ldots,\mathcal{T}} T_i = 1$. The granularity of the sub-interval duration can be made arbitrarily small by selecting large enough $\mathcal{T}$. A link's last transmission can finish at infinitesimally small duration from the boundary of a time interval wasting $R_{\max}/\mathcal{T}$ capacity (in the limit), where $R_{\max}$ is the maximum rate on a link. Therefore, $\mathcal{T}$ should be chosen according to $\mathcal{T} >> MR_{\max}$. Although $\mathcal{T}$ has no bearing on the time complexity of the maximum dispersion based algorithm presented here, such is not the case in general.

In the following, we denote the link corresponding to ρ_k by k. We denote the sender and receiver nodes of link k by s_k and r_k, respectively. To make the notation of (2) a bit more compact, let $\psi_k(i)$ denote the SINR at node r_k during the i^{th} sub-interval T_i when the transmission

$s_k \rightarrow r_k$ is taking place. We define $\psi_k(i) \triangleq 0$ if $s_k \rightarrow r_k$ is not taking place. We also define $\psi_k(i) \triangleq \infty$ if $s_k \rightarrow r_k$ causes primary interference conflict (a node is either a source or destination of more than one concurrent transmission).

The *feasibility of the rates* $\boldsymbol{\rho}$ can then be defined as

$$\rho_k \leq \sum_{i=1}^{\mathcal{T}} \frac{W}{\mathcal{T}} \lg(1 + \psi_k(i)), \ \forall k.$$

Feasibility over the unit time interval implies that the average data rate over the time horizon I approaches the target data rates. This can be seen by first scaling the time intervals and hence the unit interval to appropriate values (e.g., a slot size on a time-slotted system or average transmission time) and then partitioning I into segments of scaled unit interval size.

Establishing the intractability of the rate feasibility problem is straightforward by reducing from the multidimensional knapsack problem which is known to be NP-hard in strong sense [22].

III. Maximum dispersion based sum-rate maximization

A. Max-Rate-Greedy-Dispersion *Algorithm*

We first present the greedy sum-rate maximization algorithm and follow it up in the next subsection with a procedure that determines whether a given rate vector is feasible.

Max-Rate-Greedy-Dispersion (Algorithm 1) is based on a greedy solution of the max-sum p-dispersion problem [19]. The algorithm selects nodes that maximize the *interferer-receiver* separation. It has been shown that this greedy approach achieves 2-approximation [20].

At each step, the algorithm greedily selects the link whose sender's location maximizes the total distance from all the receivers of links in S, where S denotes the set of links already selected by the algorithm for activation (Algorithm 1, Line 4). A newly selected link l is added to set S if doing so does not cause (a) SINR at any receiver in $S \cup \{l\}$ to exceed the threshold, and (b) the sum of rates does not decrease, that is $R(S \cup \{l\}) \geq R(S)$, where $R(S)$ denotes the sum of rates achieved when all links in set S are activated simultaneously. We denote a link that satisfies both these conditions by $l \perp S$ (Algorithm 1, Line 5).

B. *Rate vector feasibility algorithm*

The rate vector feasibility algorithm Max-Dispersion-Rate-Feasibility takes $\boldsymbol{\rho}$ as input

Algorithm 1 Max-Rate-Greedy-Dispersion

Input: Set of links L ▷ Node positions, transmit power, noise, SINR threshold
1: $S := l \in L \mid l$ has the maximum SNR
2: $\bar{S} := \phi$
3: **while** $|S| + |\bar{S}| < |L|$
4: find $l \in L \backslash (S \cup \bar{S})$ that maximizes $\sum_{k \in S} d(s_l, r_k)$
5: **if** $l \perp S$ ▷ Explained in the text
6: $S := S \cup \{l\}$
7: **else**
8: $\bar{S} := \bar{S} \cup \{l\}$
9: **end if**
10: **end while**

Output: S ▷ Output S to the rate feasibility algorithm (step 5 of Algorithm 2)

and keeps track of remaining rates $\boldsymbol{\rho'}$ (initialized to $\boldsymbol{\rho}$) as links are scheduled. At step 5 the algorithm calls the sum-rate maximization algorithm Max-Rate-Greedy-Dispersion and passes it the set of links that have non-zero remaining target rate which returns a set of links S. Unless the residual rate on at-least one of the links become 0, S remains optimal. Therefore, the feasibility algorithm finds the minimum number of time intervals $\delta_{\min}$ when the target rate on at-least one of the links in S is met. The algorithm then subtracts from the residual rate vector $\boldsymbol{\rho'}$ the rate provided to the links in S during $\delta_{\min}$ time up-to a maximum ρ'_k for all k in S. Feasibility is indicated by $\boldsymbol{\rho'} = \mathbf{0}$.

C. *Time complexity analysis*

We pre-compute an $M \times M$ table Ψ whose $(k, l)^{\text{th}}$ entry is $P_{s_k} G_{s_k, r_l}$. Thus, $\Psi[k, l]$ is the signal on the receiver of link l if $k = l$, and is the interference contribution of the sender of link k otherwise. We maintain a table of the current sum of interference values $\Psi[k, l]$ of each $l \in S$ which is reset at each new call of the rate feasibility algorithm (Algorithm 2, Line 14). Thus, checking $m \perp S$ (in Algorithm 1, Line 5) takes $O(|S|)$ time. Hence the time complexity of Max-Rate-Greedy-Dispersion is $O(M^2)$. Upon each iteration of the Algorithm 2 (line 4), either residual rate becomes 0, or the rate vector is found infeasible. Thus the time complexity of Max-Dispersion-Rate-Feasibility combined with Max-Rate-Greedy-Dispersion is $O(M^3)$.

Algorithm 2 MAX-DISPERSION-RATE-FEASIBILITY

Input: $\boldsymbol{\rho}$ ▷ Rate vector $\rho_1, \ldots, \rho_M$
Input: $\mathcal{T}$ ▷ Time granularity = $1/\mathcal{T}$
Input: Algorithm $A(\boldsymbol{\rho})$ ▷ A finds a set of links S that can be scheduled simultaneously while satisfying SINR constraint at all receivers in S
$\boldsymbol{\rho'} := \boldsymbol{\rho}$ ▷ Residual rate vector
$\boldsymbol{\delta}$ ▷ Transmission duration vector $\delta_1, \ldots, \delta_M$
$T := 0$
while ($T < \mathcal{T}$) and ($\boldsymbol{\rho'} \neq \mathbf{0}$)
  $S := A(\boldsymbol{\rho'})$ ▷ Need to find a subset S that maximizes sum of rates. We'll use MAX-RATE-GREEDY-DISPERSION here.
  $\boldsymbol{\delta} := \boldsymbol{\infty}$
  do for each $k \in S$
    $\delta_k := \lceil \frac{\rho'_k \mathcal{T}}{W \lg(1+\psi_k)} \rceil$
  end do
  $\delta_{\min} := \mathrm{Min}_k \delta_k$
  do for each $k \in S$
    $\rho'_k := \mathrm{Max}\{0, \rho'_k - \delta_{\min} \frac{W}{\mathcal{T}} \lg(1+\psi_k)\}$
  end do
  $T := T + \delta_{\min}$
end while
Output: **if** $\boldsymbol{\rho'} = \mathbf{0}$, $\boldsymbol{\rho}$ is feasible

IV. EVALUATION

A. Comparison algorithms

We evaluated the performance of the rate feasibility algorithm where different algorithms were used to find the set of links that maximized sum-rate (step 5 of Algorithm 4). Note that MAX-DISPERSION-RATE-FEASIBILITY is the same as General rate vector feasibility algorithm (Algorithm 4) except that the former is optimized for use along-with algorithms like MAX-RATE-GREEDY-DISPERSION that are agnostic to the magnitude of elements of the rate vector. We compared the performance of MAX-RATE-GREEDY-DISPERSION with the algorithm of [13], referred here as as MAX-LINKS and a modified version of maximum weight scheduling algorithm [21] described below. We chose [13] because the evaluations presented in the paper show that their algorithm preforms better than algorithms proposed earlier.

Maximum weight scheduling algorithm [21] schedules links essentially in decreasing order of packet queue size in a conflict-free manner. It is one of the most studied wireless network scheduling algorithms, and as a result several variations of this algorithm exist in the literature. We define a conflict free schedule as one where SINR criteria is satisfied at the receivers of all scheduled links. We refer to this algorithm as MAX-WEIGHT-SINR. Further, similar to MAX-RATE-GREEDY-DISPERSION (Algorithm 1, Line 5), we add a link l to the scheduled link set S only if it does not cause decrease in the sum-rate. We denote a link that satisfies the SINR and monotonically non-decreasing sum-rate criteria by $l \perp S$. We refer to the algorithm along-with the later modification as MAX-WEIGHT-SINR-SR, described in Algorithm 3.

We compared the performance of both these versions of max-weight (Fig. 2). Although on less dense instances MAX-WEIGHT-SINR-SR performs better than MAX-WEIGHT-SINR which considers only conflict freeness, we found that the performance of MAX-WEIGHT-SINR-SR was only slightly better in general. We have used MAX-WEIGHT-SINR-SR in further evaluations.

Algorithm 3 MAX-WEIGHT-SINR-SR

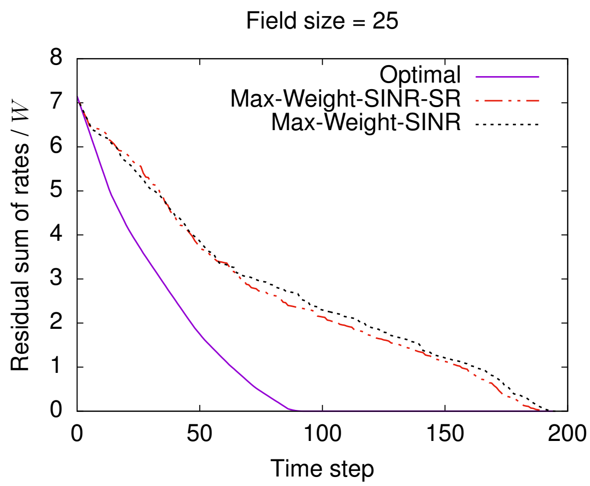
Input: Links: $L = \{l_1, \ldots l_L\}$
▷ Sorted in non-increasing residual rate order.
  $S := \phi$
  for $i = 1$ to $i \leq |L|$ **do**
    if $l_i \perp S$ ▷ Explained in the text
      $S := S \cup \{l_i\}$
    end if
    $i := i + 1$
  end for
Output: S ▷ Output S to the rate feasibility algorithm (step 4 of Algorithm 4)

B. Simulation set-up

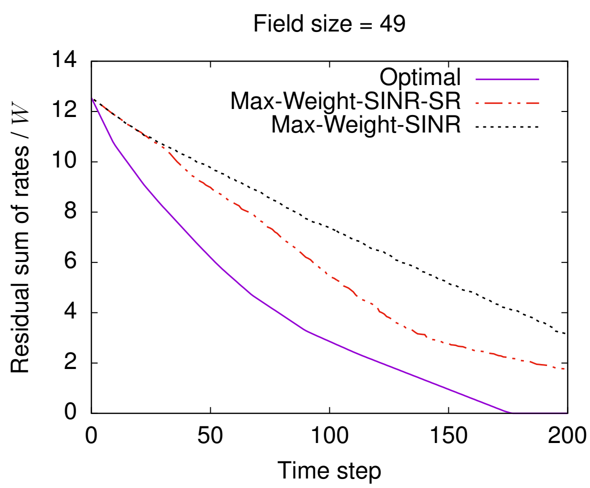
We distribute nodes in a plane according to Poisson point process. We designate node-pairs as links by selecting a random pair within a given maximum link length. The reported network dimensions are normalized in the unit of this distance. We assign randomized target rates to the links distributed uniformly in the range $0.01W$ to twice the average rate (reported with plots). In all experiments, the SINR threshold $\beta = 1$, path loss exponent $\alpha = 3$ and noise was 0.01. We logged the residual sum of rates at the end of each time interval for all the algorithms, which is shown in the figures.

C. Comparison with optimal

Figure 3 shows the performance of MAX-RATE-GREEDY-DISPERSION, MAX-LINKS and MAX-WEIGHT-SINR-SR on feasible single-hop rate vectors



(a) Average rate = $0.3W$



(b) Average rate = $0.7W$

Fig. 2: Max-Weight with and without consideration of sum-rate increase. The number of links was 20. The details of this simulation set-up was the same as that of Fig. 3, which is described below.

on a set of 20 links. We computed the set of links that each of the algorithms schedules at each time step. We also performed exhaustive search at each time step to determine the subset of links that led to the maximum reduction in the residual sum of rates. The results obtained by exhaustive search are labeled as *Optimal* in the figures. We used Optimal along-with Algorithm 4 since we were logging residual sum of rates at each time step. To lower the time complexity, Algorithm 2 can be used with Optimal but with the following modifications: replacing ceiling with floor function in step 8 and

Algorithm 4 General rate vector feasibility algorithm

Input: $\boldsymbol{\rho}$ $\triangleright$ Rate vector $\rho_1, \ldots, \rho_M$
Input: $\mathcal{T}$ $\triangleright$ Time granularity = $1/\mathcal{T}$
Input: Algorithm $A(\boldsymbol{\rho})$ $\triangleright$ A finds a set of links S that can be scheduled simultaneously while satisfying SINR constraint at all receivers in S

$\boldsymbol{\rho'} := \boldsymbol{\rho}$ $\triangleright$ Residual rate vector
$T := 0$
while ($T < \mathcal{T}$) and ($\boldsymbol{\rho'} \neq \mathbf{0}$)
  $S := A(\boldsymbol{\rho'})$ $\triangleright$ Need to find a subset S that maximizes sum of rates, e.g., MAX-WEIGHT-SINR-SR.
  do for each $k \in S$
    $\rho'_k := \mathrm{Max}\{0, \rho'_k - \frac{W}{\mathcal{T}} \lg(1 + \psi_k)\}$
  end do
  $T := T + 1$
end while

Output: if $\boldsymbol{\rho'} = \mathbf{0}$, $\boldsymbol{\rho}$ is feasible

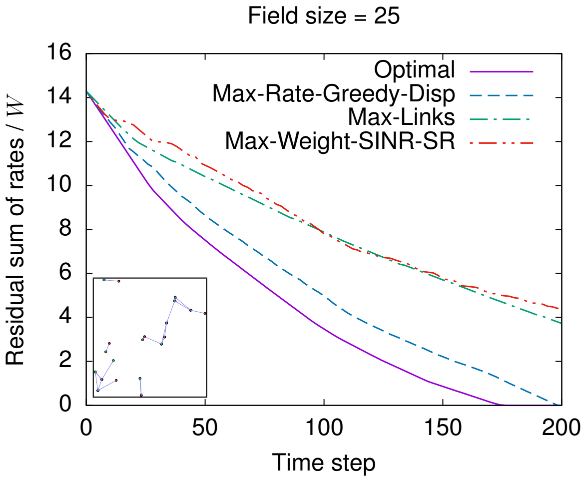
redefining $\delta_{\min}$ as $\mathrm{Max}\{1, \mathrm{Min}_k \delta_k\}$.

We report the results for three different field sizes. Increasing the field size generally implies that the links are more spread-out resulting in larger sizes of link subsets that can be activated simultaneously. The transmit power in these evaluations was set to $\beta(\mathrm{Noise} + l_{\max}^{\alpha})$, where $l_{\max}$ is the maximum link length. We present the results for non-uniform transmit power in sec. IV-F. The inset figures in the plots show the network topology.

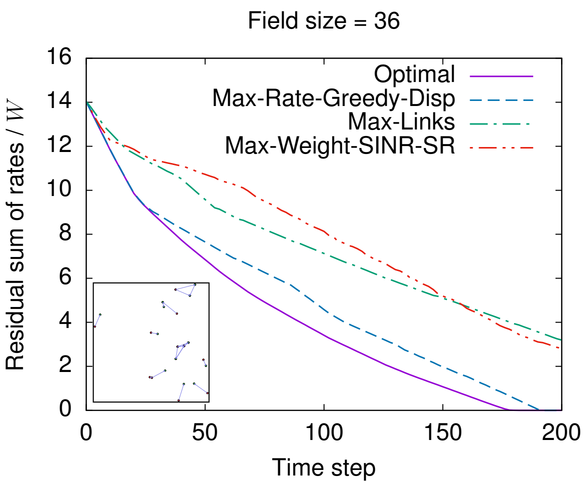
The data obtained from optimal search show that the rate vector was feasible in all three settings. MAX-RATE-GREEDY-DISPERSION gives the smallest residual sum of rates among all algorithms except the optimal. Further, its curve follows the optimal curve much more closely than the other two algorithms.

D. Performance in large networks

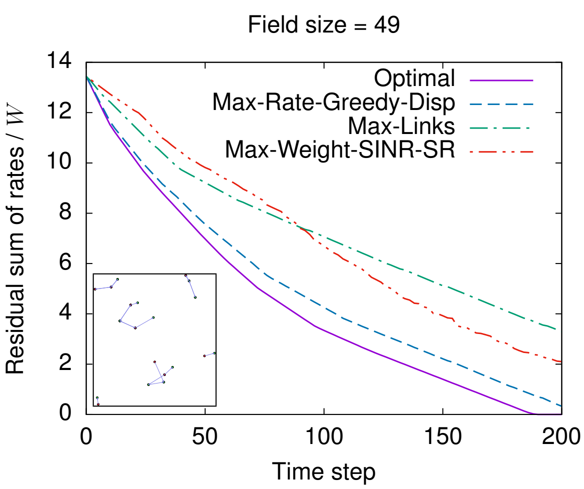
Since exhaustive search was not feasible for larger networks, here we compared the performance of MAX-RATE-GREEDY-DISPERSION with that of MAX-LINKS and MAX-WEIGHT-SINR-SR only. The data, shown in Figure 4, confirm that the maximum dispersion based strategy is effective in large networks. We also find that the comparative performance of MAX-RATE-GREEDY-DISPERSION gets better as the number of links is increased. Though Max-Weight has nice properties, it does not consider geometry at all. This seems to be reason why its modified version performs poorly here.



(a) Average rate = $0.6W$

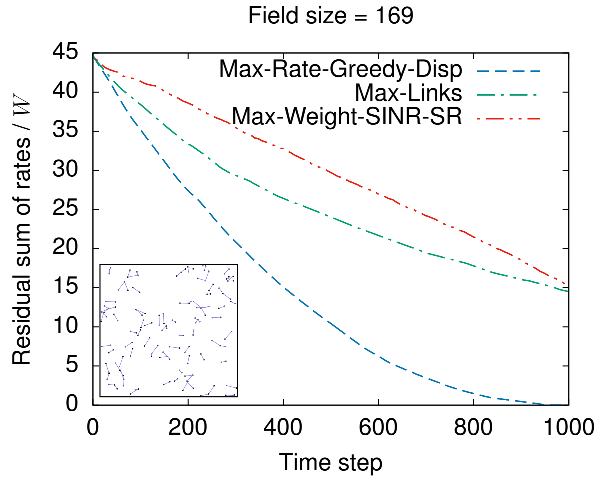


(b) Average rate = $0.7W$

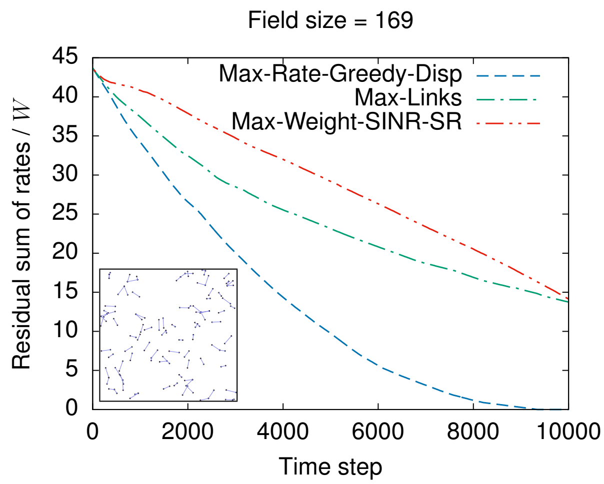


(c) Average rate = $0.75W$

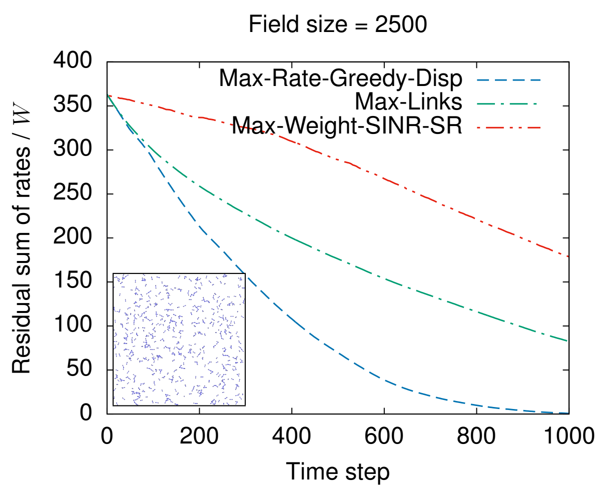
Fig. 3: No. of links = 20. Figures in the order of decreasing density of nodes. Network topology is shown in the inset.



(a) 100 links, average rate = $0.46W$.



(b) 100 links, average rate = $0.46W$. T=10000



(c) 1000 links, average rate = $0.38W$

Fig. 4: Effectiveness of maximum dispersion strategy in large networks.

E. Discretization of time

The first two plots in Figure 4 show the data obtained on the same network and link rate vector but using time granularities of $1/\mathcal{T} = 10^{-3}$ and 10^{-4}, respectively. MAX-RATE-GREEDY-DISPERSION declares the rate vector feasible at 0.956 and at 0.9334 for the time granularities 10^{-3} and 10^{-4}, respectively. These two plots were obtained for a network that had 100 links. Observe that the ratio $M/\mathcal{T}$ is only 10 in the first case.

F. Non-uniform transmit power

Fig. 5 shows the results for non-uniform transmit power setting. The transmitting node's power was set to the link length raised to the power α. This is equivalent to normalizing the signal strength on the links to 1. In the plots shown here as well as several other settings, the dispersion based strategy performs better than the case of uniform transmit power setting, primarily because of reduced interference.

V. CONCLUSION

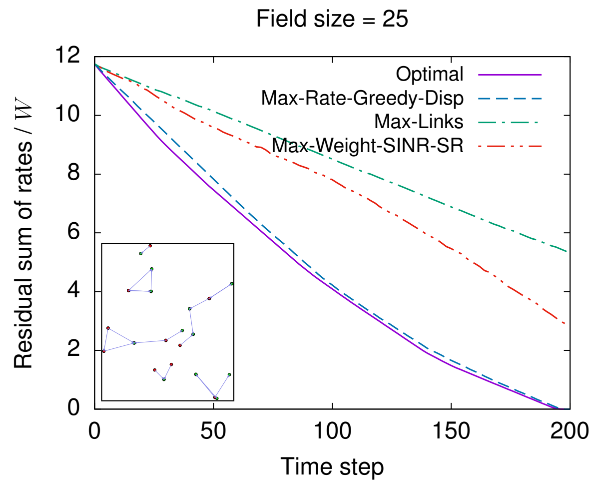
We discussed why maximizing the number of concurrently scheduled links does not necessarily maximize the sum-rate. We proposed a new polynomial time strategy inspired by max-sum p-dispersion results. Comparisons with the exhaustive search indicate that the proposed algorithm preforms better than the state-of-the-art in finding the set of links that maximize the sum-rate. Although algorithmic aspects of wireless capacity, as well as link scheduling, are well studied problems, to the best of our knowledge, any work on these problems using the idea of maximum-dispersion has not appeared in the literature. We note that the design of MAX-RATE-GREEDY-DISPERSION does not take channel into account. Study of maximum-dispersion based algorithms where channel is taken into account explicitly will appear elsewhere. Development of a distributed version of MAX-RATE-GREEDY-DISPERSION where node locations are not provided apriori is another future work.

VI. APPENDIX: MAX-LINKS [13]

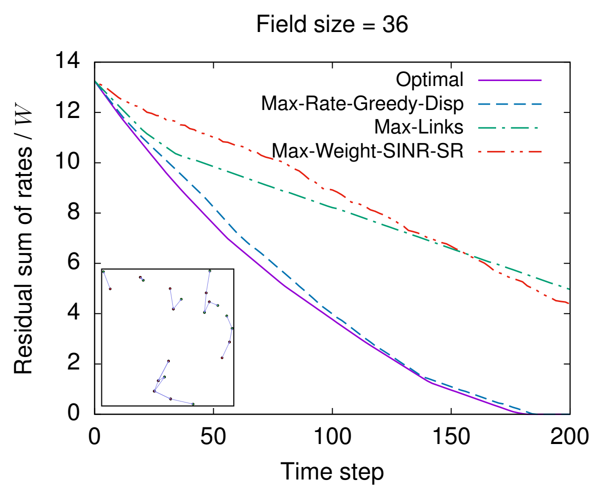
This section summarizes the algorithm of [13] that we have used in the evaluation. The algorithm assumes uniform transmit power P on all links. It defines relative interference $RI_j(k) = P_{jk}/P_{kk}$, where $P_{jk} = P/d(s_j, r_k)^\alpha$ and $\alpha > 2$. For a given set of links S, $a_S(k)$ for a link k is defined as

$$a_S(k) = \frac{1}{1 - \beta\eta/P_{kk}} \sum_{j \in S} RI_j(k).$$

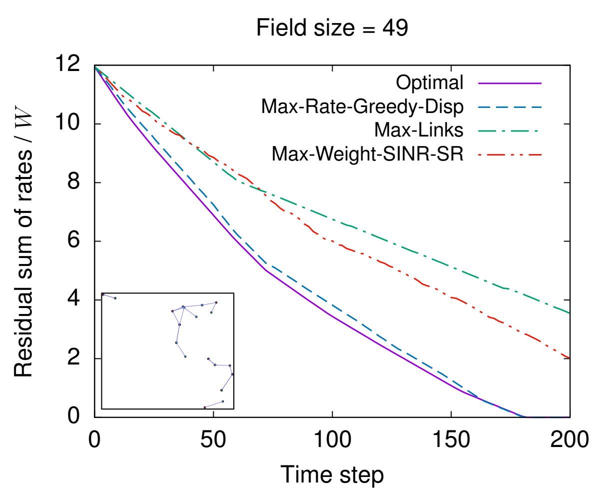
The rest of the steps is shown in Algorithm 5.



(a) Average rate = $0.50W$



(b) Average rate = $0.65W$



(c) Average rate = $0.65W$

Fig. 5: Non-uniform transmit power. No. of links = 20. Figures in the order of decreasing density of nodes.

Algorithm 5 MAX-LINKS

Input: Set of links: L

1: = Sort the links in non-decreasing length order $\{l_1, \ldots l_{|L|}\}$

2: $c = max\left(2, \left(288\beta\frac{\alpha-1}{\alpha-2}\right)^{1/\alpha}\right)$

3: $S := \phi$

4: **for** $v := 1$ to $|L|$ **do**

5: **if** $a_S(l_v) \leq 2/3$ and $d(s_w, r_v) > cd(s_w, r_w) \forall l_w \in S$

6: $S := S \cup l_v$

Output: S $\triangleright$ Output S to step 5, Algorithm 2

REFERENCES

[1] Q. He, D. Yuan, and A. Ephremides, "A general optimality condition of link scheduling for emptying a wireless network," in *2016 IEEE International Symposium on Information Theory (ISIT)*, July 2016, pp. 1446–1450.

[2] L. Zhang, Y. Xin, Y.-C. Liang, and H. V. Poor, "Cognitive multiple access channels: optimal power allocation for weighted sum rate maximization," *IEEE Transactions on Communications*, vol. 57, no. 9, pp. 2754–2762, 2009.

[3] C. W. Tan, M. Chiang, and R. Srikant, "Fast algorithms and performance bounds for sum rate maximization in wireless networks," *IEEE/ACM Transactions on Networking*, vol. 21, no. 3, pp. 706–719, June 2013.

[4] Y. Wang and G. de Veciana, "Improving user perceived QoS in D2D networks via binary quantile opportunistic scheduling," in *2016 14th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt)*, May 2016, pp. 1–8.

[5] S. A. Borbash and A. Ephremides, "Wireless link scheduling with power control and SINR constraints," *IEEE Transactions on Information Theory*, vol. 52, no. 11, pp. 5106–5111, Nov 2006.

[6] T. Moscibroda, R. Wattenhofer, and A. Zollinger, "Topology control meets SINR: the scheduling complexity of arbitrary topologies," in *Proceedings of the 6th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2006, pp. 310–321.

[7] O. Goussevskaia, Y. A. Oswald, and R. Wattenhofer, "Complexity in geometric SINR," in *Proceedings of the 8th ACM International Symposium on Mobile Ad Hoc Networking and Computing (MobiHoc)*, 2007, pp. 100–109.

[8] D. Chafekar, V. S. A. Kumar, M. V. Marathe, S. Parthasarathy, and A. Srinivasan, "Approximation algorithms for computing capacity of wireless networks with SINR constraints," in *INFOCOM 2008. The 27th Conference on Computer Communications. IEEE*, April 2008.

[9] M. Andrews and M. Dinitz, "Maximizing capacity in arbitrary wireless networks in the SINR model: Complexity and game theory," in *IEEE INFOCOM 2009*, April 2009, pp. 1332–1340.

[10] D. M. Blough, G. Resta, and P. Santi, "Approximation algorithms for wireless link scheduling with SINR-based interference," *Networking, IEEE/ACM Transactions on*, vol. 18, no. 6, pp. 1701–1712, 2010.

[11] S. Kompella, J. Wieselthier, A. Ephremides, H. Sherali, and G. Nguyen, "On optimal SINR-based scheduling in multihop wireless networks," *Networking, IEEE/ACM Transactions on*, vol. 18, no. 6, pp. 1713 –1724, Dec. 2010.

[12] T. Kesselheim, "A constant-factor approximation for wireless capacity maximization with power control in the SINR model," in *Proceedings of the Twenty-second Annual ACM-SIAM Symposium on Discrete Algorithms*, ser. SODA '11. Philadelphia, PA, USA: Society for Industrial and Applied Mathematics, 2011, pp. 1549–1559. [Online]. Available: http://dl.acm.org/citation.cfm?id=2133036.2133156

[13] O. Goussevskaia, M. Halldórsson, and R. Wattenhofer, "Algorithms for wireless capacity," *IEEE/ACM Trans. Netw.*, vol. 22, no. 3, pp. 745–755, Jun. 2014. [Online]. Available: http://dx.doi.org/10.1109/TNET.2013.2258036

[14] Q.-S. Hua and F. C. Lau, "Exact and approximate link scheduling algorithms under the physical interference model," in *Proceedings of the Fifth International Workshop on Foundations of Mobile Computing*, ser. DIALM-POMC '08. New York, NY, USA: ACM, 2008, pp. 45–54. [Online]. Available: http://doi.acm.org/10.1145/1400863.1400874

[15] Y. Zhou, X. Y. Li, M. Liu, Z. Li, S. Tang, X. Mao, and Q. Huang, "Distributed link scheduling for throughput maximization under physical interference model," in *2012 Proceedings IEEE INFOCOM*, March 2012, pp. 2691–2695.

[16] D. Qian, D. Zheng, J. Zhang, and N. Shroff, "CSMA-Based distributed scheduling in multi-hop MIMO networks under SINR model," in *2010 Proceedings IEEE INFOCOM*, March 2010, pp. 1–9.

[17] A. Pantelidou and A. Ephremides, "Scheduling in wireless networks," *Foundations and Trends in Networking*, vol. 4, no. 4, pp. 421–511, 2011. [Online]. Available: http://dx.doi.org/10.1561/1300000030

[18] D. Wang and Y.-S. Kuo, "A study on two geometric location problems," *Information processing letters*, vol. 28, no. 6, pp. 281–286, 1988.

[19] S. S. Ravi, D. J. Rosenkrantz, and G. K. Tayi, "Heuristic and special case algorithms for dispersion problems," *Operations Research*, vol. 42, no. 2, pp. 299–310, 1994.

[20] A. Borodin, H. C. Lee, and Y. Ye, "Max-sum diversification, monotone submodular functions and dynamic updates," in *Proceedings of the 31st ACM SIGMOD-SIGACT-SIGAI Symposium on Principles of Database Systems*, ser. PODS '12. New York, NY, USA: ACM, 2012, pp. 155–166. [Online]. Available: http://doi.acm.org/10.1145/2213556.2213580

[21] L. Tassiulas and A. Ephremides, "Stability properties of constrained queueing systems and scheduling policies for maximum throughput in multihop radio networks," *IEEE transactions on automatic control*, vol. 37, no. 12, pp. 1936–1948, 1992.

[22] G. V. Gens and E. V. Levner, *Computational complexity of approximation algorithms for combinatorial problems*. Berlin, Heidelberg: Springer Berlin Heidelberg, 1979, pp. 292–300.