Evolutionary Dynamics of Cooperative Sensing in Cognitive Radios Under Partial System State Information

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***Abstract*—Cooperative sensing enables secondary users to combine individual sensing results in order to attain sensing accuracies beyond those achieved by consumer RF devices. However, due to sensing costs, secondary users may prefer not to cooperate to the sensing task, leading to higher false alarm probability. In this paper, we study how information about the presence of cooperators affects the dynamics of cooperative sensing schemes. We consider two scenarios, namely the case when SUs cannot detect the presence of other potential cooperators, and the case when SUs have prior information on the presence of other SUs in radio range. Using an evolutionary game framework, we demonstrate that protocols delivering such type of information to SUs reduce cooperation and ultimately lead to degraded network performance. Finally, a learning process based on the replicator dynamics is proposed which is capable to drive the system to the evolutionary stable solution. The results of the paper are illustrated through numerical simulations.**

***Index Terms*—Cooperative sensing, cognitive radio, evolutionary games, evolutionary stable strategy, replicator dynamics**

I. Introduction

Since its introduction, cognitive radio (CR) has attracted a fair amount of interest from both academia and industry. CR is still regarded as a promising technology that enables secondary users (SUs) – also called unlicensed users – to opportunistically access spectrum resources. As highlighted by the US federal regulation authority [1], in fact, a large amount of spectrum is underutilized resulting in wasted resources. Currently, spectrum policies adopted worldwide allow licensed users, only, to operate over a fixed and predetermined spectrum bandwidth. Whereas, it is more efficient, under some appropriate conditions, to switch some traffic demand over temporary available resources. This is the case, e.g., of channels dedicated to TV broadcast services: those can be reallocated to SUs as specified by IEEE 802.22 standard [2].

One fundamental issue related to CR is the design of a reliable and robust spectrum sensing mechanism. Spectrum sensing enables SUs to accurately detect available channels and avoid severe interference with Primary Users (PUs), i.e., licensed users. Indeed, due to the random nature of wireless media, several spectrum sensing errors may occur when a single SU – possibly equipped with consumer electronics – performs sensing tasks. The sensing error rate, however, can be reduced considerably by combining channel observations of multiple SUs by means of *cooperative spectrum sensing*.

This paper tackles the cooperative spectrum sensing paradigm, briefly *cooperative sensing*, from a game theoretical perspective. Indeed, CR has been extensively modeled through game theory over the past decade [3]. Compared to existing results, the present work captures fundamental tradeoffs between false alarm probability and costs related to the sensing task when network configuration changes over time.

In particular, our analysis incorporates the knowledge available at the SUs on the network status, e.g., the presence of other SUs in radio range. Indeed, in a dynamic environment, SUs may join or leave the network at any point in time according to energy constraints, mobility, and quality of service (QoS) requirements. As a result, the evolution of SUs' behaviors need to be integrated into the analysis of cooperative sensing dynamics. To this respect, evolutionary games (EGs) provide a framework and analytical tools to study the properties of this type of systems.

The rest of the paper is organized as follows. In the next section, we revise related works. Section III describes the system model. Section IV introduces the EG formalism and models cooperative sensing as an EG. Section V studies the existence and uniqueness of ESS when SUs may know or not whether they are alone. In Section VI, the results of the paper and the performance of the replicator dynamics are discussed through numerical simulations. Finally, concluding remarks and possible extensions of this work are provided.

II. Related works

The theory of evolutionary games dates back to the 70's, when biologist John Maynard Smith [4] used game theory to model animal interactions. Compared to classical game theory, EG theory studies the diffusion and stability properties of a set of strategies adopted within a population of agents. A core concept we adopt in this work is that of an *evolutionary stable strategy* (ESS). The ESS describes a distribution of strategies which persists over time over the population. It hence represents a notion of stable Nash equilibrium. In particular, when the ESS is reached, deviant strategies are

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not adopted. Instead, they tend to vanish against the frequency of strategies which determine the ESS equilibrium. Furthermore, the system evolution is well described by the *replicator dynamics*. Such equation, as introduced later in the paper, provides a model to approach the dynamics by which strategies are adopted over time.

Evolutionary games, traditionally confined to biology and economy, have recently gained attention from the wireless communications research community. In [5], Tembine et *al.* adopt EG in order to model channel access in slotted Aloha networks and power control in W-CDMA systems. Cooperation between cognitive radio relays using an evolutionary framework is studied in [6]. In that work, SUs with different channel conditions decide whether to cooperate or not in order to select the best primary channel.

As the performance of the network depends on both spectrum sensing and spectrum access, authors of [7] design a joint sensing and access algorithm for both synchronous and asynchronous systems. The problem of spectrum sensing scheduling is also studied in [8] and [9] using EG.

In [10], an optimal listen-sleep mechanism is introduced using EG to save energy for cognitive radio sensor networks. A punishment mechanism is proposed to encourage SUs with high quality of service to perform the sensing task.

Contribution. In [11], we have considered the *sense-or-not-to-sense* game as a one-shot volunteer dilemma. In that work, a counter-intuitive property has been proved: the probability to volunteer for the sensing task decreases when the number of SUs increases. In this work, we consider the evolutionary version of the volunteer dilemma and analyze the dynamical properties of the cooperative sensing system. We are able to tie together the tradeoff between the false alarm probability, the sensing cost, the amount of knowledge available at the radio terminals, and the spatial distribution of the SUs. Overall, the main contributions of this work are resumed hereafter:

1) the *sense-or-not-to-sense* game is modeled as an EG able to incorporate the tradeoff between SU's sensing cost and the cooperative false alarm probability;
2) conditions for the existence and uniqueness of ESSs are devised for two scenarios:
   - scenario 1: the SU has no information about existing cooperators,
   - scenario 2: the SU knows when it is alone with probability one;

III. System model

We consider $M \geq 1$ SUs able to opportunistically access a single primary channel. Each SU is equipped with a transceiver: the transceiver can be tuned on the primary channel in order to check if it is free or not. The SUs adopt a distributed scheme by which each device decides on the channel's availability independently by gathering its sensing results and the results of its nearby SUs. Each cognitive radio transceiver is equipped with an energy-based spectrum sensing detector. Therefore, a licensed user is supposed to occupy the channel if its signal energy is measured above a given threshold. We suppose that the sensing results of individual SUs are shared on the same transmission channel during a coordinated *reporting phase*. In this work, we focus on the *sensing phase*; the *reporting phase* and the *access phases* are not taken into account in the analysis. More details about such phases can be found in [11].

The performance of the sensing scheme is measured by the false alarm probability: it is defined as the probability to sense the channel busy while it is idle. The false alarm probability of a single SU depends on both the time of sensing and the detection threshold. In this paper, we assume the sensing duration and the energy detection threshold to be assigned. Hence, the false alarm probability of SU i is a fixed parameter denoted f_i.

Furthermore, in order to combine the sensing results, the SUs adopt the "AND" combining rule. Thus, the cooperative false alarm probability writes

$$f^c_{\text{AND}}(\mathcal{S}) = \prod_{i \in \mathcal{S}} f_i, \tag{1}$$

where $\mathcal{S}$ denotes the set of SUs taking part to the sensing task.

A tradeoff exists between the cooperative false alarm probability and the cost of performing the sensing task. In each time slot, a cooperator SU i among SUs consumes a normalized amount of energy $0 < C_i \leq 1$ in order to sense and share its sensing results with nearby SUs. At the same time, the SU improves the complementary false alarm probability $1 - f^c_{\text{AND}}(\mathcal{S})$, thus improving sensing accuracy.

The payoff U_i of cognitive user i depends on the set of strategies $\mathcal{A} = \{\text{S}, \text{NS}\}$, where "S" is the strategy of the SU when it *senses* the channel, and "NS" the strategy when it does *not sense*. The notation (a_i, a_{-i}) indicates the strategy of cognitive user i facing the opponents' strategy vector a_{-i}. The payoff of SU i is described as follows

$$U_i(a_i, a_{-i}) = \begin{cases} -r & \text{if } \mathcal{S} = \emptyset \\ (1 - \prod_{j \in \mathcal{S}} f_j) & \text{if } a_i = \text{NS}, \mathcal{S} \neq \emptyset \\ (1 - C_i)(1 - \prod_{j \in \mathcal{S}} f_j) & \text{otherwise}, \end{cases}$$

where $0 \leq r \leq 1$ is the regret of a user when no SU senses the channel.

In the remainder of the paper, we make a homogeneity assumption: SUs have same sensing cost C and false alarm probability f. For notation's sake, we denote U instead of U_i. This seemingly simplified setup yet leads to interesting insights. The extensions to the heterogeneous case and more general combining rules are left as part of future works.

IV. Evolutionary game model

In this section, we define the formal components of the adopted evolutionary game framework and formulate the cooperative sensing as an EG.

A. Preliminaries

We consider a dynamical system where SUs, namely the players, change their strategies ("S" and "NS") depending on the strategies of the players they interact with over time. The SUs meet each other at random in a large population of χ players. We hence assume that, at each sensing time, the number of interacting players is a random variable χ that takes values in $\{0,\dots,M-1\}$; we let $M=\infty$ in the case of infinite support. Let $p_m = \mathbb{P}(\chi = m)$ be the probability to meet $0 \le m \le M-1$ players.

We denote by $\mathbf{x}(t) = (x(t), 1-x(t))$ the 2-dimensional vector that describes the state of the population over time; $x(t) \in [0,1]$ is the fraction of the population that senses the channel at time t. For notation's sake, we will remove the dependency on t and use x instead of $x(t)$. We observe that x can also be seen as a mixed strategy, i.e., the probability to choose the "S" strategy, or equivalently as the frequency to sense the channel. Next, we present some basic elements of EG.

With standard notation, the probability-generating function of random variable χ writes $\mathcal{G}_\chi(x) := \sum_{m=0}^{M-1} p_m x^m$.

1) Fitness: the fitness of a tagged player with respect to strategy $a_j \in \mathcal{A}$ and population's state x is the expected payoff it receives by playing strategy a_j and when interacting with a population of M SUs in state x. In general, χ is a random variable with a given distribution. Therefore, the fitness writes

$$g(a_j, x) = \sum_{m=0}^{M-1} p_m \, \mathbb{E}[U(a_j, x)], \tag{2}$$

where $\mathbb{E}[U(a_j,x)]$ is the expected payoff of the player with respect to the random strategies of the opponents. We define $G : [0,1]^2 \to \mathbb{R}$ the overall fitness by the following

$$G(y,x) = y\, g(\mathrm{S}, x) + (1-y)\, g(\mathrm{NS}, x). \tag{3}$$

2) ESS: is a population state that cannot be invaded by a minority group using another strategy. A formal definition can be found in [5]:

Definition 1 (Evolutionary stable strategy)**.** *[5] A population state x^* is an ESS if $\forall\, \mathrm{mut} \neq x^*$, there exists an $\epsilon_{\mathrm{mut}} \in [0,1]$ (which may depend on* mut*), such that $\forall \epsilon \in [0, \epsilon_{\mathrm{mut}}]$*

$$G(x^*, \epsilon\,\mathrm{mut} + (1-\epsilon)\,x^*) > G(\mathrm{mut}, \epsilon\,\mathrm{mut} + (1-\epsilon)\,x^*) \tag{4}$$

A sufficient pair of conditions ensures the existence of an ESS: strategy x^* is an ESS if it satisfies

1. ESS is a Nash equilibrium i.e.

$$G(x^*, x^*) \ge G(\mathrm{mut}, x^*) \quad \forall \mathrm{mut} \in [0,1] \tag{5}$$

1. and x^* is stable: if $\mathrm{mut} \neq x^*$

$$G(x^*, x^*) = G(\mathrm{mut}, x^*) \Rightarrow G(\mathrm{mut}, \mathrm{mut}) < G(x^*, \mathrm{mut}) \tag{6}$$

The first condition ensures that, at the ESS, no mutant strategy can invade the population, and the second guarantees that the mutants will vanish over time.

3) Replicator dynamics: it is a representation of the population state evolution over time. Many dynamics can be found in the literature [5]. They describe the evolution of x over time: the replicator equation, in particular, indicates that the growth of a strategy is proportional to how this strategy is successful. The replicator dynamics is given by

$$\dot{x} = \mu\, x\big(g(S,x) - G(x,x)\big), \tag{7}$$

where $\dot{x} = dx/dt$, and μ is a tuning parameter in $[0,1]$.

B. Game model

We analyze the performance of the cooperative sensing scheme under either full information or partial information about the presence of potential cooperators.

Our aim is to compare the behavior of a tagged SU in the following opposite scenarios: when a SU can detect that no other SUs are in radio range (singleton detection), and when such estimation is not possible.

1) Scenario 1 – SU with no singleton detection (NoSD): The SU has no prior knowledge about the existence of other cognitive users who may participate in the sensing task. E.g., the CR protocol employed does not require SUs to signal their presence to other SUs beforehand. From equation (2), we can derive the fitness g_1 of a given SU with respect to both strategies *sense* and *not to sense*

$$\begin{aligned} g_1(\mathrm{S}, x) &= \sum_{m=0}^{M-1} p_m \sum_{n=0}^{m} \binom{m}{n} x^n (1-x)^{m-n} (1 - f^{n+1})(1-C) \\ &= 1 - C - f(1-C)\mathcal{G}_\chi(xf + (1-x)), \end{aligned} \tag{8}$$

$$\begin{aligned} g_1(\mathrm{NS}, x) &= \sum_{m=0}^{M-1} p_m \sum_{n=1}^{m} \binom{m}{n} x^n (1-x)^{m-n} (1 - f^n) - r \sum_{m=1}^{M-1} p_m (1-x)^m \\ &= 1 - \mathcal{G}_\chi(xf + (1-x)) - r\mathcal{G}_\chi(1-x). \end{aligned} \tag{9}$$

2) Scenario 2 – SU with singleton detection: In this scenario, we suppose that the SU can detect whether it is alone or there are SUs nearby in the network. Therefore, each SU selects its strategy based on his knowledge of being or not being alone. In this case, four strategies arise $\{(\mathrm{S},\mathrm{S}), (\mathrm{NS},\mathrm{S}), (\mathrm{S},\mathrm{NS}), (\mathrm{NS},\mathrm{NS})\}$. For example, when an SU selects (S,NS), then, it chooses "S" when it is not alone, and selects "NS" when it is alone.

The fitness of a given SU with respect to strategy (S,S) writes

$$\begin{aligned} g_2((\mathrm{S},\mathrm{S}), x) &= (1-p_0) \sum_{m=1}^{M-1} \mathbb{P}(\chi = m | \chi > 0) \sum_{n=0}^{M-1} \binom{m}{n} x^n (1-x)^{m-n} \\ &\quad (1 - f^{n+1})(1-C) + p_0(1-C)(1-f) \\ &= (1-p_0)\big(1 - C - f(1-C)\mathcal{G}_{\chi|\chi>0}(xf + (1-x))\big) \\ &\quad + p_0(1-C)(1-f). \end{aligned} \tag{10}$$

Analogously, when the player adopts (S,NS)

$$\begin{aligned} g_2((\mathrm{S},\mathrm{NS}), x) &= (1-p_0)\big(1 - C - f(1-C)\mathcal{G}_{\chi|\chi>0}(xf + (1-x))\big) \\ &\quad - p_0 r. \end{aligned} \tag{11}$$

The fitness of a SU that chooses not to sense when potential cooperators are in radio range and prefers to sense when alone writes

$$\begin{aligned} g_2((\mathrm{NS},\mathrm{S}),x) = &(1-p_0)\sum_{m=1}^{M-1}\mathbb{P}(\chi=m|\chi>0)\sum_{n=1}^{M-1}\binom{m}{n}x^n(1-x)^{m-n} \\ &(1-f^n)-r(1-x)^m+p_0(1-C)(1-f) \\ =&(1-p_0)\big(1-\mathcal{G}_{\chi|\chi>0}(xf+(1-x)) \\ &-r\mathcal{G}_{\chi|\chi>0}(1-x)\big)+p_0(1-C)(1-f). \quad (12) \end{aligned}$$

Finally, when it chooses not to sense in both cases,

$$\begin{aligned} g_2((\mathrm{NS},\mathrm{NS}),x) =& (1-p_0)\big(1-\mathcal{G}_{\chi|\chi>0}(xf+(1-x)) \\ &-r\mathcal{G}_{\chi|\chi>0}(1-x)\big)-p_0 r. \quad (13) \end{aligned}$$

We observe that strategies (S,NS) and (NS,NS) are dominated by (S,S) and (NS,S) respectively. Clearly, when the SU knows it is alone, it has no choice but to sense the channel. Thus, we can restrict our analysis to strategies (S,S) and (NS,S), and use simplified notation accordingly: $g_2(S,x)$ stands for $g_2((\mathrm{S},\mathrm{S}),x)$, and $g_2(NS,x)$ stands for $g_2((\mathrm{NS},\mathrm{S}),x)$.

In the following, individual and overall fitness are indexed by 1 or 2 according to the corresponding scenario just described.

V. ESS ANALYSIS

In this section, we give a full characterization of the ESS for both scenarios, under different distributions of the number of SUs.

A. Calculation of ESSs

Proposition 1 (ESS, scenario 1). *Let $C^1_{th} := \frac{(1-f)\mathcal{G}_\chi(f)}{1-f\mathcal{G}_\chi(f)}$. In scenario 1, the evolutionary game is characterized by*

1) *a unique strictly mixed ESS given by $x^* = \Delta_1^{-1}(0)$ when $C > C^1_{th}$, with $\Delta_1(x) = g_1(S,x) - g_1(NS,x)$,*
2) *a unique pure ESS where every SU senses the channel when $C \leq C^1_{th}$.*

Proof. We are applying conditions (5) and (6) to the system state x. First, condition (5) is satisfied when $G_1(\mathrm{mut},x) - G_1(x,x) = (\mathrm{mut}-x)\Delta_1(x) \geq 0$, where

$$\begin{aligned} \Delta_1(x) =& g_1(\mathrm{S},x) - g_1(\mathrm{NS},x) \\ =& -C+(1-f(1-C))\mathcal{G}_\chi(fx+(1-x))+r\mathcal{G}_\chi(1-x). \quad (14) \end{aligned}$$

We observe that Δ_1 is a strictly decreasing and continuous function since both $G_\chi(fx+(1-x))$ and $G_\chi(1-x)$ are continuous and strictly decreasing, $(1-f(1-C)) \geq 0$ and $r \geq 0$.

Also, $\Delta_1(0) = (1-C)(1-f)+r \geq 0$. Hence, when $\Delta(1) < 0$, we can denote x^* the unique solution of $\Delta_1(x^*) = 0$, $x^* \in]0,1[$. The value $\Delta_1(1)$ writes $\Delta_1(1) = -C + (1-f(1-C))\,\mathcal{G}_\chi(f)$. Therefore, when $\Delta(1) < 0$, i.e., $C > \frac{(1-f)\mathcal{G}_\chi(f)}{1-f\mathcal{G}_\chi(f)} = C^1_{th}$, we obtain $G_1(\mathrm{mut},x^*) = G_1(x^*,x^*)$, which satisfies (5) with equality.

As equation (5) holds, we check the second condition described by equation (6), which writes

$$G_1(x^*,\mathrm{mut}) - G_1(\mathrm{mut},\mathrm{mut}) = (x^*-\mathrm{mut})\Delta_1(\mathrm{mut}) :$$

i. $\mathrm{mut} < x^*$: as Δ_1 is strictly decreasing, $\Delta_1(\mathrm{mut}) > \Delta_1(x^*) = 0$, thus $G_1(x^*,\mathrm{mut}) - G_1(\mathrm{mut},\mathrm{mut}) > 0$.
ii. $\mathrm{mut} > x^*$: as Δ_1 is strictly decreasing, $\Delta_1(\mathrm{mut}) > \Delta_1(x^*) = 0$, thus $G_1(x^*,\mathrm{mut}) - G_1(\mathrm{mut},\mathrm{mut}) > 0$.

But, it is immediate to observe that $(\mathrm{mut}-x^*)\Delta_1(x^*) < 0$ for all $\mathrm{mut} \neq x^*$: consequently, when $C > \frac{(1-f)\mathcal{G}_\chi(f)}{1-f\mathcal{G}_\chi(f)}$, $x^* \in]0,1[$ given by $\Delta_1(x^*) = 0$ is the unique ESS.

Now, suppose $C \leq C^1_{th}$, then $\forall x, \Delta_1(x) \geq 0$. In order to have $G_1(\mathrm{mut},x) - G_1(x,x) < 0, \forall \mathrm{mut} \neq x$, x must be equal to 1, meaning every SU senses the channel is the ESS. This completes the proof. ■

As expected, SUs are more likely to volunteer for the sensing task when the sensing cost is smaller: in particular, below cost threshold $C^1_{th} = \frac{(1-f)\mathcal{G}_\chi(f)}{1-f\mathcal{G}_\chi(f)}$, sensing dominates. At the same time, when the individual false alarm probability increases, the cost threshold increases, augmenting participation in the sensing task. Similar results hold in scenario 2, as showed next.

Proposition 2 (ESS, scenario 2). *Let $C^2_{th} := \frac{(1-f)\mathcal{G}_{\chi|\chi>0}(f)}{1-f\mathcal{G}_{\chi|\chi>0}(f)}$. In scenario 2, the evolutionary game is characterized by*

1) *a unique strictly mixed ESS given by $x^* = \Delta_2^{-1}(0)$ when $C > C^2_{th}$, with $\Delta_2(x) = g_2(S,x) - g_2(NS,x)$,*
2) *a unique pure ESS where every SU senses the channel when $C \leq C^2_{th}$.*

Proof. The proof develops as in Prop. 1: it is sufficient to replace $\mathcal{G}_\chi$ in $\Delta_1(x)$ by $\mathcal{G}_{\chi|\chi>0}$ to obtain $\Delta_2(x)$. Moreover, $\mathcal{G}_\chi$ and $\mathcal{G}_{\chi|\chi>0}$ hold same monotonicity properties. ■

B. Effect of singleton detection

We are comparing the performance of the system, i.e., the cooperation level in the sensing task, as it is induced under the two opposite scenarios, i.e., – we recall – when the SUs cannot perform singleton condition detection (scenario 1), and when they can, (scenario 2).

Proposition 3 (singleton detection effect). *Let C^1_{th} and C^2_{th} be the sensing cost thresholds of scenario 1 and scenario 2, respectively. Then, it holds*

$$C^2_{th} \leq C^1_{th},$$

where equality holds if and only if $p_0 = 0$.

Proof. Note that $G_\chi(x) = (1-p_0)G_{\chi|\chi>0}(x) + p_0$. Therefore,

$$G_\chi(x) - G_{\chi|\chi>0}(x) = p_0(1-G_{\chi|\chi>0}(x)) \geq 0 \quad (15)$$

Besides, function $h : y \longmapsto \frac{(1-f)y}{1-fy}$ is increasing with respect to $y \in [0,1]$, which completes our proof. ■

The result of Proposition 3 can be detailed as follows.

- $0 < C \leq C^2_{th}$: in both scenarios all SUs sense the channel regardless knowing or not about the presence of other cooperators;
- $C^2_{th} < C \leq C^1_{th}$: SUs sense all the channel only when the absence of potential cooperators cannot be assessed.

- $C_{th}^1 < C \leq 1$: the SU senses the channel with probability $x_i^* \in]0,1[$, such that $x_i^* = \Delta_i^{-1}(0)$, with $i \in \{1,2\}$.

The above reasoning applies to the mixed equilibria which are attainable for both scenarios in a given system configuration, i.e., f, r, C and given distribution of cooperators, i.e., $G_\chi(x)$, $G_{\chi|\chi>0}(x)$.

Proposition 4 (Mixed ESS)**.** *Let x_1^* and x_2^* be the strictly mixed ESSs of scenario 1 and scenario 2, respectively, attained for same set of system parameters. It holds*

$$x_2^* \leq x_1^* \tag{16}$$

Proof. First, as described by equation (15), $\mathcal{G}_\chi(x) \geq \mathcal{G}_{\chi|\chi>0}(x)$. Moreover, $\forall x \in]0,1[$, we have

$$\begin{aligned}\Delta_2(x)-\Delta_1(x) = &(1-f(1-C))(\mathcal{G}_{\chi|\chi>0}(fx+(1-x)) \\ &-\mathcal{G}_\chi(fx+(1-x))) + r(\mathcal{G}_{\chi|\chi>0}(1-x) - \mathcal{G}_\chi(1-x)) \\ &\leq 0\end{aligned} \tag{17}$$

It follows that $\Delta_2(x_1^*) \leq \Delta_1(x_1^*) = 0$, then, $\Delta_2(x_1^*) \leq \Delta_2(x_2^*)$, we recall that Δ_2 is strictly decreasing, so that $x_2^* \leq x_1^*$. ∎

It is interesting to observe that in the regime of small sensing costs, namely $0 < C \leq C_{th}^2$, SUs volunteer for the sensing task regardless the information they possess about being the only one to sense.

Remark 1 (Role of Singleton Detection)**.** *One may be tempted to try to design a protocol able to detect very reliably the* absence *of SUs in radio range. The motivation for equipping SUs with such functionality, would be that, riding on the regret (bad performance), SUs would be incentivized to sense. Unfortunately, the above results show that delivering such information to SUs actually decreases cooperation. This is apparently a paradox because by delivering more information to SUs, e.g., at the cost of extra signaling, one would drive instead the system to a lesser desirable operating point [12].*

VI. Numerical Results

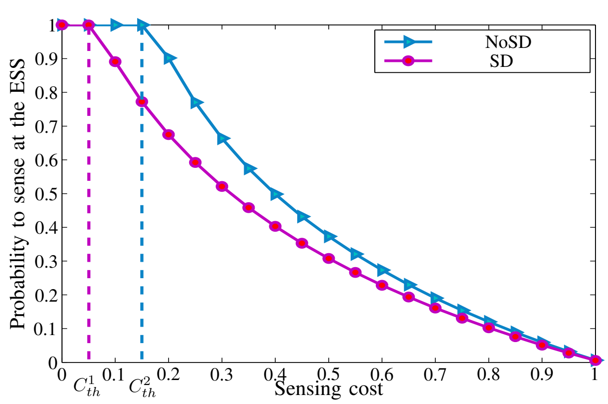


Fig. 1: ESS with respect to the sensing cost. Parameters: Poisson distribution with parameter $\lambda = 2$, $f = 0.2$, $r = 0.01$.

In this section, we present numerical results meant to provide further characterization of the performance of cooperative sensing, according to the proposed model.

Fig. 1 depicts the probability to sense at the ESS, with respect to the sensing costs. Clearly, the probability to sense decreases as the sensing cost increases. As expected, when the sensing cost is smaller than thresholds, C_{th}^1 and C_{th}^2, respectively, all SUs participate in the sensing task. Furthermore, in agreement with Prop. 4, the probability to sense under no singleton detection scenario, briefly NoSD, is always better than singleton detection scenario, briefly SD. Indeed, an SU that is able to detect the presence of other SUs is more likely to free-ride cooperation resulting in a worse sensing accuracy.

We study the impact of increasing the average number of SUs in Fig. 3(a). By increasing the Poisson intensity parameter λ, the average number of SUs in radio range of each SU increases. This lowers the probability to sense the channel at the equilibrium. The effect of the volunteer's dilemma can be mitigated by enlarging the regret parameter. Indeed, Fig. 3(a) shows that SUs are more likely to participate in the sensing task when the regret r is higher. This can be seen as a possible incentive mechanism to overcome lack of cooperation of SUs. It is worth noting that the regret can be tuned at the protocol level, e.g., the regret parameter can be the average service time of packets served in SUs' transmission buffers. Hence, SUs will improve the performance when the regret is weighted more, e.g., by means of a simple multiplicative factor.

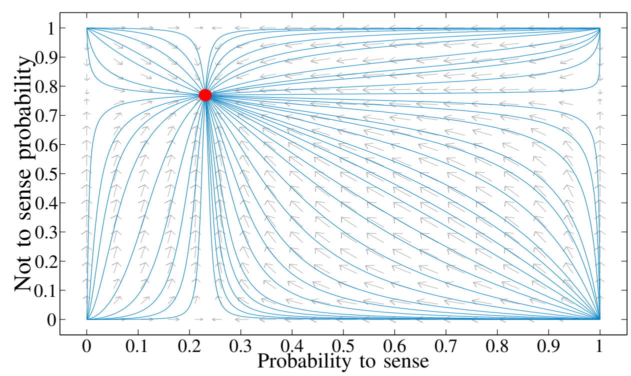
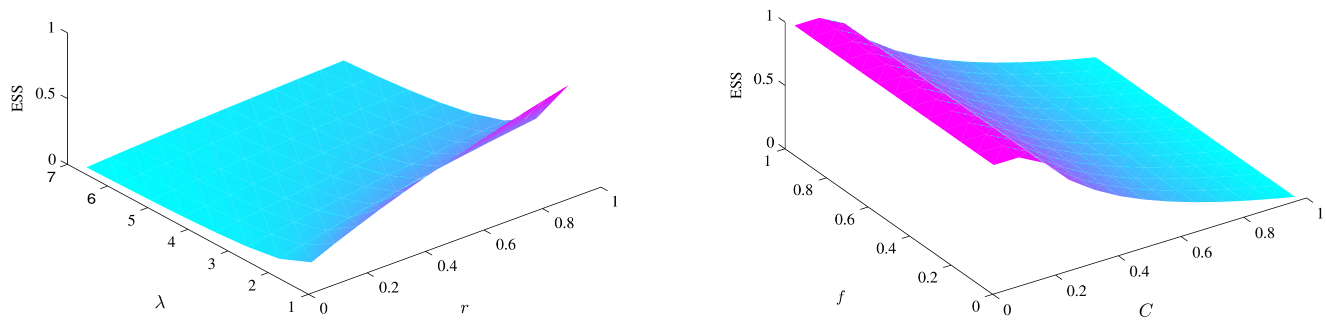


Fig. 2: Asymptotic stability for the strictly mixed ESS. Parameters: a Dirac distribution with parameter $k = 2$, $C = 0.6$, $f = 0.02$, $r = 0.01$, the strictly mixed ESS is $(0.78, 0.22)$.

Next, we assume Poisson intensity $\lambda = 2$ and regret $r = 0.01$: in Fig. 3(b) the probability to sense at the ESS is plotted with respect to the false alarm probability f and sensing costs C. As expected, when the false alarm probability increases, the SUs prefer to sense in order to achieve a better sensing accuracy. Whereas, SUs are more reluctant to cooperate when sensing is more costly. The plateau in Fig. 3(b) illustrates how before the threshold cost is attained, all SUs choose to participate in the sensing task.

The replicator dynamics represents the evolution of the



(a) (b)

Fig. 3: (a) ESS with respect to the regret r and the density of SUs with a Poisson distribution of parameter λ, $C = 0.8$, $f = 0.02$, (b) ESS with respect to the false alarm f and the sensing cost C of SUs with a Poisson distribution of parameter $\lambda = 2$, and a regret $r = 0.01$

fraction of a population that selects a given strategy. Moreover, it can also be associated to a revision protocol describing the agents' learning process. Each agent decides, at random times, to revise its strategy. The choice of the replicator dynamics fits well the limited information available to SUs. Thus, SUs need only to know the fitness related to their current strategy and that of others in proximity in order to adopt a different one. It is also worth noting that the replicator dynamics may not necessarily converge to the ESS. Nevertheless, for the studied game, it is possible to show that the replicator dynamics does converge to the ESS, and that the ESS is asymptotically stable under the replicator, i.e., it is a globally attracting point. Due to space limitation, the proof is only included in the extended version of the paper that can be found in [13]. Finally, Fig. 2 captures the vector field of the replicator dynamics for two SUs. It can be seen that the replicator dynamics is attracted by the asymptotically stable ESS at $(0.78, 0.22)$ regardless the initial configuration of the network.

VII. Conclusion

In this work, we model cooperative sensing as an evolutionary game. We study the impact of information available at the SUs on the sensing accuracy. We hence prove that SUs are more likely not to cooperate when they are able to detect the absence of potential cooperators. In future works, we will investigate the case of heterogeneous costs and regrets, and we shall also study the impact of other combining rules.

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