Optimal Geographic Caching in Cellular Networks with Linear Content Coding

Jocelyne Elias* and Bartłomiej Błaszczyszyn†

Abstract—We state and solve a problem of the optimal geographic caching of content in cellular networks, where *linear combinations of contents* are stored in the caches of base stations. We consider a general content popularity distribution and a general distribution of the number of stations covering the typical location in the network. We are looking for a policy of content caching maximizing the probability of serving the typical content request from the caches of covering stations. The problem has a special form of *monotone sub-modular* set function maximization. Using *dynamic programming*, we find a deterministic policy solving the problem. We also consider two natural *greedy caching policies*. We evaluate our policies considering two popular stochastic geometric coverage models: the Boolean one and the Signal-to-Interference-and-Noise-Ratio one, assuming Zipf popularity distribution. Our numerical results show that the proposed deterministic policies are in general not worse than some randomized policy considered in the literature and can further improve the total hit probability in the moderately high coverage regime.

Index terms— Cellular caching, Network coding, Hit probability, Coverage model, Optimization, Stochastic Geometry.

I. Introduction

The rapid proliferation of smartphones, tablets and other smart mobile devices over the last few years has come hand in hand with content-oriented services, which are actually dominating the Internet traffic. According to [1], video streaming for example accounts for 54% of the total Internet traffic, and the ratio is expected to grow to 71% by the end of 2019. This phenomenon has posed new challenges for mobile network operators and has pushed them to implement novel schemes to efficiently operate their cellular infrastructure, dealing with the explosive growth in mobile data traffic.

A promising approach to deal with this phenomenon is to introduce caching at Base Stations (BSs). Content caching at BSs is indeed very beneficial for mobile operators for several reasons, the most important are: 1) it reduces the data traffic on the backhaul links, 2) it reduces the delay experienced by cellular network's users, and 3) it contributes in reducing the congestion during the peak hours. Therefore, this issue has attracted the attention of the research and industrial communities. A comprehensive survey on *caching* in *cellular networks*, and more specifically in 5G networks, along with its benefits and challenges is given in [2].

Furthermore, very recently, some ideas taken from network coding have been applied to caching in cellular networks [3], [4], [5], [6] and it was shown that network coding-based caching policies further improve the performance obtained so far by classical caching schemes. The core idea of this technique is to use random linear network coding, where linear combination of contents (or chunks of files) are stored in the caches of BSs.

In this work we combine *linear content coding* techniques and cellular network coverage models from *stochastic geometry*, to propose and evaluate some novel geographic caching policies that further improve the hit probability obtained in previous works. The idea is to store in the caches of base stations *linear combinations of contents* so as to increase the cache-hit probability by leveraging the probability of covering the request location by more than one base station. For a general content popularity distribution and a general distribution of the number of stations covering a typical location in the network, we formulate a problem of the *optimal deterministic policy of content caching with network coding*, maximizing the probability of serving the typical content request from the caches of covering stations. We find the solution to this problem using the *dynamic programming* approach. We also consider two natural *greedy caching policies*. Theoretical bounds can be given on the sub-optimality of one of these greedy policies leveraging the classical theory of monotone sub-modular set functions.

We evaluate numerically our policies considering two popular stochastic geometric coverage models: the Boolean one and the Signal-to-Interference-and-Noise-Ratio (SINR) one, and compare their performance (the hit probability) to those offered by the caching of the most popular content in all base stations and an optimal random, independent, caching strategy from [7], both considered as reference strategies in the literature. Our numerical results show that the proposed policies employing network coding are in general not worse than the two reference policies and can further improve the total hit probability offered by the independent caching policy in the moderately high coverage regime.

Related Work

There is a considerable number of papers dealing with cellular caching. In what follows we mention only the most relevant to our approach.

Bastug et al in [8] provide some early stochastic geometry results on the user outage probability and average delay experienced in cache-enabled cellular networks, further developed in [9].

---

*Jocelyne Elias is with Inria/ENS, Paris and LIPADE Laboratory, Paris Descartes University, France. E-mail: jocelyne.elias@parisdescartes.fr. Bartłomiej Błaszczyszyn is with Inria/ENS, Paris, France. E-mail: Bartek.Blaszczyszyn@ens.fr.

The optimized independent caching policy, used here as a reference one, was proposed in [7] and evaluated under the same Boolean and SINR coverage models. Networks of wireless caches in the plane with geometric constraints and linear network coding were first studied by Altman et al in [5].

Avrachenkov et al in [6] considered an approach with linear network coding very similar to ours, addressing the hit probability maximization as a generalized unbounded knapsack problem. Having proved that the problem can be solved in general using dynamic programming, they mainly focus on the explicit solution for a special case when no coding is applied (each file is stored or not in a single chunk). They also consider a relaxation of the problem in which the average constraints on the storage space are imposed. This latter model with one chunk happens to be equivalent to the one considered in [7]. Our formulation of the hit maximization problem as a monotone sub-modular set function maximization, while sub-optimal with respect to the generalized knapsack problem of [6], cf Remark 2, has a two-fold advantage: (1) the optimal (dynamic programming) solution has much simpler structure allowing one for efficient numerical evaluation of its performance in a general case and (2) it admits natural approximations by greedy policies. Both approaches prove to be competitive with respect to the reference policies considered so far in the literature.

Further related works include [10], where some auction-based collaborative caching mechanism for wireless streaming is studied. In [11], the authors studied the on-line collaborative caching problem for a multicell-coordinated system from the point of view of minimizing the total cost paid by the Content Providers. Similar scenarios are considered in [12], [13]. Cooperative caching and cooperative redundancy elimination models were proposed in [14] for an intra-Autonomous System.

The remaining part of the paper is organized as follows. Section II introduces the caching and network coding as well as the user coverage models used in the paper. Section III describes the optimal and greedy caching policies that we propose to solve the hit probability maximization problem. Evaluation of the proposed policies under the considered coverage models is done in Section IV. Finally, concluding remarks and future research are discussed in Section V.

II. Caching with Network Coding Model

A. Network Coverage Distribution

We consider a general cellular-network germ-grain type [15, Section 6.5] coverage model generated by stationary marked point process $\Phi = \{(X_i, \mathcal{C}_i)\}$, where X_i model the positions of Base Stations (BSs) and $X_i \oplus \mathcal{C}_i$ their cells (service zones) [1]. In fact, the only characteristics of this model, used in the remaining part of this paper, is the *coverage (number) distribution* defined as
$$p_k := \mathbf{P}\{N = k\}, \qquad k = 0, 1, \ldots,$$
where $N := \sum_i \mathbf{1}(0 \in X_i \oplus \mathcal{C}_i)$ is the number of cells covering the typical location, assumed without loss of generality (due to

stationarity of the model) to be the origin 0 of the plane. We consider $\mathcal{P} := \{p_k : k = 0, 1, \ldots\}$ as a (given) probability distribution on $\mathbb{N}_0 := \{0, 1, \ldots\}$. We shall use also the following notation for the tail-distribution function of the coverage number
$$\overline{P}(k) := \mathbf{P}\{N \geq k\} = \sum_{n=k}^{\infty} p_n.$$

1) Boolean Model: An important special case of the coverage model considered in the literature and in this paper is the *Boolean Model* (BM), where Φ is a homogeneous Poisson point process and $\{\mathcal{C}_i\}$ are independent, identically distributed closed sets of finite mean surface $\mathbf{E}[|\mathcal{C}_i|] < \infty$. In this case the coverage number N has a Poisson distribution with parameter $\lambda' := \lambda \mathbf{E}[|\mathcal{C}_i|]$, where $\lambda > 0$ is the intensity of Φ (corresponding to the density of BSs); cf [16, Lemma 3.1]. Consequently, for the BM we have
$$p_k = e^{\lambda'} \frac{\lambda'^k}{k!}, \qquad k = 0, 1, \ldots \tag{1}$$

2) SINR model: In a more adequate model for cellular network, called *Signal-to-Interference-plus-Noise Ratio (SINR) coverage model* [2], the coverage number distribution is given by a more complicated expression; cf [17]. For this model we have
$$p_k := \sum_{n=k}^{\lceil 1/\tau \rceil} (-1)^{n-k} \binom{n}{k} \mathcal{S}_n(\tau), \tag{2}$$
where
$$\mathcal{S}_n(\tau) = \tau_n^{-2n/\beta} \mathcal{I}_{n,\beta}((W) a^{-\beta/2}) \mathcal{J}_{n,\beta}(\tau_n) \tag{3}$$
represents the expected number of n-tuples of BSs the typical user can select among those which cover it with the SINR greater than τ. The notation used in the above expressions is as follows: $\tau_n := \frac{\tau}{1-(n-1)\tau}$, β is the path-loss exponent [3], W is the external noise power, and the two special functions
$$\mathcal{I}_{n,\beta}(x) := \frac{2^n \int_0^\infty u^{2n-1} e^{-u^2 - u^\beta x \Gamma(1-2/\beta)^{-\beta/2}} du}{\beta^{n-1} \Gamma(1-2/\beta)^n \Gamma(1+2/\beta)^n (n-1)!} \tag{4}$$
with $a = \frac{\lambda \pi \mathbf{E}[(PS)^{\frac{2}{\beta}}]}{K^2}$, where λ is the density of BSs, S is the fading/shadowing random variable, P is the BS transmission power, K is the path-loss constant, and
$$\mathcal{J}_{n,\beta}(x) := \frac{(1+nx)}{n} \int_{[0,1]^{n-1}} \frac{\prod_{i=1}^{n-1} v_i^{i(2/\beta+1)-1} (1-v_i)^{2/\beta}}{\prod_{i=1}^{n} (x+\eta_i)} dv_1 \ldots dv_{n-1}, \tag{5}$$
with $\eta_1 = v_1 v_2 \ldots v_{n-1}$, $\eta_2 = (1-v_1) v_2 \ldots v_{n-1}$, $\eta_3 = (1-v_2) v_3 \ldots v_{n-1}$, ..., $\eta_n = 1 - v_{n-1}$. Note that in contrast to the Boolean model, the coverage number distribution in the SINR model has a bounded support; N is not larger than the constant $\lceil 1/\tau \rceil$ depending on the required SINR threshold τ.

---

[1] $\mathcal{C}_i$ are assumed to be random closed sets and $X_i \oplus \mathcal{C}_i$ means the translation of the set $\mathcal{C}_i$ by X_i.

[2] Also called shot-noise germ-grain model in [15, Section 6.5.4].

[3] The path-loss function is $(Kr)^{-\beta}$, with constants $K > 0$ and $\beta > 2$, and r is the distance between the BS and the user.

B. Content Popularity Distribution

We consider a *finite* [4] set of contents, indexed by a subset of integers $\mathbb{J} := \{1, 2, \ldots, J\}$, $J < \infty$. Popularity of these contents is modeled by a probability distribution $\{a_j : j \in \mathbb{J}\}$, called *(content) popularity distribution*. The value a_j is interpreted as the probability that a typical user (located at the origin 0) requests the content $I = j$ from the network; $a_j := \mathbf{P}\{I = j\}$, where I is the index of the requested content. Without loss of generality we assume that the content items are indexed according to the decreasing popularity: $a_1 \geq a_2 \geq \ldots \geq a_J$. In what follows, we always assume that the requested content I and the coverage number N are independent random variables.

An important special case of the content popularity distribution, having some empirical justifications, is the truncated Zipf distribution, with

$$a_j = A^{-1} j^{-\gamma}, \qquad j = 1 \ldots J, \qquad (6)$$

where γ is the (Zipf) exponent and $A = \sum_{j=1}^{J} j^{-\gamma}$.

C. Content Placement and Recovery Using Network Coding

We assume that a cache memory consisting of $L \geq 1$ *blocks* is available at each BS. The size of each block corresponds to the size of exactly one content item (all content items are assumed to have the same size). In this paper we assume that *all BSs store exactly the same subset of contents*. The spatial diversity (leveraging multiple coverage) is achieved by using some network coding techniques in the contents storage implementation, allowing one to store linear combinations of *more content items than the number of memory blocks*.

More precisely, in each block $i = 1, \ldots, L$ a linear combination of the content items from $C_i \subset \mathbb{J}$ of cardinality $|C_i| := \#(C_i) \geq 1$ is stored. All base stations encode in their memory blocks $i = 1, \ldots, L$ exactly the same subsets C_i of content items using *mutually independent linear combinations of the contents*. Motivated by this, we assume that a user (say located at the origin) requesting some content item $j \in \mathbb{J}$ can effectively recover it from the caches of the BSs covering it when j is encoded in some block C_i of contents of cardinality $|C_i|$ not greater than the coverage number N, i.e., when

$$\min\{|C_i| : j \in C_i, i = 1, \ldots, L\} \leq N. \qquad (7)$$

When the condition (7) is satisfied, we say the requested content item j *is hit* in the network.

Denoting by δ_j, $j \in \mathbb{J}$, the indicator of the event (7) we can write the *hit probability* of the content item I randomly selected according to the popularity distribution as a function of the choice of the subsets $\{C_i\}_{i=1}^{L}$ of contents encoded in the memory blocks of BSs as

$$\mathbb{P}_{\text{hit}} = \mathbb{P}_{\text{hit}}(\{C_i\}) := \mathbf{P}\{\delta_I = 1\}$$
$$= \sum_{j=1}^{\infty} a_j \overline{P}\Big(\min\{|C_i| : j \in C_i\}\Big), \quad (8)$$

with $\min\{\emptyset\} = \infty$.

[4]Finiteness of the content set simplifies the model analysis.

III. HIT PROBABILITY OPTIMIZATION PROBLEM

By a *content caching policy* we mean in what follows a configuration of L sets of contents $\{C_i\}_{i=1}^{L}$ to be encoded in L memory blocks of all BSs. Our main goal in this section consists in finding a caching policy maximizing the hit probability, that is solving

$$\max_{C_1, \ldots, C_L \subset \mathbb{J}} \mathbb{P}_{\text{hit}}(\{C_i\}). \qquad (9)$$

We shall also present a few reasonable sub-optimal content caching policies. Let us first remark the following relations to some previously considered caching policies.

Remark 1. Caching the L *most popular contents* corresponds to taking $C_j = \{j\}$, $j = 1, \ldots, L$. This policy is obviously the optimal one in the case of the *1-coverage* regime ($p_k = 0$ for $k \geq 2$.). *Independent caching* proposed in [7] leverages multiple coverage to increase the hit probability, without using network coding. In contrast to the policies considered in this paper, it is a *randomized policy* providing all BSs with some probability distribution on the set of content items (in fact the sequence of caching probabilities for all contents) and letting BSs independently sample the composition of their cached contents from this distribution. This distribution is calculated (as in the current paper) in function of the content popularities and BS coverage probabilities so as to maximize the average (cache) hit probability. Note this policy maximizes the hit probability but in a different class of policies and hence, in general, one cannot easily compare it to policies considered in this paper.

Remark 2. Our current optimization problem is a restriction of the knapsack problem stated in [6]. The restriction comes from the fact that our policy $\{C_i\}$ assumes coding content items separately for each memory block $i = 1, \ldots, L$ and using for all contents $j \in C_i$ present in a given block i the same number of equations $n_i = M/|C_i|$, where M is the number of chunks of each item considered in [6]. While this latter assumption is not restrictive ($n_i \neq M/n$ for some n can be shown sub-optimal), coding separately different memory blocks is indeed sub-optimal. This assumption however has an important consequence: it transforms the generalized knapsack problem to a simpler sub-modular set function maximization problem, as will be shown in what follows.

A. Properties of the Optimal Caching Policies

In what follows we present some properties satisfied by any policy maximizing (9).

Lemma 3. *There exists a policy $\{C_i\}$ maximizing* (9) *having the following properties:* $\bigcup_{i=1}^{L} C_i = \{1, \ldots, j_{\max}\}$, $|C_1| \leq \ldots \leq |C_L|$, *and all elements of C_i precede those of C_{i+1}; i.e.,* $C_i = \{|C_1| + \ldots + |C_{i-1}| + 1, \ldots, |C_1| + \ldots + |C_{i-1}| + |C_i|\}$.

Proof: Let $\{C_i\}$ maximizing (9). Suppose an item $x \in \bigcup_{i=1}^{L} C_i$ is present in more than one set C_i. Keeping x only in one set of smallest cardinality does not decrease the hit probability. Assume hence that $\{C_i\}$ are pairwise disjoint and suppose there exist $x \in \dot{\bigcup}_{i=1}^{L} C_i$ and $y \notin \dot{\bigcup}_{i=1}^{L} C_i$, such that $y < x$. Replacing x by y does not decrease the hit

probability. Consider hence the case $\dot{\bigcup}_{i=1}^{L} C_i = \{1, \ldots, j_{\max}\}$. Assume now that $\{C_i\}$ is indexed in the increasing order of cardinalities. Suppose that $x \in C_i$, $y \in C_j$ with $|C_i| \leq |C_j|$ and $x > y$. Then, swapping x and y does not decrease the hit probability and we can construct a new partition $\{C_i\}$ of $\{1, \ldots, j_{\max}\}$ in which all elements of C_i precede those of C_{i+1}. ∎

Note that a policy $\{C_i\}$ satisfying the conditions of Lemma 3 has pairwise disjoint sets C_i. It is easy to see that this simplifies the expression (8) of the hit probability to the following one.

Lemma 4. *For pairwise disjoint C_i, $i = 1, \ldots, L$.*

$$\mathbb{P}_{\text{hit}}(\{C_i\}) = \sum_{k=1}^{L} A(C_k)\overline{P}(|C_k|), \qquad (10)$$

where $A(C_k) = \sum_{j \in C_k} a_j$.

In view of Lemma 3 we can simplify the problem (9) restricting ourselves to the policies of the form $C_i = [m_1 + \ldots + m_{i-1} + 1, m_1 + \ldots + m_i]$, where $[k, l] := \{k, k+1, \ldots, l\}$ for integers k, l and m_i the number of contents cached in block i. By Lemma 3 and Lemma 4:

Proposition 5. *We have*

$$\max_{C_1, \ldots, C_L \subset \mathbb{J}} \mathbb{P}_{\text{hit}}(\{C_i\}) \qquad (11)$$

$$= \max_{1 \leq m_1 \leq \ldots \leq m_L} \sum_{k=1}^{L} A([m_1 + \ldots + m_{k-1} + 1, m_1 + \ldots + m_k])\overline{P}(m_k).$$

B. Dynamic Programming Solution of the Optimal Caching Problem

The idea consists in finding first the optimal cardinality m_L of the last block C_L in function of the assumed (unknown beforehand) total number n of contents cached in previous blocks. Then proceed recursively with cardinalities m_l of blocks C_l, $2 \leq l \leq L-1$, maximizing them in function of the assumed total number n of contents cached in $C_1, \ldots, C_{l-1}$ while taking into account already calculated contribution of the blocks $l+1, \ldots, L$. This leads to the following Dynamic Program (DP):

We express the optimal number of contents cached in the L-th block as a function of the total number $n \in \mathbb{J}$ of contents cached in previous blocks and the corresponding hit probability as:

$$m_L(n) := \arg\max_{x} A([n+1, n+x])\overline{P}(x),$$

$$\mathbb{P}_{\text{hit}}(L, n) := \max_{x} A([n+1, n+x])\overline{P}(x).$$

By the induction, for a block l, with $2 \leq l \leq L-1$, we define:

$$m_l(n) := \arg\max_{x} \Big(A([n+1, n+x])\overline{P}(x) + \mathbb{P}_{\text{hit}}(l+1, n+x)\Big),$$

$$\mathbb{P}_{\text{hit}}(l, n) := \max_{x} \Big(A([n+1, n+x]) + \mathbb{P}_{\text{hit}}(l+1, n+x)\Big).$$

Finally, for the first block we consider only $n = 0$ since in this case we should start with the first content item

$$m_1 := \arg\max_{x} \Big(A([1, x])\overline{P}(x) + \mathbb{P}_{\text{hit}}(2, x)\Big),$$

$$\mathbb{P}_{\text{hit}}(1) := \max_{x} \Big(A([1, x]) + \mathbb{P}_{\text{hit}}(2, x)\Big).$$

The above DP approach leads to the following solution of our optimal caching problem (11).

Proposition 6. *The maximal hit probability in* (11) *is equal to* $\mathbb{P}_{\text{hit}}(1)$ *and it is achieved on* $(m_1^*, \ldots, m_L^*)$ *defined as*

$$
\begin{aligned}
m_1^* &:= m_1 \\
m_2^* &:= m_2(m_1^*) \\
&\ldots \\
m_{L-1}^* &:= m_{L-1}(m_1^* + m_2^* + \ldots + m_{L-2}^*) \\
m_L^* &:= m_L(m_1^* + m_2^* + \ldots + m_{L-1}^*).
\end{aligned}
\qquad (12)
$$

C. Greedy Sub-Optimal Caching Policies

The DP approach to the hit probability maximization (9) completely solves the problem but presents a considerable numerical complexity. Greedy algorithms are supposed to propose simpler policies, reasonably approaching the maximal hit probability (9). Depending on whether we apply the greedy approach to the class of general policies $\{C_i\}$, using the expression (8) as the definition of the hit probability, or we restrict ourselves to the class of structured policies $C_i = [m_1 + \ldots + m_{i-1} + 1, m_1 + \ldots + m_i]$, using the expression (10) for this probability, we obtain two, in general different, greedy policies, both in general suboptimal. The former one, using general $\{C_i\}$, has interesting theoretical bounds regarding its sub-optimality, but still represents considerable numerical complexity. The latter one, assuming the structured policies, is numerically much simpler, but we do not have any theoretical guarantees regarding its performance. Numerical evidences suggest its utility. Finally, note that, even if the optimal policy is known to have the structured form, a greedy algorithm operating in this class is in general worse than the greedy algorithm operating in the set of all non-structured policies.

1) Greedy Caching Policy with General Blocks: Let us choose the first set C_1^g of items as a subset maximizing the one-block hit probability $\mathbb{P}_{\text{hit}}(\{C\})$. Since a_i are decreasing C_1^g has the form $C_1^g = [1, \ldots, m_1^g]$ for some $m_1^g \geq 1$. Thus

$$C_1^g \in \arg\max_{C \subset \mathbb{J}} A(C)\overline{P}(|C|) = \arg\max_{m_1 \geq 1} A([1, m_1])\overline{P}(m_1). \qquad (13)$$

Then recursively, let us choose sets C_l^g, $2 \leq l \leq L$ maximizing the increment of the hit probability they offer, *without assuming mutual disjointness of the sets*

$$C_l^g \in \arg\max_{C \subset \mathbb{J}} \mathbb{P}_{\text{hit}}(\{C_1^g, \ldots, C_{l-1}^g, C\}) - \mathbb{P}_{\text{hit}}(\{C_1^g, \ldots, C_{l-1}^g\})$$

$$= \arg\max_{C \subset \mathbb{J}} \sum_{j \in C}^{\infty} a_j \Big(\overline{P}(|C|) - \overline{P}\big(\min\{|C_i^g| : j \in C_i^g, i \leq l-1\}\big)\Big)^+, \qquad (14)$$

where $(x)^+ = \max(x, 0)$.

The following result not only gives a lower bound on the hit probability achieved by $\{C_i^g\}$ but also allows one to mitigate the decrease of this probability by increasing the number of memory blocks.

Proposition 7. *Let* $\{C_i^g\}_{i=1}^{K}$ *be contents sets selected by the greedy caching policy* (13), (14) *applied for* $K \geq L$ *memory blocks and* $\{C_i^*\}_{i=1}^{L}$ *an optimal solution of the problem* (9) *for* L *memory blocks. Then*

$$\mathbb{P}_{\text{hit}}(\{C_i^g\}_{i=1}^{K}) \geq (1 - e^{-L/K})\mathbb{P}_{\text{hit}}(\{C_i^*\}_{i=1}^{L}). \qquad (15)$$

Proof: Consider $\mathbb{P}_{\text{hit}}(\cdot)$ defined by (8) as a set function on the space of finite subsets $\{C_1, \ldots, C_i\}$ of finite subsets $C_i \subset \mathbb{J}$, $i \geq 1$. Clearly $\mathbb{P}_{\text{hit}}(\cdot)$ is non-negative, increasing

$$\mathbb{P}_{\text{hit}}(\{C_1, \ldots, C_i\}) \leq \mathbb{P}_{\text{hit}}(\{C_1, \ldots, C_i, C_{i+1}, \ldots, C_{i+k}\})$$

and sub-modular

$$\mathbb{P}_{\text{hit}}(\{C, C_1, \ldots, C_i\}) - \mathbb{P}_{\text{hit}}(\{C_1, \ldots, C_i\})$$
$$\geq \mathbb{P}_{\text{hit}}(\{C, C_1, \ldots, C_i, C_{i+1}, \ldots, C_{i+k}\}) -$$
$$\mathbb{P}_{\text{hit}}(\{C_1, \ldots, C_i, C_{i+1}, \ldots, C_{i+k}\}).$$

Indeed, for this latter property observe that the right hand side of (14) is decreasing with respect to l. The result follows thus by the classical result [18] for sub-modular set functions. ∎

2) Greedy Caching Policy with Disjoint Blocks: The first set $C_1^{gd} := C_1^g = [1, m_1^g]$ is chosen by this policy as for the previous greedy policy (13). Then recursively, let us choose sets $C_l^{gd} = [m_1^g + m_2^g + \ldots + m_{l-1}^g + 1, m_1^g + m_2^g + \ldots, m_l^g]$, $2 \leq l \leq L$ maximizing the increment of the hit probability they offer, *assuming mutual disjointness of the sets*, thus using expression (10) for this probability

$$C_l^{gd} \in \arg\max_{C \subset \mathbb{J}} \mathbb{P}_{\text{hit}}(\{C_1^g, \ldots, C_{l-1}^g, C\}) - \mathbb{P}_{\text{hit}}(\{C_1^g, \ldots, C_{l-1}^g\})$$

$$= \arg\max_{m_l \geq m_1 + \ldots + m_{l-1}} A([m_1 + \ldots + m_{l-1} + 1, m_1 + \ldots + m_l]) \overline{P}(m_l).$$

(16)

IV. Numerical Results

In this section, we evaluate the performance of the proposed caching policies described previously considering the BM and SINR coverage models. In particular we study the dependence of our policies on the mean number of stations $\mathbf{E}[N]$ covering a given location and the Zipf exponent γ of the content popularity distribution. We first describe the numerical setup, including adopted coverage models (Sec. IV-A), and then we analyze and discuss the numerical results (Sec. IV-B).

A. Numerical Setup

Let us first give more details about the coverage models used in our results.

1) Boolean Model: We assume that the interference is small compared to noise (*noise-limited case*) and hence we use the Boolean model to calculate the probability of user coverage by k BSs (p_k in (1)). The signal-to-noise ratio can be expressed as $\frac{P(Kr)^{-\beta}}{W}$, where P is the BS transmit power, K is the path-loss constant, r is the distance between the BS and the user, β is the path-loss exponent and W is the noise power. Let $B = K(\frac{P}{W})^{1/\beta}$. Hence, we have $\mathbf{E}[|\mathcal{C}_i|] = \pi\tau^{-2/\beta}B^{-2}$ and $\lambda' = \lambda\pi\tau^{-2/\beta}B^{-2}$ (see Section II-A1). Note, the mean coverage number is equal to $\mathbf{E}[N] = \pi\lambda\tau^{-2/\beta}B^{-2}$.

2) Signal-to-Interference Ratio (SIR) Model: For general shadowing conditions, the coverage probability p_k is calculated in (2), Section II-A2. We use a package developed in Matlab, available at [19], to compute the numerical values of the probabilities p_k for this model. The mean coverage is equal to $\mathbf{E}[N] = \mathcal{S}_1(\tau)$.
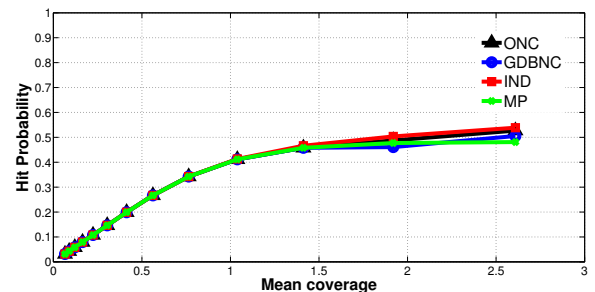
To evaluate the effectiveness of the proposed content caching policies, we conduct calculations using Matlab. Default values of key numerical parameters are as follows:
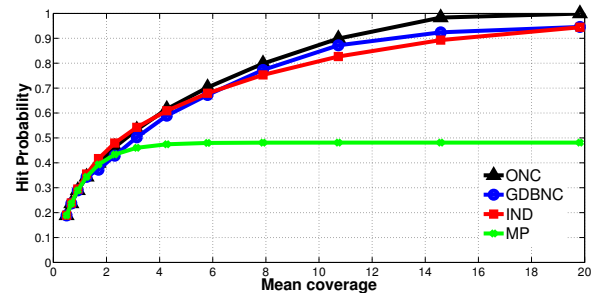
(a) $L = 5$, $\lambda = 1$, $\gamma = 0.9$, Boolean model
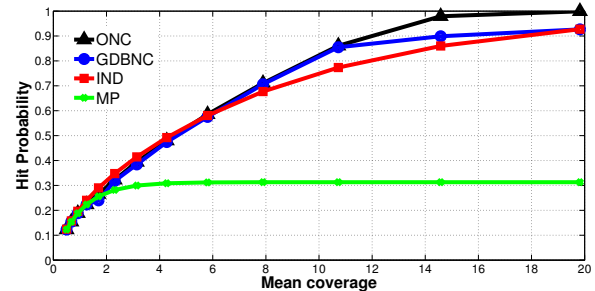
(b) $L = 5$, $\lambda = 1$, $\gamma = 0.56$, Boolean model

(c) $L = 5$, $\lambda = 1$, $\gamma = 0.9$, SINR model

Figure 1. Hit probability $\mathbb{P}_{\text{hit}}$ versus the mean coverage for $\tau \in [-12 \text{ dB}, 12 \text{ dB}]$, cache size $L = 5$, $\lambda = 1$ and $\gamma \in \{0.9, 0.56\}$.

density of BSs $\lambda = 1$, number of cache blocks $L = 5$, total number of content items $J = 40$, Zipf exponent $\gamma \in \{0.9, 0.56\}$, path-loss exponent $\beta = 3$ and constant $K = 1$, noise power $W = 0$ in SIR model and $P/W = 1$ in BM.

B. Performance Evaluation

We evaluate the optimal policy considered in Section III-B, called in what follows the Optimal Network Coding (ONC) policy, and the greedy policy proposed in Section III-C2 called the Greedy Disjoint-Blocks Network Coding (GDBNC) policy. We plot the hit probability $\mathbb{P}_{\text{hit}}$ versus the *mean coverage* $\mathbf{E}[N]$ for the BM and the SINR models.

We further compare our policies to the Most Popular (MP) and Independent (IND) caching policy discussed in Remark 1.

The results are discussed in the next subsections.

1) Hit probability under the Boolean model:

Figure1 (a) and Figure1 (b) show the total hit probability versus the mean coverage, when τ varies in the range [-12 dB, 12 dB], $L = 5$, $\lambda = 1$, and for $\gamma = 0.9$ and 0.56, respectively. It can be observed that both the optimal and the greedy policies

give us a very good performance compared to MP. This is especially true for a mean coverage value higher than 2. For small mean coverage values (or equivalently for high τ values), all considered policies perform similarly to MP.

Furthermore, ONC performs better than IND for larger mean coverage values. The gain is more important when the content popularity distribution is more flat (smaller γ). The greedy policy GDBNC is close to the optimal ONC in some intermediate coverage regime and joins IND when the coverage ultimately increases. It is an open question whether the same holds true for the greedy policy with general bocks considered in Section III-C1, which is in between GDBNC and ONC.

2) Hit probability under the SINR model:
We now evaluate our policies under the SINR model. In this case, the number of BSs covering the user is very much limited (i.e., $\mathbf{E}[N] < 3$). As before, we plot in Figure1(c) $\mathbb{P}_{\text{hit}}$ versus the *mean coverage*. So, it is not surprising to obtain the same behavior under all policies, except for mean coverage bigger than 1.5. In fact, in this latter case, some performance gains could be obtained by our policies with respect to the MP policy. However, for the considered parameters of the SINR model, there is no much enough room to improve $\mathbb{P}_{\text{hit}}$ with respect to MP.

V. CONCLUSION

In this paper we show how network coding ideas can be used to improve the performance of caching in cellular networks. Specifically, we study a geographic caching problem in cellular networks allowing for linear content coding at base stations. Three policies are proposed: an optimal for our model and two natural greedy ones. We show that all considered policies are equivalent to caching in every base station the most popular content (without any coding) when there is no coverage diversity. However, when the number of stations covering a typical user increases, our policies perform much better than this trivial policy and sometimes even better, in terms of the hit rate, than the randomized caching policy [7] previously proposed for such regimes.

REFERENCES

[1] CISCO, "Cisco Visual Networking Index: Global Mobile Data Traffic Forecast Update, 20152020," http://www.cisco.com/c/en/us/solutions/collateral/service-provider/visual-networking-index-vni/mobile-white-paper-c11-520862.pdf, February 3, 2016.

[2] X. Wang, M. Chen, T. Taleb, A. Ksentini, and V. C. Leung, "Cache in the air: exploiting content caching and delivery techniques for 5g systems," *IEEE Communications Magazine*, vol. 52, no. 2, pp. 131–139, 2014.

[3] A. G. Dimakis, P. B. Godfrey, Y. Wu, M. J. Wainwright, and K. Ramchandran, "Network coding for distributed storage systems," *IEEE Transactions on Information Theory*, vol. 56, no. 9, pp. 4539–4551, 2010.

[4] N. Golrezaei, A. F. Molisch, A. G. Dimakis, and G. Caire, "Femtocaching and device-to-device collaboration: A new architecture for wireless video distribution," *IEEE Communications Magazine*, vol. 51, no. 4, pp. 142–149, 2013.

[5] E. Altman, K. Avrachenkov, and G. Jasper, "Distributed storage in the plane," in *IFIP Networking'14*. IFIP, June 2014.

[6] K. Avrachenkov, B. Xinwei, and G. Jasper, "Optimization of caching devices with geometric constraints," in *arXiv preprint arXiv:1602.03635*, 2016.

[7] B. Blaszczyszyn and A. Giovanidis, "Optimal geographic caching in cellular networks," in *Communications (ICC), 2015 IEEE International Conference on*. IEEE, London, UK, June 2015, pp. 3358–3363.

[8] E. Baştuğ, M. Bennis, M. Kountouris, and M. Debbah, "Cache-enabled small cell networks: Modeling and tradeoffs," *EURASIP Journal on Wireless Communications and Networking*, vol. 2015, no. 1, pp. 1–11, 2015.

[9] E. Baştuğ, M. Kountouris, M. Bennis, and M. Debbah, "On the delay of geographical caching methods in two-tiered heterogeneous networks," in *Signal Processing Advances in Wireless Communications (SPAWC), 2016 IEEE 17th International Workshop on*. IEEE, 2016, pp. 1–5.

[10] J. Dai, F. Liu, B. Li, B. Li, and J. Liu, "Collaborative caching in wireless video streaming through resource auctions," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 2, pp. 458–466, 2012.

[11] A. Gharaibeh, A. Khreishah, B. Ji, and M. Ayyash, "A provably efficient online collaborative caching algorithm for multicell-coordinated systems," *IEEE Transactions on Mobile Computing*, vol. 15, no. 8, pp. 1863–1876, 1 August 2016.

[12] X. Li, X. Wang, S. Xiao, and V. Leung, "Delay performance analysis of cooperative cell caching in future mobile networks," in *IEEE International Conference on Communications (ICC)*. IEEE, 2015, pp. 5652–5657.

[13] X. Wang, X. Li, V. Leung, and P. Nasiopoulos, "A framework of cooperative cell caching for the future mobile networks," in *System Sciences (HICSS), 2015 48th Hawaii International Conference on*. IEEE, 2015, pp. 5404–5413.

[14] J. M. Wang, J. Zhang, and B. Bensaou, "Intra-as cooperative caching for content-centric networks," in *Proceedings of the 3rd ACM SIGCOMM workshop on Information-centric networking*. ACM, 2013, pp. 61–66.

[15] S. N. Chiu, D. Stoyan, W. S. Kendall, and J. Mecke, *Stochastic Geometry and its Applications*, ser. Wiley Series in Probability and Statistics. Chichester, UK: John Wiley & Sons, Ltd, aug 2013.

[16] F. Baccelli, B. Blaszczyszyn, and B. Błaszczyszyn, *Stochastic Geometry and Wireless Networks: Volume I Theory*, ser. Foundations and Trends in Networking. NoW Publishers, 2009, vol. 3, No 3–4.

[17] B. Błaszczyszyn and H. P. Keeler, "Studying the sinr process of the typical user in poisson networks using its factorial moment measures," *IEEE Transactions on Information Theory*, vol. 61, no. 12, pp. 6774–6794, 2015.

[18] G. L. Nemhauser, L. A. Wolsey, and M. L. Fisher, "An analysis of approximations for maximizing submodular set functions—I," *Mathematical Programming*, vol. 14, no. 1, pp. 265–294, 1978.

[19] H. P. Keeler, "SINR-based k-coverage probability in cellular networks," http://www.mathworks.fr/matlabcentral/fileexchange/40087-sinr-based-k-coverage-probability-in-cellular-networks, last update, 29 January 2014.