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Entry and Investment in CBRS Shared Spectrum

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***Abstract*—The Citizens Broadband Radio Service (CBRS) recently adopted in the U.S. enables commercial users to share spectrum with incumbent federal users. This sharing can be assisted by Environmental Sensing Capability operators (ESCs), that monitor the spectrum occupancy to determine when the use of the spectrum will not harm incumbents. An important aspect of the CBRS is that it enables two tiers of spectrum access by commercial users. The higher tier corresponds to a spectrum access (SA) firm that purchases a priority access license (PAL) in a competitive auction. The PAL holder obtains dedicated licensed access to a portion of the spectrum when the incumbent is not present. The lower tier, referred to as generalized Authorized Access (GAA), does not request a PAL and is similar to unlicensed access, in which multiple firms share a portion of the spectrum. Entry and investment in such a market introduces a number of new dimensions. Should an entrant bid for a PAL? How does the availability of a PAL impact their investment decisions? We develop a game-theoretic model to study these issues in which entrant SAs may bid in a PAL auction and decide on their investment levels and then compete downstream for customers.**

I. Introduction

Recently, the U.S. FCC has finalized plans for the Citizens Broadband Radio Service (CBRS) that enables commercial users to share the 3.5 GHz band with incumbent users (e.g., federal users and fixed satellite users) [1]. Accessing this band in a given location is controlled by one or more *Spectrum Access Systems* (SASs), which are geographic databases that contain information about the spectrum utilization of users of this spectrum. Spectrum access firms (SAs) wishing to offer service in that band must then register with one SAS. Additionally, each SAS can utilize an *Environmental Sensing Capability operator* (ESC). An ESC will deploy a network of sensors to detect the presence of federal incumbent users, enabling firms to better utilize the spectrum than would be possible under more conservative exclusion zones.

A key aspect of CBRS is that there are two different tiers of commercial access: a *Priority Access* (PA) tier and a *General Authorized Access* (GAA) tier. The PA tier provides a form of licensed access in that a SA with a *Priority Access License* (PAL) is given the exclusive right to use a portion of the spectrum in a given location when incumbent users are not present. The GAA tier allows for a type of unlicensed access: any SA may utilize spectrum that is not needed by an incumbent or PA user. The guidelines for this band also limit the portion of spectrum that can be allocated a PAL, so that when incumbents are not present, a portion of the band will be available for GAA use. The CBRS policy specifies that in a given area PALs will be assigned via a competitive auction.

In a given location, different entrant SAs can invest different amount and may belong to different tiers. This raises important questions. Can different tiers SAs co-exist with different levels of investment? How will the SAs bid in the PAL auction? How does a SA's investment level influence the PAL auction? How much bandwidth should be made available for each tier? How often does the PA tier SA also utilizes the GAA tier spectrum? In this paper, we study a stylized model of the CBRS ecosystem to gain insight into these questions.

We consider a market in which SAs seek to utilize a band of spectrum in a given area which is shared in a manner similar to that in the CBRS system. To keep our analysis tractable, we focus on a duopoly scenario with two SAs. Each SA must invest in infrastructure in order to utilize the spectrum and then compete to serve a common pool of customers. Additionally, we assume that a single PAL is available and is allocated via a second price auction and that there is a single SAS/ESC serving the given area, which both SAs utilize.

We formulate a multi-stage game in which the SAs first decide how much to invest and then how much to bid in the PAL auction (if they are willing to). Our model for the SAs' competition for customers is based on the literature for price competition with congestable resources, e.g., [2]–[5]. In these models, firms compete for customers by announcing prices; customers in turn select firms based on a *delivered price* given by the sum of the announced price and a congestion cost that depends on the number of users using a firm's resources. This type of model has been widely used to study competition among wireless service providers, e.g. [6]–[15]. Similar to [14], we assume that a PAL tier SA can utilize both the unlicensed (GAA) and licensed (PA) band. We model this by assuming that this SA's customers are served on the PA band with probability α and on the GAA band with probability $1-\alpha$. An important distinction in our work from the above is that we assume that the investment reduces the congestion cost incurred by the subscribers and consider SA's bids in the PAL auction. Thus, a SA needs to determine how much to invest, how much to bid and their impact on the downstream market. Adding these considerations significantly complicates the analysis and leads to very different conclusions. Models similar to ours with investment have been considered, e.g. [16]. However, this work did not consider multiple tiers of SAs or

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the auctioning of a PAL. These considerations significantly changes the model and conclusions. For example, [16] finds that a monopoly exists when the SAs invest in a single tier of unlicensed spectrum. However, our result shows that SAs in different tiers can co-exist under some circumstances.

We initially characterize the price equilibrium under different scenarios. We show that when both the SAs do not participate in the auction, (i.e., both belong to the GAA tier), only the SA who invests more can charge a positive price (Theorem 1). This shows that only one of the SAs can enter the market if one of them does not get a PAL. Subsequently, we characterize the price equilibrium when one of the SAs has priority access and the other does not. Our analysis reveals that if the fraction α with which the PA tier SA routes its traffic through the unlicensed band increases and the investment of the PA tier SA is higher than that of the GAA tier SA, then, the GAA tier SA's price will be competed to zero, causing it to not enter the market. This suggests that regulation may be needed to enable a competitive GAA tier. Our result also shows that when the number of subscribers of both SAs reaches the maximum limit, higher investment may lead to a decrease in profit.

Subsequently, we characterize the equilibrium bidding strategies (Theorem 2). Our analysis reveals that if the GAA tier SA's profit is *not small*, the SA with a larger investment would not bid in the auction and remain a GAA tier SA. The SA would only bid for a PAL if the gain from being a GAA tier SA is significantly small. Hence, the SA who has invested more may opt to remain at the GAA tier rather than obtaining a PAL. Further, our analysis shows that if a SA has invested more and gains a PAL, it would select α and price in such a manner that the other SA would have it price competed to 0 (Lemma 5). Again this suggests that *a regulator should consider the investment of the SAs along with the bidding values when distributing a PAL.*

Finally, we characterize the equilibrium investment strategy. However, because of the non-convex nature of the problem, we focus on some special scenarios. We show that when the ratio, η, between the licensed bandwidth and unlicensed bandwidth is large, only one of the SAs invests and a monopoly arises. On the other hand, when η is small both the SAs invest and the SA who invests *less* gets the PAL (Theorem 3). This suggests that a regulator should limit the PAL spectrum if it wants to encourage competition. Our analysis also characterizes that if both the SAs have incentive to invest a positive amount, in an equilibrium they will invest such a way that they serve the entire market when the cost scales linearly.

II. System Model

We consider a scenario in which two SAs denoted by SA 1 and SA 2 seek to serve users at a given location using a band of spectrum with bandwidth W. In order to use the spectrum, each SA must acquire spectrum measurements from the ESC which indicates that it is available with probability (w.p.) q, i.e., it is not used by the federal incumbents. The SAs may bid to obtain a fixed licensed bandwidth of L out of W for prioritized access (PA). We assume that only one of the SAs will be granted PA access. We designate the SA who has PA access as the PAL tier SA and the SA which does not have PA access as the GAA tier SA.

If a SA (say 1) gets prioritized access, it gets an exclusive access to the bandwidth of L when the spectrum is available, the rest of the bandwidth, $W-L$, can be used by both the SAs. Note that the GAA tier SA (SA 2) can only use this $W-L$ bandwidth. The amount of licensed bandwidth L is predetermined.

A. SAs

We, first, describe the decision variables of the SAs and their revenue models.

1) Auction for the Prioritized Access: The CBRS architecture mandates that PALs are to be auctioned. The auction format is yet to be finalized. However, because of its popularity and strategic simplicity, we assume that the auction will be similar to a second price auction. In the *second price auction*, the SA who bids the highest price wins the auction and pays the second highest price. We further assume that there is a *reservation price* c, i.e., this is minimum price for a PAL. Hence, if one SA bids less than c and the other SA bids larger than c then the other SA wins the auction and must pay c. If both the SAs bid less than c, no one will get prioritized access.

One or both the SAs may not participate in the auction. Since c is the reservation price, if a SA does not want to participate in the auction, we assume that the SA bids $c-1$. We denote the bidding prices of SA i as μ_i.

2) Prices of SAs: Each SA i selects a price p_i it will charge the users. As in the wireless market today, we view the price p_i as representing the amount users pay for receiving long-term service from SA i (e.g.,the monthly service price). As such these prices represent the service from an SA averaged over this service period. Here, we view these as flat-rate prices, and assume that each SA only offers a single service plan (which is reasonable as our user population is homogeneous).

3) Both the SAs are GAA tier: If both the SAs do not participate in the auction and are GAA tier, they both will share the entire bandwidth W if the spectrum is available and there is no dedicated licensed bandwidth.

4) One of the SAs has prioritized access: If one of the SAs gets the PAL (say, SA 1), then, it can use both the licensed and unlicensed band when the channel is available [**?**]. We model this by assuming that SA 1 assigns users to the licensed band with probability $1-\alpha$, and the unlicensed band with probability α. On the other hand, SA 2 can only use the unlicensed bandwidth when the spectrum is available. [1]

SA 1 decides α to maximize its profit. Alternatively α can be specified by a regulator or determined by some underlying

[1] We assume that SA 1 offers a single price for service and serves all customers using both bands over time based on the parameter α. Alternatively, SA 1, could offer two different services on the two bands with different prices as in [6]. Our assumption stems from the following reasons: (1) offering one service that uses multiple bands is in-line with current practice (e.g. providers offer services and then utilize which ever bands are available to provide this); (2) [12] shows that offering separate services may be harmful to both of the SPs compared to a single service.

technology. For example, a regulator may restrict the traffic on the unlicensed band to improve the social welfare.

5) Investment Model: SA i decides how much to invest $I_i \in \{0\} \cup [1, \infty)$ for providing wireless service irrespective of whether it belongs to the PAL tier or GAA tier. Each SA must invest at least 1 unit in order to serve users, which includes the cost of registering with the SAS/ESC inorder to use the spectrum. If a SA invests 0 units, the SA can not enter the market. SA i incurs a cost $h_i(\cdot)$ for investment, where $h_i(\cdot)$ is strictly increasing and convex, which is a standard assumption in the literature.

6) SA's revenue model: Let λ_i be the number of users of SA i. If both the SAs belong to the GAA tier, then, SA i obtains a payoff of

$$p_i\lambda_i - h_i(I_i). \tag{1}$$

If a SA does not invest, its payoff is zero.

If SA i belongs to the PAL tier, it obtains a payoff of

$$p_i\lambda_i - h(I_i) - \max(\mu_j, c) \tag{2}$$

where recall that μ_j is the bidding price of SA j, $j \neq i$ and c is the minimum reservation price that must be paid by the winning SA.

B. User's Subscription Model

We consider a mass Λ of non-atomic users, so that $\lambda_1+\lambda_2 \leq \Lambda$. The users are assumed to be homogeneous so that each user obtains a value v for getting service from either SA over the service period. However, as in [6]–[8] users also incur a *congestion cost* when using this service. The congestion cost models the degradation in service due to congestion of network resources. Here, we model the congestion cost for using a band of spectrum with bandwidth B by x/B where x is the total mass of users using that band. More generally, the congestion cost could be given by $g(x/B)$, where $g(\cdot)$ is an increasing, convex function. Here, we assume this function is linear to simplify the analysis, similar to [6]. The dependence on B models the fact that a larger band of spectrum is able to support more users.

Similar to [7], we assume that if a SA invests I units, the congestion cost will be given by $x/(BI)$. This models the decrease in congestion due to an increased investment, e.g., adding more access points. We assume that the resulting congestion is inversely proportional to the investment.

We, now, describe the expected payoffs of the subscribers of the SAs depending on the tiers the SAs belong to.

1) One of the SAs has Priority access: Without loss of generality, assume that SA 1 belongs to the PAL tier. If a user subscribes to SA 1, its payoff depends on whether it is served by the GAA band (unlicensed band) or the licensed band. If the subscriber is served in the licensed band, it will face congestion from $(1-\alpha)$-fraction of the subscribers of SA 1 on an average, thus, its *ex-post* payoff would be

$$v - \frac{(1-\alpha)\lambda_1}{LI_1} - p_1. \tag{3}$$

On the other hand, if a subscriber is served by the GAA band, on an average it will face congestion from the α-fraction of subscribers of SA 1 and all the subscribers of SA 2, resulting in an expected pay-off of

$$v - \frac{\alpha\lambda_1 + \lambda_2}{(W-L)I_1} - p_1. \tag{4}$$

Let $\Pi_{1S}(\lambda_1, \lambda_2)$ be the expected payoff of the subscribers of SA 1. A subscriber can only access the spectrum w.p. q since the ESC renders the channel available w.p. q. A subscriber is served using the licensed band w.p. $1-\alpha$ and using the unlicensed band w.p. α. Thus, the expected payoffs of subscribers of SA 1 are

$$\begin{aligned}\Pi_{1S}(\lambda_1, \lambda_2) =& qv - \frac{q(1-\alpha)^2 r_1\lambda_1}{L} - \\ & \frac{qr_1\alpha(\alpha\lambda_1+\lambda_2)}{W-L} - p_1\end{aligned} \tag{5}$$

where for the ease of exposition, we denote $1/I_i$ as r_i.

On the other hand, the users of SA 2 can only use the GAA band when it is available. On an average it faces congestion from α-fraction of the subscribers of SA 1 and faces congestion from all the subscribers of SA 2. Thus, the expected payoff of a subscriber of SA 2 is

$$\Pi_{1S}(\lambda_1, \lambda_2) = qv - q\frac{r_2(\alpha\lambda_1+\lambda_2)}{W-L} - p_2. \tag{6}$$

Recall that $r_2 = 1/I_2$.

Hence, the expected payoff of subscribers of SA 2 are

$$\Pi_{2S}(\lambda_1, \lambda_2) = qv - q\frac{r_2(\alpha\lambda_1+\lambda_2)}{W-L} - p_2. \tag{7}$$

2) Both the SAs belong to the GAA tier: If both the SAs belong to the GAA tier, they will be served using the unlicensed bandwidth W. Hence, the subscribers of both the SAs will face congestion from each other. Thus, the ex-ante payoff of a subscriber of SA i, $i = 1, 2$ would be

$$\Pi_{iS}(\lambda_1, \lambda_2) = qv - q\frac{r_i(\lambda_1+\lambda_2)}{W} - p_i. \tag{8}$$

Note that in this case, the expected payoffs of the subscribers differ in the payment that is made to the SA i.

C. Multi-Stage Market Equilibrium

We model the overall setting as a game with the SAs and the users as the players. Each SA's pay-off in this game is its profit (cf. (1)), while each user's objective is maximizing the expected pay-off described in Section II-B. This game consists of the following stages:

1) In the first stage, each SA decides how much to invest.
2) In the second stage, SA 1 and SA 2 decide how much to bid depending on the outcome of the first stage.
3) In the third stage, if a SA has a licensed access, decides the fraction α, the traffic at the unlicensed band.
4) In the fourth stage, SA i selects its price p_i knowing the decisions made in the previous stages.
5) In the last stage, given the first two stages' decisions, the subscribers will choose one of the SAs from which to

receive serve or choose not to receive service. We seek to characterize *Wardrop equilibrium*.

In a Wardrop equilibrium, a user subscribes to one of the SAs, if its expected payoff is the highest from that SA. We refer to a sub-game perfect Nash equilibrium of this game as a *market equilibrium*.

If a SA i decides not to invest in the first place (or, $I_i = 0$) it can not participate in the auction. Note that, alternatively, a SA might first bid in the auction and then invest. However, in practice, the bidding value depends on the the payoff of a SA which depends on the investment level. Thus, without knowing the investment level it may be difficult to bid in the auction. Nevertheless, the analysis remain the same irrespective of whether the first two stages of the game are interchanged.

We will introduce another notation before characterizing the equilibrium.

Definition 1. *Let $p_i\lambda_i$ be the revenue of the SA i.*

The revenue of a SA denotes the total money gathered by the SA from the subscribers. Note that the payoff of a SA is the difference between the revenue and the total cost (investment cost and bidding cost). It is worthwhile to note that a SA prefers to be in the market if its revenue exceeds the total cost.

III. Both the SAs belong to the GAA tier

In this section, we characterize the sub-game equilibrium prices where neither of the SAs have the priority access, thus, they both use the entire bandwidth W. We characterize the sub-game equilibrium given the decisions in the stages 1 and 2. Recall that both the SAs belong to the GAA tier when both the SAs decide not to participate in the auction at stage 2. Thus, there is not need of any decision in stage 3 since none of the SAs is PAL. Note that this setting is similar to the one studied in [16]. We characterize the equilibrium here following the work in [16].

Before characterizing the equilibrium, we make the following assumption:

Assumption 1. *Any SA j with price $p_j = 0$ receives $\lambda_j = 0$ whenever the expected payoff of a user from SA j is the same as that of the other SA i with price $p_i > 0$.*

The purpose of Assumption 1 is to guarantee a pure strategy Nash equilibrium. The conditions for this assumption will never occur on the equilibrium path, since an active SA with a price of zero will make zero profit and thus should not invest in the first place. However, it is needed to guarantee the existence of a pure strategy equilibrium for pricing sub-games off the equilibrium path.

Note from (8) that when both the SAs belong to the GAA tier, both the SAs can serve a positive mass of customers when

$$qv - q\frac{r_i x}{W} - p_i = qv - q\frac{r_j x}{W} - p_j \tag{9}$$

where $\lambda_1 + \lambda_2 = x$. Note that if $r_i = r_j$, i.e., both the SAs invest equally, the congestion cost becomes the same. In this case, the SAs engage in a price war as in [2,3], with each trying to undercut the other's price to capture the entire market. Hence, when $r_i = r_j$, the equilibrium price becomes 0.

On the other hand if $r_i < r_j$, i.e., SA i has invested more compared to SA j, the price of SA j must be smaller compared to SA i for it to attract a positive mass of customers. Again, the SAs will again engage in a price war causing SA j to lower its price to 0. However, SA i can sustain a positive price via Assumption 1- the resulting equilibrium is characterized next:

Theorem 1. *If $r_i \leq r_j$,*

$$p_j = 0$$

$$p_i = \begin{cases} \min\{\frac{q(r_j - r_i)v}{r_j}, \frac{q(r_j - r_i)\Lambda}{W}\}, & \textit{if } r_i \leq r_j \leq 2r_i, \\ \min\{\frac{qv}{2}, qv - \frac{qr_i\Lambda}{W}\} & \textit{otherwise}. \end{cases}$$

The above theorem entails that when $r_i \leq r_j$, in the equilibrium $p_j = 0, p_i \geq 0$. p_i only becomes 0 when $r_i = r_j$. Note that when r_j is very high compared to r_i, SA i's price does not depend on the investment made by SA j. Rather, SA i will select the *monopoly* price, i.e., the price it would select if SA j were not in the market. Thus, if the investment of SA j is very high compared to SA i, SA i enjoys the monopoly.

IV. One of the SAs has priority access

We, now, characterize the sub-game equilibrium setting when one of the SAs has licensed access. We characterize the last stage Wardrop equilibrium and the price equilibrium, when it is decided that one of the SAs has priority access. Without loss of generality, we assume that the SA 1 has the priority access. Note that in this scenario, SA 1 must also select α the ratio of the traffic that is routed via the unlicensed bandwidth. When $\alpha = 1$, the scenario becomes exactly equal to the one where both the SAs share the unlicensed bandwidth. Thus, we consider the scenario with $0 < \alpha < 1$.

A. Last Stage Equilibrium

In this section, we characterize the Wardrop equilibrium. In the Wardrop equilibrium, if a user subscribes to a SA it can not get a higher expected payoff from the other SA or not choosing any SA. Thus, in the Wardrop equilibrium, if both SA's serve traffic, then

$$\Pi_{1S}(\lambda_1, \lambda_2) = \Pi_{2S}(\lambda_1, \lambda_2) \geq 0.$$

If on the other hand, SA 1 does not serve traffic, then it must be that

$$\Pi_{1S}(0, \lambda_2) < \Pi_{2S}(0, \lambda_2)$$

and the corresponding equation would hold if SA 2 does not serve traffic. Finally it must also be the case that if some users are not served ($\lambda_1 + \lambda_2 < \Lambda$) then

$$\max_i \Pi_{iS}(\lambda_1, \lambda_2) = 0.$$

This last condition shows that the users' expected payoffs are positive only when the the total number of users of both the SAs is equal to the total number of users present in the system,

Λ. Intuitively, users will avail service from the SAs as long as the expected payoff is positive. Thus, if the total number of users is less than Λ, the expected payoff must be zero.

We, now, compute the Wardrop equilibrium. First, we characterize the Wardrop equilibrium in the regime when $\lambda_1 + \lambda_2 = \Lambda$, and $\lambda_1, \lambda_2 \geq 0$.

Lemma 1. *In the Wardrop equilibrium,*

$$\lambda_1 = \frac{A}{\frac{q(1-\alpha)^2 r_1}{L} + \frac{q(1-\alpha)(r_2 - \alpha r_1)}{W-L}}$$
$$\lambda_2 = \frac{B}{\frac{q(1-\alpha)^2 r_1}{L} + \frac{q(1-\alpha)(r_2 - \alpha r_1)}{W}} \tag{10}$$

where

$$A = p_2 - p_1 + \frac{q(r_2 - \alpha r_1)\Lambda}{W}$$
$$B = p_1 - p_2 + \frac{q(1-\alpha)^2 r_1 \Lambda}{L} - \frac{q\Lambda\alpha(r_2 - \alpha r_1)}{W} \tag{11}$$

if $A \geq 0, B \geq 0$ *and* $\Pi_{1S}(\lambda_1, \lambda_2) \geq 0$.

The sum of $\lambda_1 + \lambda_2$ is Λ. Note that λ_1 increases as the difference between p_2 and p_1 increases. Both λ_1 and λ_2 are independent of v in this regime.

Note that as r_1 decreases λ_1 increases, i.e., as SA 1 invests more, it attracts more subscribers. However, note that the λ_1 and λ_2 are functions of $\alpha r_1 - r_2$. Hence, the number of subscribers only depend on the weighted difference between the level of investment, rather than individual investment level. Intuitively, if one SA invests more compared to the other it will attract more subscribers. However, if the investment of both the SAs is large, the subscribers will be indifferent between the SAs.

We, now, consider the regime where $\lambda_1 + \lambda_2 < \Lambda$. Note that in this regime, user's expected payoff is zero.

Lemma 2. *In the Wardrop equilibrium,* $\lambda_1 > 0, \lambda_2 > 0$ *are given by the unique solution of the following equation–*

$$\begin{bmatrix} \frac{q(1-\alpha)^2 r_1}{L} + \frac{q\alpha^2 r_1}{W} & \frac{q\alpha r_1}{W} \\ \frac{q\alpha r_2}{W} & \frac{q r_2}{W} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} qv - p_1 \\ qv - p_2 \end{bmatrix} \tag{12}$$

if $\lambda_1 + \lambda_2 < \Lambda$ *and* $\lambda_1 > 0, \lambda_2 > 0$

B. Price Equilibrium

In this section we characterize the equilibrium prices set by the SAs to the users. First, we start with the regime where $\lambda_1 + \lambda_2 = \Lambda$. Consider the following price strategies-

$$p_1 = \frac{q(r_2 - \alpha r_1)(2-\alpha)\Lambda}{3(W-L)} + \frac{q(1-\alpha)^2 r_1 \Lambda}{3L},$$
$$p_2 = \frac{q(r_2 - \alpha r_1)(1-2\alpha)\Lambda}{3(W-L)} + \frac{2q(1-\alpha)^2 r_1 \Lambda}{3L} \tag{13}$$

Under the above price strategies, the last-stage user's equilibrium becomes

$$\lambda_1 = \frac{p_1}{\frac{q(1-\alpha)^2 r_1}{L} + \frac{q(1-\alpha)(r_2 - \alpha r_1)}{W-L}}$$
$$\lambda_2 = \frac{p_2}{\frac{q(1-\alpha)^2 r_1}{L} + \frac{q(1-\alpha)(r_2 - \alpha r_1)}{W-L}} \tag{14}$$

Now, we show that the above price strategies constitute a NE price strategies under certain conditions.

Lemma 3. *If*

$$\frac{q(r_2 - \alpha r_1)(2\alpha - 1)}{W-L} \leq \frac{2q(1-\alpha)^2 \Lambda r_1}{L} \tag{15}$$

and $\Pi_{1S}(\lambda_1, \lambda_2) \geq 0$ *where* λ_1, λ_2 *are given in (14), then the strategy profile defined in (13) constitute a Nash equilibrium in the sub-game.*

If the inequality in (15) is not *satisfied, the equilibrium price* p_2^* *must be zero.*

Note that the inequality in (15) is *not* satisfied when r_1 becomes small compared to r_2 and α is large. Thus, when r_1 is small and SA 1 uses the unlicensed band with high probability, SA 2 will be out of the market as SA 2 must select 0 price and also incurs a positive cost for investment. Thus, the SA 2 will prefer to opt out of market. Small r_1 indicates that SA 1 has invested a lot. If α is large, SA 1 selects the unlicensed band with a higher probability. Thus, when α is high and r_1 is small compared to r_2, SA 2 would engage in a price which drives down the price of SA 2 at 0. Note that even when $W - L$ is very high, the inequality in (15) is less likely to be satisfied. This is because as $W - L$ becomes high, the competition becomes more intense in the unlicensed band.

The inequality in (15) is always satisfied when $r_2 > \alpha r_1$ and $\alpha \leq 1/2$. If $\alpha = 0$, the right hand side of the inequality becomes zero, and the left hand side of the inequality is negative. Hence, the inequality is always satisfied. Thus, if α is small, a small amount of investment from SA 2 is sufficient to remain in the market.

If $2\alpha > 1$, the price of SA 2 increases as r_2 decreases. Thus, when SA 1 also uses the unlicensed band with a higher probability, the SA 2's price increases as the amount of investment increases. However, when $2\alpha \leq 1$, the price of SA 2 decreases as r_2 decreases. Thus, in this regime, SA 2 sets a higher price when the investment level is high only when SA 1 also routes a significant portion of the traffic via the unlicensed band. Intuitively, when α is small, the SA 2 does not face any stiff competition from the PAL tier SA. Further, since $\lambda_1 + \lambda_2 = \Lambda$, thus, either of the SAs can not increase the demand by investing more. Hence, SA 2 does not have any incentive to invest more when α is small.

The prices of both the SAs increase as q increases since the subscribers can achieve higher expected payoff as q increases.

We ,now, describe the equilibrium price strategy when $\lambda_1 + \lambda_2 < \Lambda$.

Lemma 4. *The unique price strategies are given by the following equations*

$$p_1^* = \frac{qv}{2} - \frac{\alpha q v r_1}{2r_2} + \frac{p_2^* \alpha r_1}{2r_2}$$
$$p_2^* = \frac{qv}{2} - \frac{\frac{\alpha r_2 q}{W-L}(qv - p_1^*)}{\frac{2q(1-\alpha)^2 r_1}{L} + \frac{2q\alpha^2 r_1}{W-L}} \tag{16}$$

The last stage user's equilibrium is given by

$$\lambda_1 = \frac{p_1^*}{\frac{q(1-\alpha)^2 r_1}{L}}$$
$$\lambda_2 = \frac{p_2^*(\frac{(1-q)}{L} + \frac{q(1-\alpha)^2}{L} + \frac{q\alpha^2 r_1}{W-L})}{\frac{r_2 q}{W}\frac{q(1-\alpha)^2}{L}} \tag{17}$$

The above strategy is the unique equilibrium when $\lambda_1 + \lambda_2 < \Lambda$.

Solving equations (13), we obtain The equilibrium prices are

$$p_1^* = qv\left(\frac{\frac{2(1-\alpha)^2}{L}(2 - \frac{\alpha}{r_2}) + \frac{\alpha^2}{W-L})(1 - \frac{\alpha}{r_2})}{4\frac{(1-\alpha)^2}{L} + 3\frac{\alpha^2}{W-L}}\right)^+$$
$$p_2^* = qv\left(\frac{(\frac{2(1-\alpha)^2 r_1}{L} + \frac{\alpha^2 r_1}{W-L}) - \frac{\alpha r_2}{W-L}}{4\frac{(1-\alpha)^2 r_1}{L} + 3\frac{\alpha^2 r_1}{W-L}}\right)^+ \tag{18}$$

Note that the SA 2 can select a positive price only when

$$\frac{2(1-\alpha)^2}{L} \geq \frac{\alpha(r_2 - \alpha r_1)}{W-L} \tag{19}$$

Thus, if $\alpha = 0$, SA 2 always select a positive price. On the other hand, if $\alpha = 1$, SA 2 can select a positive price only when $r_2 < r_1$. As α increases, SA 2 can select positive price only when the investment of SA 2 is higher compared to SA 1. Intuitively, as α increases, the SAs use the unlicensed band more often. Thus, they face intense competition as the congestion cost to the subscribers become increasingly similar for both the SAs. Hence, the SA 2 must invest more in order to select a positive price.

Note that if L is small compared to $W - L$, the price of SA 2 is also likely to be positive. Intuitively, when L is small SA 1 does not enjoy much advantage because even when it has a priority access. Thus, even a small amount of licensed spectrum can make the SA 2 to stay in the market.

The price of SA 2 increases as r_2 decreases for all values of α. Thus, unlike the scenario where the total number of subscribers reach the maximum value Λ (i.e., $\lambda_1 + \lambda_2 = \Lambda$), in this scenario, the value of p_2 increases as r_2 decreases for all the positive values of α.

From (18), the price of SA 1 increases as r_1 decreases for all values of α unlike in the regime when $\lambda_1 + \lambda_2 = \Lambda$. On the other hand if $\alpha > 0$ and r_2 is very small compared to r_1, the price of SA 1 may go down to 0. Thus, if r_2 is very small compared to r_1, SA 1 would prefer smaller α in order to avoid a price war against SA 2. If $\alpha = 1$, SA 1 can select positive price only if $r_1 < r_2$.

Note from (15) and (19) we have

Corollary 1. *If $r_1 \leq r_2$, there exists $\alpha_d \in (0, 1]$ such that the price of SA 2 becomes zero for $\alpha \geq \alpha_d$.*

Hence, if the GAA tier SA invests less compared to the PAL tier, there exists a α such that the GAA tier SA must select 0 price. Hence, the GAA tier SA would not invest in the first place.

V. Equilibrium Bidding Prices

In this section, we first characterize the equilibrium bidding strategies for a fixed investment I_i and α-the probability with which a PAL tier SA will route its traffic via the unlicensed bandwidth. Note that the payoff functions of the SAs are non-convex in α and r_i. Thus, it is difficult to obtain a closed-form expression. Hence, we first describe the equilibrium bidding prices when α and r_i are assumed to be fixed for all the SAs. In the following section, we characterize optimal α for a PAL tier SA. We also characterize the properties of equilibrium investment strategy profile for the SAs in an asymptotic limit when the ratio between the licensed and unlicensed bandwidth becomes infinite or zero.

Before characterizing the equilibrium, we, first, introduce some notations.

Definition 2. *Let $\bar{v}_i$ be the revenue (recall Definition 1) of SA i in an equilibrium when it has the priority access and the other SA belongs to the GAA tier.*

Let $\tilde{v}_i$ be the revenue of SA i in an equilibrium when it belongs to the GAA tier and the other SA belongs to the PAL tier.

Let $v_{g,i}$ be the revenue of SA i in an equilibrium when both the SAs belong to the GAA tier.

Note that the revenue of a SA inherently depends on the strategy employed by the other SA since the number of subscribers depends on the prices selected by the other SA , investment made by the other SA as well as whether the other SA belongs to the PAL tier or not. Note that we have fully characterized the equilibrium p_i, λ_i for different scenarios. Thus, the values of $\bar{v}_i$, $\tilde{v}_i$ and $v_{g,i}$ are readily obtained.

Without loss of generality, we assume that $r_1 \leq r_2$. Note that when both the SAs belong to the GAA tier, the revenue of SA 2 is 0 from Theorem 1. Thus, $v_{g,2} = 0$. Thus, SA 2 would try to avoid the above scenario. The value of $v_{g,1}$ inherently depends on the values r_1 and r_2. In order to avoid triviality we assume that $\bar{v}_i > c$ for $i = 1, 2$, i.e., the revenue obtained by the SA who has the priority access must exceed the reservation price; otherwise, either of the SAs would not participate in the auction. We now characterize the equilibrium.

Theorem 2. *If $r_1 \leq r_2$, in the first stage equilibrium,*

1) *If $\tilde{v}_i < \bar{v}_i - \bar{v}_j$, in an equilibrium SA i bids $\bar{v}_i$.*
2) *If $\tilde{v}_1 \geq \bar{v}_1 - \bar{v}_2$ and $\tilde{v}_2 < \bar{v}_2 - c$, in an equilibrium, SA 2 bids $\bar{v}_2$; SA 1 does not participate in the auction.*
3) *If $\tilde{v}_1 \geq \bar{v}_1 - \bar{v}_2$, $\tilde{v}_2 \geq \bar{v}_2 - c$, and $v_{g,1} < \bar{v}_1 - c$, in an equilibrium, SA 1 bids $\bar{v}_1$, and SA 2 does not participate in the auction.*
4) *If $\tilde{v}_1 \geq \bar{v}_1 - \bar{v}_2$, $\tilde{v}_2 \geq \bar{v}_2 - c$, and $v_{g,1} \geq \bar{v}_1 - c$, SA 2 bids $\bar{v}_2$, and SA 1 does not participate in the auction.*

When $\tilde{v}_i$ is small, SA 1 both the SAs participate in the auction and bid aggressively. This is because if a SA belongs to the GAA tier its payoff would be small. Hence, both will bid aggressively to achieve the priority access. When $\tilde{v}_1$ is reasonably high, SA 1 does not have any incentive to be the PAL tier and pay the bidding price of SA 2. The condition in case (2) is likely to arise when SA 1 has invested a lot compared to SA 2 and α is small.

If SA 2 does not obtain value by gaining priority access rather using only only the GAA band, it would prefer to do as long as the SA 1 obtains a higher payoff by gaining PAL compared to staying as a GAA tier. This gives rise to the case 3 in the result. Since SA 2 obtains 0 revenue when both the SAs belong to the GAA tier, thus, SA 2 participates in the auction even when revenue gain from the PAL is small. This behavior is characterizes in case 4. Note that the case 4 is likely to arise when α is small Thus, our result shows that in an equilibrium the SA who invests the maximum amount does not necessarily bid for the priority access.

VI. Generalization

We, now, characterize the equilibrium strategy where α and the investment are both decision variables rather fixed parameters. We start with finding the optimal α.

A. Generalization: Choosing α

First, we characterize optimal α when the investment decision is fixed and one of the SAs has PAL. Without loss of generality, we assume that the SA 1 has the priority access. Note that optimal α is obtained by SA 1 by finding the value of α for which SA 1 optimizes its profit $p_1\lambda_1$. The expression may not be concave, hence, it is difficult to obtain a closed form expression. In the following we characterize the properties of optimal α under a certain values of the investments r_1 and r_2.

Lemma 5. *If $r_1 < r_2$, optimal α is α_d where α_d is given in Corollary 1.*

Thus, when SA 1 has the priority access and invests more compared to the SA 2, it will use the unlicensed spectrum with a high probability such that SA 2's revenue will be 0. Hence, SA 2 would not invest in the first place. Thus, *surprisingly* even though the SA 1 has the licensed bandwidth, it will use the unlicensed band with a high probability in order to kick SA 2 out of the market.

The above result tells that in order to maintain competition, the regulator should regulate whether the SA who has the priority access does not invest more than the GAA tier SA. In other words, *the regulator should not provide licensed access to the one who has invested more.*

We, now, characterize the value of α when $r_1 \geq r_2$.

Lemma 6. *If $r_1 \geq r_2$, there exists an $\alpha_c \in [0, 1)$ such that optimal $\alpha = \alpha_c$.*

Further, α_c is a decreasing function of the ratio $\dfrac{r_1}{r_2}$; if $r_1 > 2r_2$, $\alpha_c = 0$.

When $r_1 \geq r_2$, thus, SA 2 invests more amount compared to the SA 1. Thus, in order to avoid competition, SA 1 would prefer not to utilize the unlicensed band much. Hence, optimal α decreases as the investment of SA 1 becomes smaller compared to that of SA 2. Ultimately, when the investment of SA 1 is smaller than half of SA 2, SA 1 only uses its own licensed bandwidth.

B. Generalization: Bidding Prices

The equilibrium bidding prices is fully characterized by Theorem 2 for given investment values r_i, $i = 1, 2$. Unlike Theorem 2, in this scenario we consider that α is a decision variable that is set by the SA that wins the PA auction. Note that if $r_i < r_j$ and SA i obtains the PAL, then SA j's revenue would be 0 from Lemma 5. Thus, $\tilde{v}_j = 0$. Hence, cases 3 and 4 of Theorem 2 do not arise in this scenario. Hence, we have the following corollary of Theorem 2

Corollary 2. *If $r_i \leq r_j$, in the first stage equilibrium*

1) *If $\tilde{v}_i < \bar{v}_i - \bar{v}_j$, SA i and j bids v_i and v_j, respectively.*
2) *If $\tilde{v}_i \geq \bar{v}_i - \bar{v}_j$, SA j bids v_j, and SA i does not participate in the auction.*

Case (1) is likely to arise when $W - L$ is small or the difference between r_j and r_i is small. In case (1) if $\bar{v}_i > \bar{v}_j$, SA i has the PAL since SA i bids higher compared to SA j. Since SA i has also invested a higher amount, it would compete SA 2 out of the market by Lemma 5. Thus, SA j would not invest in the first place. Hence, *this shows that the regulator may also have to consider the investment level when allocating a PAL to the SAs in order to maintain competition.*

In case (2), the gain from obtaining PAL is small for SA i. Thus, it will not participate in the auction. In this case, both the SAs may co-exist. Case (2) is likely to arise when the difference between r_j and r_i is large.

C. Generalization: Equilibrium Investment

We, now describe how much the SAs should invest in an equilibrium when one of them has a priority access. It is difficult to obtain a closed form expression. We characterize the properties of equilibrium (r_1, r_2) when the ratio $\eta = \dfrac{L}{W - L}$ either goes to ∞ or 0 .

First, note from Theorem 1 that when both the SAs belong to the GAA tier, at most one of the SAs can achieve a positive revenue, and so the other SA would not invest at all.

Theorem 3.
- *When $\eta \to 0$ where L is finite, i.e., $W - L \to \infty$, both the SAs invest positive amount, the one who invests more does not participate in the auction.*

- *When $\eta \to \infty$, where $L \to \infty$, $W - L$ is finite, monopoly exists where only one of the SAs invests positive amount.*
- *When $\eta \to 0$ and $L \to 0$, both the SAs invest positive amount, the SA who has invested more does not participate in the auction.*
- *When $\eta \to \infty$ and $W - L \to 0$, only one of the SAs invests a positive amount.*

Note that when $W - L$ is infinite, a SA who invests more does not participate in the auction as it would gain a higher payoff from not participating. The other SA who invests less procures the licensed access by participating in the auction.

When L is infinite, the SA who has the priority access can access the huge licensed bandwidth. Thus, the congestion cost for the PAL tier SA is negligible which forces the GAA tier SA to select 0 price. Thus, the GAA tier SA would prefer not to invest.

When $L \to 0$, the SA who belongs to the GAA tier can sustain using non-zero GAA band. Further, since L is small, the priority access does not add much value to a SA. Thus, similar to the case 1, the SA who invests more will not participate in the auction and rather only use the unlicensed band. Thus, both the SAs can invest positive amount in an equilibrium.

When $W - L \to 0$, the priority access adds immense value to a SA. The SA who does not have the priority access will get 0 revenue. Thus, only of the SAs will invest in the first place.

This shows that the ratio of the licensed band and the unlicensed band should not be large in order to maintain competition. Rather, a small portion of licensed band can increase the competition. Note that we have not specified the characteristics of the equilibrium for other scenarios which we have left for the future analysis.

We, now, characterize an important property of the equilibrium when the cost of investment is linear.

Lemma 7. *If the cost of is linear then in an equilibrium investment regime where both the SAs invest at least 1 unit, $\lambda_1 + \lambda_2 = \Lambda$.*

This shows that if both the SAs have incentive to invest at least 1 unit they will invest such that the total number of subscribers reach the entire market demand. Note that with the advancement of technology it is expected that the cost will decrease. Thus, both the SAs have incentive to invest at least 1 unit when the ratio of the licensed and unlicensed bandwidth is not very high. Our analysis reveals that if the cost only scales linearly with the investment, the total subscribers of both the SAs reach the maximum amount. Thus, *competition can drive the investment to serve the maximum demand.*

VII. Conclusion and Future Work

We consider a scenario where two SAs compete for customers in the CBRS architecture. Before competing the SPs decide on investment levels and whether to compete in a PAL auction. We characterize the impact of investment on the bidding prices and the resulting downstream competition. Our analysis shows that if the SA who invests more compared to the other SA gains a PAL and controls the traffic it can route through the GAA band, a monopoly arises where the GAA tier SA achieves zero revenue. Thus, a regulator need to consider the investment of different SAs while granting PAL. The regulator may also regulate the traffic routed via the GAA band if the SA who has invested more gains the PAL.

Future directions include extending the model to include different ESCs which can differ in providing spectrum availability information to the SAs. The consideration of more than 2 SAs is also an important future direction.

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