Performance Evaluation of Full Duplex Communications in 5G Networks: The Impact of Interference and Traffic Asymmetry

Yigit Ozcan and Catherine Rosenberg
University of Waterloo, Canada
{yozcan,cath}@uwaterloo.ca

***Abstract*—Full-duplex communications (FDC) is one of the enabling technologies for 5G systems and its true impact on performance needs to be evaluated by taking into account the new sources of interference inherent to this technology. We focus on the realistic scenario where FDC is only enabled at the base-stations (BS). The coexistence of uplink and downlink transmissions calls for a *joint uplink and downlink scheduling problem* and to take all interference into account, a *multi-cell* system needs to be considered. We also argue that traffic asymmetry (TA), i.e., the fact that the downlink traffic is much larger than uplink traffic, cannot be ignored. Specifically, we formulate a system-wide joint scheduling problem that incorporates all sources of interference and TA, and show how to solve it. The main engineering insights are that in a realistic setting, i) FDC does not *double capacity* as often mentioned; ii) TA has the largest (negative) impact on the FDC performance; iii) imperfect self interference cancellation has the second largest impact and iv) inter-base station interference, i.e., the interference caused by co-channel BSs, has the third. FDC cannot double the performance of a TDD (time division duplex) system. The gain is rarely above 50% even though it is higher in rural or heterogeneous networks.**

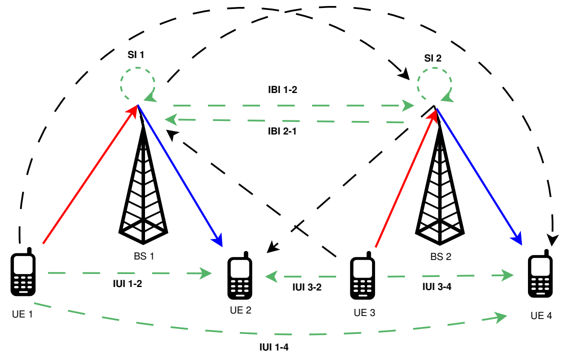

Fig. 1. Sources of interference: An example FDC scenario in a two-cell system where straight lines show data transmissions and dashed lines show the interference (all on the same subchannel). The green dashed lines are new types of interference (i.e., not present in a TDD system)

I. Introduction

Full-duplex communications (FDC) is one of the enabling technologies for 5G systems [1] and its true impact on performance needs to be evaluated carefully. In the following, we focus on the realistic scenario where FDC is only enabled at the base-stations (BS). FDC allows simultaneous downlink and uplink transmissions on the same subchannel. This calls for a *joint uplink and downlink scheduling problem*. The coexistence of uplink and downlink transmissions in co-channel cells creates new sources of interference (i.e., sources that are not present in a time division duplex (TDD) system) [2] and to take all interference into account, a multi-cell system needs to be considered when evaluating the performance. Another important characteristic of any multi-cell OFDMA system is its *traffic asymmetry* (TA), i.e., the fact that the traffic is in general much larger on the downlink than on the uplink. Today's cellular traffic is dominated by downlink [3], e.g., about 70% of the traffic is downlink. This characteristic has not been taken into account in the evaluation of FDC so far.

Operating an FDC-enabled system is going to be very complex due to the need for careful interference management and for jointly scheduling the uplink and the downlink. Hence, it is important to show that the performance gain that an FDC-enabled multi-cell OFDMA network has over a TDD network when taking into account all sources of interference as well as the traffic asymmetry will be substantial, to make the added complexity worth it. Our first research question RQ1 is: *how much better is the performance of a FDC-enabled multi-cell OFDMA network than the performance of a TDD network?*

There are three new types of interference (i.e. not present in TDD systems) in an FDC-enabled multi-cell OFDMA network (we assume in the following that all BSs are co-channel):

- *Self-interference (SI)* at each BS since the transmission of a BS interferes with the simultaneous reception at the same BS. Note that while a BS can cancel some of this interference, it has limited cancellation capability and hence there will be a residual self interference;
- *Inter-BS interference (IBI)* between the BSs since each BS interferes with the reception of the other BSs while it is transmitting;
- *Inter-UE interference (IUI)* between user equipments (UEs) in the same cell and in different cells (every transmitting UE interferes with every receiving UE on the same subchannel).

An example full-duplex scenario is shown in Fig. 1, where only two co-channel BSs are represented. In the figure, BS 1 and BS 2 are working in FDC mode and the red lines show uplink transmissions from some user equipments (UE) and the blue lines show downlink transmissions towards other UEs on the same subchannel. Recall that UEs operate in half-duplex mode. The dashed lines show the interference created to the other receivers. The new sources of interference are shown in green while the ones in black are types of interference already present in a traditional TDD system. Hence our second research question, RQ2, can be stated as follows: *What is the relative impact on performance of each new source of interference?*

Traffic asymmetry (TA) is a characteristic of a cellular network that has not been taken into account when evaluating FDC. Each application running on a user device yields a certain ratio of uplink to downlink throughput (this is what we call traffic asymmetry in the following). For example, in a video streaming application, most of the traffic would be on the downlink, which results in a very low value of

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uplink to downlink throughput ratio. A specific value of TA would yield a constraint coupling the uplink throughput to the downlink throughput and this coupling would mean that the performance of the uplink and the downlink cannot be computed independently. This is true for both a TDD system (our benchmark) and an FDC-enabled system. Our third and last research question, RQ3, is: *What is the impact of TA on the performance of an FDC-enabled system and how different are the results when the TA is ignored?*

Specifically, we analyze the performance of a multi-cell OFDMA network both for FDC and TDD, when traffic asymmetry and all sources of interference are taken into account assuming that scheduling can be done centrally for all cells, i.e., using a centralized scheduler that has access to all the channel state information. By performing the scheduling centrally, we can manage the interference optimally and thus, we obtain upper bounds on the performance of practical systems. *Note that we do not propose a centralized real-time FDC operation, but an offline study to assess the performance gain of FDC over TDD.*

Our contributions can be summarized as follows:

- We formulate a multi-cell joint uplink/downlink proportional fair scheduling problem for FDC-enabled multi-cell networks *that includes all types of interference* as well as power management and a realistic rate function (corresponding to an adaptive modulation and coding scheme) and takes traffic asymmetry into account.
- This is a very large mixed integer non-linear programming (MINLP) problem that cannot be solved with regular commercial solvers. To tackle this, we propose two non-trivial transformations to obtain a more tractable upper bounding problem. The resultant problem is a *signomial programming* problem that we solve using the algorithm proposed in [4], which most of the times converges to the global optimal solution. We then extract a feasible solution to the original problem using the result of the upper bounding problem and show that the performances of the feasible solution and the upper bound are very close to each other and hence the upper bound is tight. We also formulate and solve a similar problem for TDD.
- We study the impact of the different types of interference and the traffic asymmetry on the performance gain of FDC over TDD for an urban homogeneous network.
- We also show how the performance gain depends on the network scenario (urban/rural or homogeneous/heterogeneous). Heterogeneous networks (HetNets) include low power small cells together with macro BSs. They are harder to operate than homogeneous networks due to the heterogeneity among the BSs. They require careful channel allocation and user association (these processes are simpler in homogeneous networks).
- Finally, we show how the results produced by models not considering traffic asymmetry are biased and favor FDC.

We give the literature review in Section II. We give the system model in Section III. In Section IV, we formulate the system-wide scheduling problem for FDC in Section IV and we do the same for TDD in Section IV-C. Then, we show how to transform it into a signomial problem and how to solve it in Section IV-D. We give all the numerical results first for homogeneous networks and then for HetNets in Section V and we conclude the paper in Section VI.

II. Related Work

We review the recent papers on FDC in terms of the system model they use and the research question(s) they pose. We first discuss the system models.

A. System models

Many papers in the literature, such as [5], [6], [7], [8], consider a single cell model that neglects the inter-cell interference and more importantly the IBI caused by the full-duplex communications. This is unrealistic as it biases the results greatly as shown later.

Since FDC is based on self interference cancellation, an important issue to consider is the residual self interference since it has been shown in previous studies, such as [9], [10], that a BS cannot completely cancel its own interference. Some of the earlier papers, such as [11], [12], [13], and [14], consider perfect interference cancellation. Again this is unrealistic as will be shown later.

Scheduling is the process where the BSs allocate resources (resource blocks and power) to the UEs both on the uplink and the downlink. Careful scheduling calls for power management and fairness [15]. Scheduling is easier in a TDD system because the uplink and the downlink can be processed independently once the fraction of the frame dedicated to the uplink has been fixed. Several papers ignore scheduling, fairness or power management. In a single channel case, power management is simpler but real systems are OFDMA-based.

B. Research questions

We now discuss the research questions addressed by the recent papers. We classify these papers in two categories, i.e., those addressing performance analysis and those addressing operational problems such as scheduling or power control.

The first category includes papers that focus on FDC from an information theory or a stochastic geometry perspective. There have been many studies on FDC based on information theory [16], [17], followed more recently by physical layer studies [9], [10], [18], which focus mostly on sophisticated interference cancellation techniques that allow FDC to be implemented in real systems. [11] and [14] are information theoretical studies that do not focus on scheduling but achievable rates in a single-channel network.

Stochastic geometry is used in [19] and [20] in a multi-cell setting. However, user scheduling is not included in any of these papers. Similarly, [21] considers a multi-cell network without scheduling and also neglects power control on the downlink. [22] is a study on user association (UA) in an FDC-enabled network. It neglects power control and uses a round-robin scheduler. Authors of [13] analyze the spectral efficiency of a single channel multi-cell FDC-enabled network with a stochastic geometry approach.

In the second category are more recent papers on FDC that focus on MAC layer and scheduling.

A game theoretical scheduling approach is used in [23] to maximize the sum-rate throughput. Note that maximizing the sum-rate ignores fairness among the UEs and zero resource might be assigned to UEs with poor channel conditions. Dual decomposition method is used in [24] for scheduling with imperfect channel estimation. The model used in the paper is a two-tiered macro cell, where FDC is allowed only at the small cells. Their approach is to limit the interference created to the other small cells while scheduling. This paper also considers sum-rate maximization.

A multi-cell system is considered in [25]. A sub-optimal heuristic scheduler is proposed to maximize the weighted sum-rate. However, there is no power control, which is a very crucial part of FDC. Similarly, [7] deals with the resource allocation problem using matching theory. However, it fixes the transmit power of the uplink UEs to simplify the model. Authors of [22], [20] also deal with a multi-cell network without power control. Without a detailed power control, it has been shown in [26] that the uplink performance can be significantly deteriorated due to high interference received from the downlink transmissions.

A multi-cell system is considered in [27] for a single channel TDMA-like system. Authors of [1] present some simulation results on a heterogeneous network where FDC is enabled at the small cells. It considers a single channel network with little details on the scheduling method. Note that a multi-channel system is more complex since power management among channels is also important.

Our paper evaluates the performance of an FDC-enabled multi-cell network while considering all sources of interference, traffic asymmetry, power management, a realistic rate function (corresponding to an adaptive modulation and coding scheme) and fairness.

III. System Model

We consider a multi-cell OFDMA homogeneous network that consists of K BSs (the set of cells is $\mathcal{K}$), all of which are using the M subchannels (the set of subchannels is $\mathcal{M}$). We will study HetNets in Section V-D.

A physical resource block (PRB) is the smallest scheduling unit that consists of one subchannel and one subframe. Hence, scheduling allocates PRBs to UEs along with the transmit power and the modulation and coding scheme (MCS) to use.

We consider a snapshot approach, where we compute the performance of the system for a realization ω characterized by the set of all UEs in the system $\mathcal{U}(\omega)$, and all the necessary information (e.g., channel gains) to perform scheduling in a centralized fashion. We assume that the user association is given so that the set of UEs associated to BS k, called $\mathcal{U}_k(\omega)$, is known. We will remove the index ω in the following to keep the notations simpler.

For TDD, we use parameter β to denote the fraction of subframes used for UL transmissions ($1-\beta$ for DL). We consider a full buffer traffic model on UL and DL. We define $\lambda^{DL}_{u,k}$ and $\lambda^{UL}_{u,k}$ as the DL and UL throughput of UE $u \in \mathcal{U}_k$.

Each UE has a power budget P_{UE} and each BS has a power budget P_{BS} to be spent in a given subframe.

FDC introduces new types of interference that do not exist in a TDD system. The only interfering transmissions for TDD on the UL (resp. DL) are the UL (resp. DL) transmissions in the neighboring cells. This interference can be computed on a per PRB basis, using the channel gains between the BSs and the UEs and the transmit power on that subchannel. We represent the channel gain between UE u and BS k with $G_{u,k}$. We consider symmetric channels, i.e., $G_{u,k} = G_{k,u}$.

The three new types of interference that occur due to FDC are self interference (SI), inter-UE interference (IUI), and inter-BS interference (IBI).

SI: It is created on the UL of a BS (due to the DL transmission on the same subchannel by the same BS). The BSs can cancel this interference partially using self-cancellation and the degree to which it can do so is represented by the parameter $\mathcal{C}$ (in dB). In this paper, we model the residual self interference as an additional noise [6].

IUI: IUI occurs due to the UL transmissions of other UEs while a UE is receiving a DL transmission from its BS. We represent the channel gain between UEs u and v with $L_{u,v}$ [28].

IBI: IBI is the interference between BSs, i.e., a BS transmitting interferes with the reception of all other BSs. To compute IBI at a BS k, we need the channel gains between k and all other BSs. Let $H_{j,k}$ be the channel gain between BSs j and k. Note that $H_{j,k}$ would certainly depend on many network parameters such as antenna heights, directions, and being in line of sight or not. Our objective in this paper is *not* to propose a propagation model but to evaluate the impact of a generic PL model on the performance. We use the free space PL model as our default model since the BS antennas are generally mounted to higher locations, hence there might be no obstacles between them.

Let $P^{c,t}_{u,k}$ (resp. $p^{c,t}_{u,k}$) denote the power used by BS k to transmit to UE $u \in \mathcal{U}_k$ on the DL on PRB $\{c,t\}$, corresponding to subchannel c and subframe t (resp. used by u to transmit to k on the UL). On the DL, the total interference seen at UE $u \in \mathcal{U}_k$ on PRB $\{c,t\}$ is denoted by $I^{c,t}_{u,k}$ and on the UL the total interference seen at BS k is denoted by $Q^{c,t}_{u,k}$.

The signal to interference plus noise ratio (SINR) on the DL, $S^{c,t}_{u,k}$, (resp. UL, $s^{c,t}_{u,k}$) when BS k transmits to (resp. receives from) UE $u \in \mathcal{U}_k$ on PRB $\{c,t\}$ is computed as:

$$S^{c,t}_{u,k} = \frac{P^{c,t}_{u,k} \times G_{u,k}}{\mu^{DL} + I^{c,t}_{u,k}}, \qquad s^{c,t}_{u,k} = \frac{p^{c,t}_{u,k} \times G_{u,k}}{\mu^{UL} + Q^{c,t}_{u,k}}, \tag{1}$$

where μ^{DL} (resp. μ^{UL}) is the noise on the DL (resp. UL).

We use an adaptive modulation and coding scheme and the corresponding rate function $f(\gamma)$ that maps the SINR γ to data rates is a piece-wise constant function with L steps ($\mathcal{L}$ is the set of the L MCS levels) [29]. Specifically, $f(s^{c,t}_{u,k})$, where $s^{c,t}_{u,k}$ is the SINR at BS k when UE $u \in \mathcal{U}_k$ transmits on PRB $\{c,t\}$, can be written as:

$$\texttt{if} \quad \eta_l \leq s^{c,t}_{u,k} < \eta_{l+1}, \texttt{then} \quad r^{c,t}_{u,k} = \vartheta_l \quad \forall l \in \mathcal{L}. \tag{2}$$

where $r^{c,t}_{u,k}$ are the UL rate of UE $u \in \mathcal{U}_k$ on PRB $\{c,t\}$, η_l is the minimum decoding threshold of MCS level l and ϑ_l is the rate achieved using MCS level l. Note that same is true on DL with $S^{c,t}_{u,k}$ and $R^{c,t}_{u,k}$.

Finally, we use the binary variable $x^{c,t,l}_{u,k}$ to indicate if PRB $\{c,t\}$ of BS k is allocated to UE $u \in \mathcal{U}_k$ using MCS level l on the DL and we use $y^{c,t,l}_{u,k}$ to denote the same on the UL.

IV. System-wide User Scheduling Problem Formulation

A. Key variables and constraints

We start by defining several key variables along with their constraints. All the variables in the formulation are non-negative. To include the discrete rate function in our optimization problem, we use the following constraints on the UL:

$$s^{c,t}_{u,k} \geq y^{c,t,l}_{u,k}\eta_l, \ \forall u \in \mathcal{U}_k, k \in \mathcal{K}, c \in \mathcal{M}, t \in \mathcal{T}, l \in \mathcal{L}, \tag{3}$$

$$r^{c,t}_{u,k} = \sum_{l \in \mathcal{L}} y^{c,t,l}_{u,k}\vartheta_l, \quad \forall u \in \mathcal{U}_k, k \in \mathcal{K}, c \in \mathcal{M}, t \in \mathcal{T}, \tag{4}$$

$$\sum_{l\in\mathcal{L}} y_{u,k}^{c,t,l} \le 1, \quad \forall u \in \mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (5)$$

We make sure that only one MCS level is selected on each PRB using Eq. (5). When the SINR $s_{u,k}^{c,t}$ on a PRB $\{c,t\}$ is lower than the decoding threshold η_l, $y_{u,k}^{c,t,l}$ is set to zero. Otherwise, it can be either one or zero. Then, the rate is computed by summing over all the MCS levels using Eq. (4). Since the objective is to maximize the throughput, the maximum MCS that satisfies $s_{u,k}^{c,t} \ge \eta_l$ will be selected on each PRB. Equivalent constraints exist for the DL with $x_{u,k}^{c,t,l}$.

Then, we first define the UL throughput of $u \in \mathcal{U}_k$ as the sum of the rate that UE sees on each PRB:

$$\lambda_{u,k}^{UL} = \sum_{c\in\mathcal{M}}\sum_{t\in\mathcal{T}} r_{u,k}^{c,t}, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}. \qquad (6)$$

Recall that the UL SINR is defined by Eq. (1). To compute it, we need the UL interference on each PRB, i.e.,

$$Q_{u,k}^{c,t} = \sum_{j\in\mathcal{K}, j\neq k}\sum_{v\in\mathcal{U}_j} P_{v,j}^{c,t} H_{k,j}(\xi) + \sum_{j\in\mathcal{K}, j\neq k}\sum_{v\in\mathcal{U}_j} p_{v,j}^{c,t} G_{v,k}$$

$$+\frac{1}{\mathcal{C}}\sum_{v\in\mathcal{U}_k, v\neq u} P_{v,k}^{c,t}, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}. \qquad (7)$$

The first term of Eq. (7) represents the IBI, where $H_{k,j}(\xi)$ is the inter-BS channel gain and ξ determines the PL parameters. The second term is the inter-cell interference received from the UL transmission in the neighboring cells, which also exists in TDD. The constant $\mathcal{C}$ is the self interference cancellation parameter introduced earlier.

We then define the DL throughput seen by UE $u \in \mathcal{U}_k$ as:

$$\lambda_{u,k}^{DL} = \sum_{c\in\mathcal{M}}\sum_{t\in\mathcal{T}} R_{u,k}^{c,t}, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}, \qquad (8)$$

$$R_{u,k}^{c,t} = \sum_{l\in\mathcal{L}} x_{u,k}^{c,t,l}\vartheta_l, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (9)$$

$$S_{u,k}^{c,t} \ge x_{u,k}^{c,t,l}\eta_l, \ \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, l\in\mathcal{L}, \qquad (10)$$

$$\sum_{l\in\mathcal{L}} x_{u,k}^{c,t,l} \le 1, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (11)$$

$$I_{u,k}^{c,t} = \sum_{j\in\mathcal{K}, j\neq k}\sum_{v\in\mathcal{U}_j} P_{v,j}^{c,t} G_{u,j} + \sum_{j\in\mathcal{K}}\sum_{v\in\mathcal{U}_j, v\neq u} p_{v,j}^{c,t} L_{v,u} \qquad (12)$$

The interference seen at each UE on each PRB is defined by Eq. (12). The first term in Eq. (12) is due to the DL transmissions in the neighboring cells, which is the only interference term for a TDD system. The second term is the IUI, which includes all the UL transmissions in all cells.

Constraints (13-17) enforce the PRB allocation constraints.

$$P_{u,k}^{c,t} \le \sum_{l\in\mathcal{L}} x_{u,k}^{c,t,l} P_{BS}, \ \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (13)$$

$$p_{u,k}^{c,t} \le \sum_{l\in\mathcal{L}} y_{u,k}^{c,t,l} P_{UE}, \ \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (14)$$

$$\sum_{u\in\mathcal{U}_k}\sum_{l\in\mathcal{L}} x_{u,k}^{c,t,l} \le 1, \ \forall k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (15)$$

$$\sum_{u\in\mathcal{U}_k}\sum_{l\in\mathcal{L}} y_{u,k}^{c,t,l} \le 1, \ \forall k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}, \qquad (16)$$

$$\sum_{l\in\mathcal{L}} (x_{u,k}^{c,t,l} + y_{u,k}^{c,t,l}) \le 1, \ \forall u\in\mathcal{U}_k, k\in\mathcal{K}, c\in\mathcal{M}, t\in\mathcal{T}. \qquad (17)$$

Eq. (13) enforces that a BS k cannot allocate power to serve u on PRB $\{c,t\}$ if that PRB is not allocated to that UE on the DL. Eq. (14) is the same for the UL. Constraints (15) and (16) enforce that a PRB can be allocated to only one UE on the DL and the UL, respectively in a given cell. Eq. (17) indicates that a UE cannot transmit and receive at the same time.

Finally, we have the following two power budget constraints, which ensure that the BSs and UEs cannot use more power than their power budget at a given time.

$$\sum_{u\in\mathcal{U}_k}\sum_{c\in\mathcal{M}} P_{u,k}^{c,t} \le P_{BS}, \quad \forall k\in\mathcal{K}, t\in\mathcal{T}, \qquad (18)$$

$$\sum_{c\in\mathcal{M}} p_{u,k}^{c,t} \le P_{UE}, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}, t\in\mathcal{T}. \qquad (19)$$

Next, we introduce the *UL/DL traffic asymmetry*. Let $\theta_{u,k} \ge 0$ be the targeted ratio of UL/DL throughput of UE $u \in \mathcal{U}_k$. We can then add the following constraint in our formulation:

$$\lambda_{u,k}^{UL} = \theta_{u,k} \times \lambda_{u,k}^{DL}, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}. \qquad (20)$$

We define Θ as the vector consisting of the $\theta_{u,k}$ values for all the UEs in the system, i.e., $\Theta = \{\theta_{u,k}\}$.

We define the overall throughput $\lambda_{u,k}$ of UE u in cell k as the sum of the throughput it sees on the UL and the DL, i.e.,

$$\lambda_{u,k} = \lambda_{u,k}^{UL} + \lambda_{u,k}^{DL}, \quad \forall u\in\mathcal{U}_k, k\in\mathcal{K}. \qquad (21)$$

Our objective is to maximize the GM Γ of the overall throughputs of all the UEs in the system, by computing the power allocations and the PRB allocations jointly for the UL and the DL, where:

$$\Gamma = \Big(\prod_{k\in\mathcal{K}}\prod_{u\in\mathcal{U}_k} \lambda_{u,k}\Big)^{1/N}, \qquad (22)$$

and N is the total number of UEs in the system.

Without Eq. (20), the optimization problem can select the ratio of UL to DL throughput without any restriction and it does so in a non-realistic manner as will be discussed later.

B. Problem formulation for FDC

We are now ready to formulate our centralized scheduling problem $\mathcal{P}_{FDC}(\Theta)$ that maximizes the GM of the overall throughput each UE sees. Its input parameters are $\Theta = \{\theta_{u,k}\}$, the realization ω, inter-BS PL model ξ, and SC value $\mathcal{C}$ at the BSs. Its variables are $\{x_{u,k}^{c,t,l}$, $y_{u,k}^{c,t,l}$, $P_{u,k}^{c,t}$, $p_{u,k}^{c,t}$, $S_{u,k}^{c,t}$, $s_{u,k}^{c,t}$, $I_{u,k}^{c,t}$, $Q_{u,k}^{c,t}$, $R_{u,k}^{c,t}$, $r_{u,k}^{c,t}$, $\lambda_{u,k}^{DL}$, $\lambda_{u,k}^{UL}$, $\lambda_{u,k}\}$.

$$\mathcal{P}_{FDC}(\Theta): \quad \max \ \Gamma \qquad (23)$$
$$\text{s.t.} \quad \text{constraints (1), (3-22)}$$

In Section IV-D, we will propose a two-step transformation to transform $\mathcal{P}_{FDC}(\Theta)$ into a more tractable upper bounding problem and then show the method to solve it.

C. Problem Formulation for TDD

Recall that β is the proportion of time spent on the UL. In this case, $\beta \times T$ of the subframes are allocated to the UL and $(1-\beta)\times T$ subframes are allocated to the DL. In the UL subframes, no DL transmissions can occur and hence $x_{u,k}^{c,t,l}$ should be equal to zero for the UL subframes and similarly,

$y_{u,k}^{c,t,l}$ should be equal to zero for the DL subframes. For a given β and a given Θ, we can formulate $\mathcal{P}_{TDD}(\Theta,\beta)$ that jointly schedules UL and DL under TDD, as follows:

$$\mathcal{P}_{TDD}(\Theta,\beta):\quad \max\ \Gamma \tag{24}$$

$$\text{s.t.}\quad \text{constraints (1), (3-22)}$$

$$x_{u,k}^{c,t,l}=0,\quad \forall t\in\{1\ldots\beta T\} \tag{25}$$

$$y_{u,k}^{c,t,l}=0,\quad \forall t\in\{(\beta T+1)\ldots T\} \tag{26}$$

In this formulation, the first βT subframes are allocated to the UL and the rest of the subframes are allocated to the DL. $\mathcal{P}_{TDD}(\Theta,\beta)$ gives a solution for a given value of Θ and β.

Note that the multi-cell centralized scheduling problems for FDC and TDD are very large size MINLP problems and hence cannot be solved easily. Next, we will propose a method to solve these problems quasi-optimally. We will describe it for $\mathcal{P}_{FDC}(\Theta)$ it can also be applied to $\mathcal{P}_{TDD}(\Theta,\beta)$.

D. Transformations yielding the upper bound signomial programming problem

Recall that the discrete rate function $f(\gamma)$ is given in Eqs. (2) and to include it in the optimization problem, we have used the constraints (3), (4), (5), (9), (10), (11). We propose to replace the rate function $f(\gamma)$ with a tight upper bound $g(\gamma)$. For the MCS function given in [29], we select $g(\gamma)$ as:

$$g(\gamma)=\min(a\gamma^{0.43},R_L), \tag{27}$$

where R_L is the highest rate achievable by $f(\gamma)$ and $a=0.168$. We can then replace the binary variables $x_{u,k}^{c,t,l}$ and $y_{u,k}^{c,t,l}$ by $x_{u,k}^{c,t}$ and $y_{u,k}^{c,t}$, respectively, and eliminate many binary variables. The tightness of the upper bound is shown in [30].

Binary variables are used in $\mathcal{P}_{FDC}(\Theta)$ for PRB allocation since a PRB can be allocated to only one UE by a BS. We can eliminate those variables altogether if we replace Constraints (13-17) with the following three constraints:

$$P_{u,k}^{c,t}P_{v,k}^{c,t}\le\sigma,\quad \forall\{u\neq v\}\in\mathcal{U}_k, c\in\mathcal{M}, k\in\mathcal{K}, t\in\mathcal{T}, \tag{28}$$

$$p_{u,k}^{c,t}p_{v,k}^{c,t}\le\sigma,\quad \forall\{u\neq v\}\in\mathcal{U}_k, c\in\mathcal{M}, k\in\mathcal{K}, t\in\mathcal{T}, \tag{29}$$

$$P_{u,k}^{c,t}p_{u,k}^{c,t}\le\sigma,\quad \forall u\in\mathcal{U}_k, c\in\mathcal{M}, k\in\mathcal{K}, t\in\mathcal{T}, \tag{30}$$

where σ is a very small positive number. Eq. (28) ensures that a PRB can be allocated for only one DL transmission in a given cell. Similarly, Eq. (29) ensures that only one UE can use a PRB for an UL transmission in a given cell. Eq. (30) ensures that a UE cannot use a PRB for a DL and an UL transmission simultaneously.

With these two modifications, we obtain a new problem that finds an upper bound to the original problem $\mathcal{P}_{FDC}(\Theta)$. With some further modifications, we can obtain a signomial problem that we call $\mathcal{P}_{UB}(\Theta)$. Specifically, we first replace the equality signs in constraints (1), (8), (12), (6), (7), and (21) with $\le$ sign, which does not change the optimal point of the problem since this is a maximization problem. We also add a constraint to make all the variables strictly positive.

After obtaining the signomial programming problem, we use the iterative algorithm proposed in [4] to solve it. The algorithm is explained in the Appendix.

E. Deriving a feasible solution to $\mathcal{P}_{FDC}(\Theta)$

$\mathcal{P}_{UB}(\Theta)$ provides an upper bound to the original scheduling problem $\mathcal{P}_{FDC}(\Theta)$. Using the results of that upper bound problem, we can derive a feasible solution to the original problem as follows:

$$x_{u,k}^{c,t,l}=\begin{cases}1, & \text{if } \arg\max_{i\in\mathcal{U}_k} P_{i,k}^{c,t}=u\ \ \&\ \ \eta_l\le S_{u,k}^{c,t}<\eta_{l+1}\\ 0, & \text{otherwise}\end{cases} \tag{31}$$

$$y_{u,k}^{c,t,l}=\begin{cases}1, & \text{if } \arg\max_{i\in\mathcal{U}_k} p_{i,k}^{c,t}=u\ \ \&\ \ \eta_l\le s_{u,k}^{c,t}<\eta_{l+1}\\ 0, & \text{otherwise}\end{cases} \tag{32}$$

$$x_{u,k}^{c,t,l}=0\ \text{ if }\ P_{u,k}^{c,t}<p_{u,k}^{c,t}\quad (y_{u,k}^{c,t,l}=0\ \text{ otherwise}) \tag{33}$$

Then, we set $P_{u,k}^{c,t}$ (resp. $p_{u,k}^{c,t}$) to zero if $x_{u,k}^{c,t,l}=0$, $\forall l\in\mathcal{L}$ (resp. $y_{u,k}^{c,t,l}=0$, $\forall l\in\mathcal{L}$). This yields a feasible solution since a PRB is allocated for only one DL transmission within a cell thanks to Eq. (31). The second eq. ensures that a PRB can be used for only one UL transmission within a cell. We prevent the same UE to transmit simultaneously on the UL and DL using the last equation.

V. Numerical Results

Here, we evaluate the performance of FDC and TDD for various parameters including the interference parameters ξ and $\mathcal{C}$. To simplify our discussion, we use the same value of $\theta_{u,k}$ for all the UEs, i.e., we use $\theta_{u,k}=\theta$ to denote the traffic asymmetry for all the UEs. We will compare TDD and FDC for different realizations characterized by the total number of UEs in the system and all the necessary channel gains. All the results are given as an average of 100 realizations.

We consider a cellular network composed of 3 BSs and we use a wrap-around model to avoid border effects. To verify that our results hold for bigger networks, we performed some computations for a 5 cell network. The performance gains of FDC over TDD were similar than those for the 3 cell network.

We will consider two homogeneous scenarios, an urban one and a rural one. We will discuss the HetNet case later. The main differences between the two scenarios are the inter-BS distance and the propagation models. For the urban scenario, the inter-BS distance is 500 meters while it is 1732 meters for the rural scenario [31].

We set the total number of UEs in the system to 30, corresponding to an average of 10 UEs per cell, except when we evaluate the impact of different numbers of UEs on FDC performance. The 30 users are uniformly distributed over the 3 cell area. Each UE is associated to the BS yielding the highest channel gain. The total number of subchannels M is set to 30. The number of subframes T in a frame is set to 10.

We use the channel model between the UEs and the BSs recommended by 3GPP [31]. $G_{u,k}$ is the channel gain between UE u and BS k, that accounts for the path loss, shadow fading, antenna gains, and equipment losses, all expressed in dB. The antenna gains are 15 dBi for the BSs and 0 dBi for the UEs. We obtain the channel gains by further applying a log-normal shadowing of 8 dB standard deviation. There is an additional 20 dB penetration loss for the urban scenario and 9 dB penetration loss for the rural scenario. The path loss between the BS and the UEs for the urban scenario is computed with the following formula: $128.1+37.6\times\log_{10}(d)$ dB, where d is the distance measured in kilometers between the UE and the BS. For the rural scenario, we use $117.5953+38.6334\times\log_{10}(d)$ to compute the path loss between a UE and a BS.

For **IUI**, we use a path loss between two UEs separated by a distance of l equal to $148+40\times\log_{10}(l)$ dB [28].

We use $\mathcal{C} = 110$ dB as the default value for the self-interference cancellation parameter [32]. However, we will later use different values for $\mathcal{C}$ to see its effects on FDC performance. For **IBI**, we use the free space loss model as our default model, which occurs when the antennas are in line of sight without any obstacles. The free space loss model is a reasonable model since the BS antennas are typically mounted to high locations and hence there might be no obstacles between them. We use the following formula to compute the path loss: $128.1 + 20 \times \log_{10}(d)$. We will also consider other path loss models later in this paper.

We set P_{BS} to 46 dBm and P_{UE} to 24 dBm. μ^{DL} and μ^{UL} are -112.44 dBm and -116.44 dBm, respectively. The piece-wise constant rate function $f(\gamma)$ is given in Table III in [29].

A. *Tightness of the upper bound*

Recall that we solve, for each realization, a signomial optimization problem, $\mathcal{P}_{UB}(\Theta)$, that finds an upper bound to the original scheduling problem $\mathcal{P}_{FDC}(\Theta)$ and we derive a feasible solution to the original problem from the results of the upper bound problem as explained in the previous section. The differences between the GM throughput (i.e., the objective function) of the upper bound problem and the feasible solution are reported in Table I for the homogeneous urban setting for different values of θ when we use $\mathcal{C} = 110$ dB and the free space loss model to compute IBI.

TABLE I
GM THROUGHPUT (MB/S) OF THE UPPER BOUND AND THE FEASIBLE SOLUTION FOR $\mathcal{P}_{FDC}(\Theta)$ (HOMOGENEOUS URBAN SETTING, FREE SPACE INTER-BS PATH LOSS, $N = 30$, AND $\mathcal{C} = 110$ DB)

	$\theta = 0.2$	$\theta = 0.5$	$\theta = 1$
Upper Bound	1.9755	2.3951	2.7915
Feasible Solution	1.8639	2.2642	2.6144

The difference between the upper bound and the feasible solution is about 5% which shows that the upper bound we obtain is tight. In the following, we will show results based on the upper bound.

B. *Performance comparison of FDC and TDD*

In the following, we will compare the performance of FDC and TDD by investigating the impact of each type interference and traffic asymmetry as well as the network setting (i.e., homogeneous urban/rural or heterogeneous). We will also show that when traffic asymmetry is not enforced, it biases the results by selecting unrealistic values of θ.

1) Quantifying the effects of each interference type and traffic asymmetry: We begin with comparing the performance of FDC and TDD in the urban homogeneous network. Here, we assume that we can disable certain types of interference so that we can understand how much they contribute to the FDC performance. We consider the 9 FDC schemes shown in Table II. In the table, a cross mark corresponds to disabling that type of interference.

TABLE II
LIST OF FDC VARIATIONS

	IBI	IUI	SC			IBI	IUI	SC
Scheme 1	✗	✗	∞		Scheme 6	✗	✓	110 dB
Scheme 2	✗	✓	∞		Scheme 7	✓	✗	110 dB
Scheme 3	✓	✗	∞		Scheme 8	✓	✓	110 dB
Scheme 4	✓	✓	∞		Scheme 9	✓	✓	50 dB
Scheme 5	✗	✗	110 dB					

Scheme 1 is the most optimistic one as it does not consider any IBI, IUI, or SI. Therefore, it is an upper bound for all other schemes. Among he nine schemes, Scheme 8 is the most realistic one as it considers IBI and IUI while having a realistic SC capability at the BSs. For the given 9 FDC schemes, we perform computations for 100 realizations and compute the average gain in GM throughput of each scheme over TDD for different values of θ. Note that the gains for the average throughput over all users are similar. The results are given in Fig. 2. Note that for TDD, we choose the best β for each θ, i.e., the one that maximizes the GM of the overall throughput averaged over 100 realizations.

Fig. 2 shows the gain as a function of θ for the 9 schemes. The first remark is that traffic asymmetry is crucial to the performance of FDC. The gain of FDC over TDD strongly depends on the value of θ. When $\theta = 0$, FDC and TDD are the same and the larger θ the more uplink traffic there is and hence the more there is to gain with FDC. Note that $\theta = 1$ means that there is as much uplink traffic as downlink traffic and that the practical range of interest for θ is around the lower values, i.e., (0.3-0.65), based on real network measurements [3], [33]. For example, if 70% of the traffic is downlink, then $\theta = 0.43$.

For Scheme 1, which is the most optimistic scheme, the gain reaches 100% (i.e., the "capacity is doubled") when all new types of interference are disabled and when uplink and downlink traffic have the same volume. However, even for Scheme 1, the gain drops to 50% from 100% when we consider a realistic value of traffic asymmetry, e.g., $\theta = 0.5$. Hence, the traffic asymmetry has a significant impact on the FDC performance (in fact it has the largest impact).

The most dominant source of interference among the new types of interference introduced by FDC is self interference and the amount of interference cancellation $\mathcal{C}$ has a great impact on the FDC performance. When we compare Schemes 4, 8, and 9, where $\mathcal{C}$ is infinity, 110 dB, and 50 dB, respectively, we see that there is a huge difference between the gains of the three schemes. Among the new sources of interference, IUI has the least effect since adding IUI reduces the gain the least. However, its impact is still not negligible as can be seen by comparing Schemes 1 and 2.

All studies that are based on single cell ignore IBI and part of IUI and hence correspond at best to Scheme 2 which is highly optimistic.

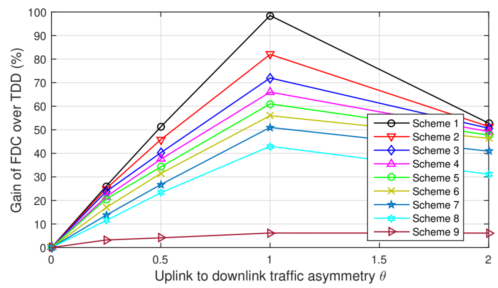

Fig. 2. Performance gain of FDC over TDD in GM throughput for the homogeneous urban scenario with $N = 30$ as a function of θ for the schemes in Table II

Next, we compare the performance of FDC and TDD further for different values of SC capability, for different inter-BS path loss models, and different numbers of UEs, still focusing on the homogeneous urban scenario.

2) Impact of SC capability at the BSs: We first focus on the impact of the self-interference cancellation capability BSs on system performance since residual self interference has the largest impact among the new types of interference as explained in the previous subsection. Recall that we have included this effect in our formulation via the self interference cancellation parameter $\mathcal{C}$ (see Constraint (7)). In Fig. 3, we show the performance gain in GM throughput of FDC over TDD as a function of $\mathcal{C}$ for $\theta = 0.6$ when IBI is modeled using the free space loss model.

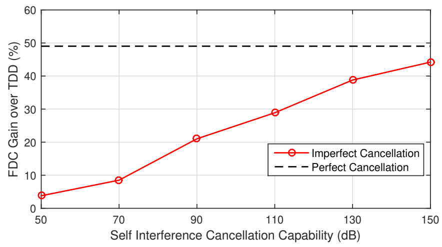

Fig. 3. Performance gain in GM throughput of FDC over TDD as a function of self interference cancellation parameter $\mathcal{C}$ when $\theta = 0.6$ and $N = 30$ for the homogeneous urban scenario

Clearly, the capability to cancel self interference greatly impacts the system performance. When the BSs can only cancel 50 dB self interference, FDC performs not better than TDD. A good performance can be achieved only when the BSs can cancel high level of self interference. In order to see a gain of 40% over TDD, we need a SC of 130 dB for the free space loss model. According to [32], a realistic value of SC is 110 dB, which yields around 30% gain. A $\mathcal{C}$ value of 150 dB would yield results very similar to perfect SI cancellation.

3) Impact of inter-BS path loss on FDC performance: We analyze the impact of inter-BS path loss on FDC performance in the urban homogeneous setting, assuming $\mathcal{C}$ =110 dB. We perform the computations for $\theta = 0.2$ and 0.6.

We use the path loss model, ξ, defined as $PL(d) = a_\xi + b_\xi \times \log(d)$, where d is the inter-BS distance. Note that d is the same for each BS pair in our homogeneous system. Fig. 4 shows the performance gain in GM throughput of FDC over TDD for different values of inter-BS path loss values computed for different values of the parameters determined by ξ. The first point in the graph (around 120 dB) corresponds to the free space loss model which is our default model.In this subsection, we only consider path loss values that are yielding less interference since obstacles or multi-path fading would only create less interference.

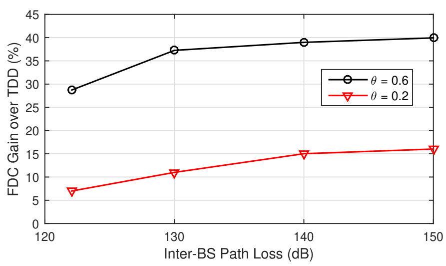

Fig. 4. Performance gain in GM throughput of FDC over TDD for the homogeneous urban scenario as a function of inter-BS path loss for θ=0.2 and 0.6 with $\mathcal{C} = 110$ dB and $N = 30$

It is clear that a better IBI (i.e., a higher inter-BS path loss) yields better gains for FDC over TDD. Indeed, the gain for the free space loss model is 29% for $\theta = 0.6$ and 7% for $\theta = 0.2$ while it is 40% for $\theta = 0.6$ and 16% for $\theta = 0.2$ for an inter-BS path loss of 150 dB, which is very optimistic.

4) Impact of the number of UEs on FDC performance: Next, we present the FDC performance for different numbers of UEs. Fig. 5 shows the results for the homogeneous urban scenario as a function of the total number of UEs.

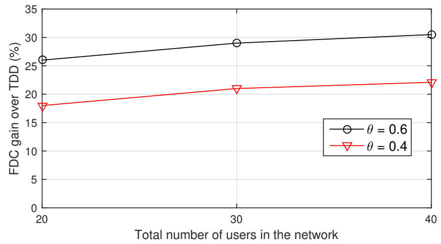

Fig. 5. Performance gain in GM throughput of FDC over TDD as a function of the total number of UEs in the network for the homogeneous urban scenario with $\mathcal{C} = 110$ dB and the free space inter-BS path loss

Increasing the number of UEs in the network slightly improves the gain of FDC over TDD. This can be explained by the fact that with more UEs, the number of opportunities to pair uplink and downlink UEs to share the same PRB increases. Therefore, a better FDC gain can be achieved in a more crowded network.

5) Performance of FDC without traffic asymmetry: Next, we present the performance of FDC when the traffic asymmetry between the uplink and downlink is not enforced unlike what we did with Constraint (20) for the homogeneous urban network. It is important to note that almost all the papers in the literature ignore the asymmetry.

When using the defaults values corresponding to Scheme 8 in Table II and $N = 30$, the gain in GM throughput of FDC over TDD is expected to be around 29% when TA is enforced and $\theta = 0.6$ while a gain of 47% is obtained when we do not enforce the traffic asymmetry. Hence, not taking traffic asymmetry into account biases the results in favor of FDC and recent papers have most probably over-estimated the gain of FDC over TDD by ignoring TA.

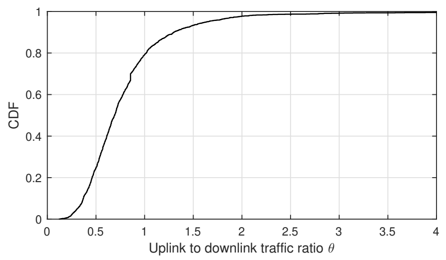

Fig. 6. CDF of the uplink to downlink throughput ratio without Constraint (20) for the homogeneous urban scenario with free space loss model, $N = 30$, and $\mathcal{C} = 110$ dB

When we do not enforce traffic asymmetry, each UE sees a different ratio between its uplink and downlink throughputs. Moreover, the average uplink to downlink throughput ratio across all UEs is around 0.8, which is unrealistically high considering today's applications that are downlink dominated. To illustrate this, we plot in Fig. 6 the cumulative distribution function (CDF) of the per user traffic asymmetry that is obtained from the solution of the problem without the constraint that enforces that asymmetry. The CDF is computed over 100 realizations for the urban scenario.

Clearly, a problem without a constraint on TA gives a higher throughput to the uplink than the downlink for almost 20% of the UEs, which is not a realistic way to allocate resources in today's networks. Also, only 40% of the UEs have a realistic value of traffic asymmetry, i.e., between 0 and 0.6.

C. Performance comparison of urban and rural scenarios

So far, we have studied the performance of FDC only in urban homogeneous networks. In this section, we will compare urban and rural homogeneous network scenarios.

Fig. 7 shows the FDC over TDD gain in terms of GM throughput as a function of θ for the urban and rural scenarios with $\mathcal{C} = 110$ dB, $N = 30$, and the free space loss model for inter-BS channels. It shows that the gain of FDC over TDD is higher in rural networks than in urban networks. This can be explained with the fact that the distances in the rural networks are higher and hence the impact of new types of interference is lower. However, even in the rural setting, the gain is limited.

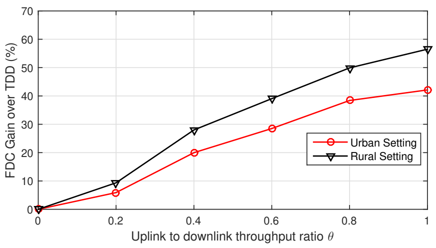

Fig. 7. Performance gain in GM throughput of FDC over TDD as a function of θ with free space path loss, $N = 30$, and $\mathcal{C} = 110$ dB

For a realistic value of θ between 0 and 0.6, the gain in GM throughput of FDC over TDD is at most 30% for the urban scenario and 40% for the rural scenario.

D. Performance of FDC in heterogeneous networks

Finally, we evaluate the performance of FDC in the context of HetNets consisting of macro BSs and small cells.

Due to the heterogeneity among the BSs, we need to be careful how we select processes such as user association (UA) and channel allocation [29]. We use a simple UA method, called *small cell first* that was proven to be very efficient in a TDD context and only focuses on the downlink [29]. To use this method, we compute the SINR per subchannel each UE receives from each BS *assuming that the BSs allocate equal power to their subchannels.* Then, a UE associates to the small cell that provides the highest SINR if it is greater than a threshold ϕ. Otherwise, it associates with the cell that yields the highest SINR (even if it is a macro or a small cell).

For channel allocation, we use the simple orthogonal deployment (OD) method, where k subchannels are allocated to the small cells and the remaining $M - k$ subchannels are allocated to the macro BSs. In this case, the macro BSs and the small cells do not create interference to each other. Hence, for a given UA and for a given k, we can consider the macro BSs and the small cells as two separate networks.

To evaluate the performance of FDC in HetNets, we consider the same 3-cell network as before, however in this case, there are two small cells installed at 230 meters right and left of each macro BS. W We set the power budget of the small cells to 30 dBm. For the small cell channels, we use the 3GPP channel model described in [29]. Similar to what we have done for the homogeneous case, we assume the same traffic asymmetry θ for all UEs in order to simplify our discussion. Then, for a given realization, UA (i.e., ϕ), channel allocation (i.e., k), and traffic asymmetry θ, we solve the centralized scheduling problem after performing user association according to ϕ.

In homogeneous networks, we have used the free space path loss as our default path loss model for inter-BS channels since macro BS antennas are generally in high locations. However, in the case of small cells, the antennas are generally located indoors and hence there might be many obstacles between each small cell antenna pair. To take this fact into account, we use three different inter-BS path loss models for the small cells. The first one, PL1, is the free space path loss model. In the second one, PL2, we introduce an additional 20 dB penetration loss on top of the free space path loss since small cells are generally located indoors and there will be walls between these antennas. In the third model, PL3, we assume an infinite path loss between small cell antennas, i.e., we ignore the IBI between them. Note that we do not need a channel model between the small cell antennas and the macro BS antennas since we consider OD as our channel allocation method and hence they operate on separate bands.

The FDC gain over TDD in GM throughput (over 100 realizations) is given in Table III for different inter-BS path loss models when θ is 0.6 and $\mathcal{C}$ is 110 dB with $N = 30$ UEs.

TABLE III
PERFORMANCE GAIN OF FDC OVER TDD IN GM THROUGHPUT FOR THE HOMOGENEOUS URBAN SCENARIO, ($\theta = 0.6$, $N = 30$, $\mathcal{C} = 110$ dB)

	Homoge.	HetNet PL1	HetNet PL2	HetNet PL3
Gain (%)	27.81	41.13	45.62	49.19

Table III shows the performance gain of FDC over TDD in HetNets for the three different inter-BS path loss models for small cells as well as for an homogeneous network (i.e., without small cells). Note that for each path loss model, we pick the best k and ϕ, i.e., the values that maximizes the average GM throughput for FDC and TDD separately, and then compute the gain. It is seen that the gain can be increased with small cell deployment. Depending on the inter-BS channel model, the gain can go up to 49%, whereas it was around 28% for the homogeneous networks. As a result, the gain of FDC is higher in HetNets than in homogeneous networks when the HetNets are well parametrized, i.e., when k and ϕ are selected appropriately.

For reference, the gain of HetNet TDD over homogeneous TDD is 23% while the gain of HetNet FDC with PL2 over homogeneous FDC is 36%.

VI. CONCLUSION

In this paper, we study the performance of FDC in multi-cell OFDMA networks. We present an offline study that compares the performance of FDC with a TDD network. We show that:

- In a multi-cell network with a realistic value of uplink/downlink traffic asymmetry, FDC performance is far from doubling the performance of a TDD system. In fact, for realistic network settings and traffic asymmetry, the gain is rarely above 50%.

- Traffic asymmetry, which is mostly ignored in the literature, has the largest impact on performance.
- Among the interference sources, residual self-interference has the largest impact followed by IBI.
- Finally, performance gains are slightly better for HetNets and for rural networks.

Based on these results, we believe that studies that neglect traffic asymmetry and that use a single cell model can lead to unreliable conclusions.

Appendix: The solution method

In order to solve the signomial programming problem, $\mathcal{P}_{UB}(\Theta)$, we use the method proposed in [4]. It is an iterative algorithm where we convert the problem into a geometric programming problem at each iteration by approximation. In a geometric programming problem, all inequality constraints should be of the form $g(x) \leq 1$, where $g(x)$ has to be a posynomial. In a signomial programming problem, a constraint can be of the form $g_1(x)/g_2(x) \leq 1$, with both $g_1(x)$ and $g_2(x)$ being posynomials, even though the ratio of two posynomials is not a posynomial. In the algorithm proposed in [4], such constraints are modified at each step by approximating $g_2(x)$ with a monomial. Then, the problem in each step becomes a geometric programming problem. The point around which the approximation is done changes at each step.

The best monomial approximation, say $\bar{h}(x,y)$, around a point $x = y > 0$, for a posynomial $h(x) = \sum_i g_i(x)$, where the $g_i(x)$'s are monomials, is [34]:

$$\bar{h}(x,y) = \prod_i \big(\frac{g_i(x)}{\alpha_i(y)}\big)^{\alpha_i(y)}, \tag{34}$$

where $\alpha_i(y)$ is equal to:

$$\alpha_i(y) = \frac{g_i(y)}{h(y)}. \tag{35}$$

Hence, at a given point y, Constraints (6) and (8) can be approximated using Eqs. (34) and (35). Then, we obtain the problem $\mathcal{P}^*_{UB}(\Theta, y)$, which is a geometric program that can be solved using a log transformation as shown in [34].

The purpose of the iterative algorithm is to find y. The point, around which the approximation is made, changes in each iteration. The overall algorithm is explained in Alg. 1.

Algorithm 1 Solving $\mathcal{P}_{UB}(\Theta)$ iteratively

1: Find a feasible initial solution for $\mathcal{P}_{UB}(\Theta)$. Let that be $s^{(0)}$.
2: At the t^{th} iteration, compute the monomial approximations for the constraints of $\mathcal{P}_{UB}(\Theta)$ that are not posynomials using equations (34) and (35), with $y = s^{(t-1)}$, and obtain $\mathcal{P}^*_{UB}(\Theta, s^{(t-1)})$.
3: Solve $\mathcal{P}^*_{UB}(\Theta, s^{(t-1)})$ with the new constraints by converting it into a convex problem using a log transformation as explained in [34] and obtain $s^{(t)}$
4: **if** $||s^{(t-1)} - s^{(t)}|| < \varepsilon$ **then**
5: Algorithm terminates
6: **else**
7: Go to step 2
8: **end if**

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