Fiber Cost Reduction and Wavelength Minimization in Multifiber WDM Networks

Christos Nomikos[1], Aris Pagourtzis[2], Katerina Potika[2], and Stathis Zachos[2,3]

[1] Department of Computer Science, University of Ioannina, 45110, Greece, cnomikos@cs.uoi.gr
[2] Computer Science, ECE, National Technical University of Athens, 15780, Greece, {pagour,epotik,zachos}@cs.ece.ntua.gr
[3] CIS Department, Brooklyn College, CUNY, NY, US

Abstract. Motivated by the increasing importance of multifiber WDM networks we study two routing and wavelength assignment problems in such networks:

- *Fiber Cost Minimization*: the number of wavelengths per fiber is given and we want to minimize the cost of fiber links that need to be reserved in order to satisfy a set of communication requests; we introduce a generalized setting where network pricing is *non-uniform*, that is the cost of hiring a fiber may differ from link to link.
- *Wavelength Minimization*: the number of available parallel fibers on each link is given and we want to minimize the wavelengths per fiber that are needed in order to satisfy a set of communication requests.

For each problem we consider two variations: undirected, which corresponds to full-duplex communication, and directed, which corresponds to one-way communication. Moreover, for rings we also study the problem in the case of pre-determined routing. We present exact or constant-ratio approximation algorithms for all the above variations in chain, ring, star and spider networks.

1 Introduction

All-optical networks make it possible to transmit data at very high speed. The technology that enables transmitting more than one signal along a single optical fiber is called *Wavelength Division Multiplexing (WDM)*; many signals can be simultaneously carried over the same physical link by light beams of different wavelengths. Recent developments make it possible to use multiple fibers on each link, allowing any signal to switch fiber at any node; however, it is preferred for each signal to remain on the same wavelength from transmitter to receiver, in order to avoid wavelength conversion.

A multifiber network can be described by a graph $G = (V, E)$ and a function $\mu : E \rightarrow \mathbb{N}$ that defines the multiplicity of fibers on each link. The set of requests $\mathcal{R}$ is a set of pair of nodes. A routing and path multicoloring[4] for $\mathcal{R}$ (w.r.t. $\mu(e)$)

[4] Color collisions between paths that use the same edge are allowed, so we use the term "path multicoloring", as opposed to classical "path coloring" where paths that share an edge must receive different colors.

is valid w.r.t. μ (or simply valid) if all requests of $\mathcal{R}$ are satisfied, i.e. there is a colored path for each request, and for each edge e any color is used at most $\mu(e)$ times among paths that pass through e. The function μ may be given in advance, representing the number of available fibers on each link, or may be sought, representing the number of fibers that should be reserved on each link in order to satisfy a set of connection requests.

In the first part of this paper, we deal with the case where $\mu(e)$ is sought. Here we follow a more general setting where fiber costs are not the same everywhere; we call such a situation *non-uniform* pricing, as opposed to *uniform pricing* where the cost of a fiber on any link is the same. We consider networks where each fiber has a limited bandwidth (number of wavelengths) w and each link has a cost, representing the cost of using a fiber on this link for a certain time period T. For a given set of communication requests with duration at most T, we want to satisfy all requests minimizing the total cost of active fibers in the network. The number of fibers needed between two adjacent nodes of the network is the maximum number of connections that use the same wavelength and pass through the link between the two nodes. An example with two different solutions is shown in Figure 1 (left and right).

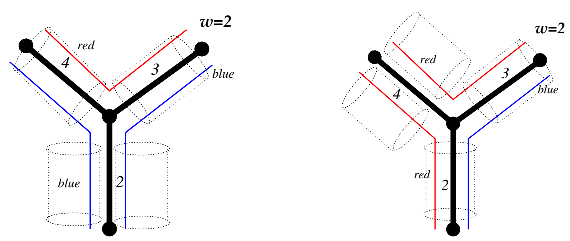

Fig. 1. An instance of *Fiber Cost Minimization Path Multi-Coloring* with 2 colors per fiber, a solution with cost 11 (left) and a solution with cost 13 (right).

We formalize this problem as the Minimum Fiber Cost Routing and Path Multi-Coloring (MinFibCost-RPMC) problem: *Given an undirected graph $G = (V, E)$, a cost function $c : E \to \mathbb{N}$, a set of requests $\mathcal{R}$ and w wavelengths (colors), assign paths to requests and colors to paths, so that the objective function $\sum_{e \in E} c(e) \cdot \mu(e)$ is minimized, where $\mu(e)$ is the maximum multiplicity of any color on edge e.*

In the second part of this paper we study the Minimum Wavelengths Routing and Path Multi-Coloring (MinWav-RPMC) problem. This problem describes the situation where the number of available fibers is given and the goal is to minimize the number of wavelengths needed to satisfy all requests. Two examples are shown in Figure 2.

Formally, the problem MinWav-RPMC is defined as follows: *Given a graph $G = (V, E)$, a function $\mu : E \to \mathbb{N}$ and a set of requests $\mathcal{R}$, find a valid routing and path multicoloring such that the number of colors used is minimized.*

We also consider, for both problems, the variation in which the routing is pre-determined, i.e. a set of paths is given instead of a set of requests. The varia-

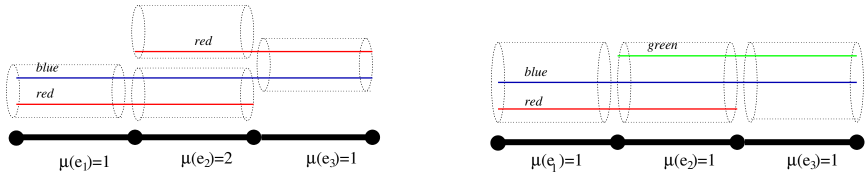

Fig. 2. Two instances of *Minimum Wavelengths Path Multi-Coloring*, the minimum number of colors needed is $w = 2$ for the left one and the minimum number of colors needed is $w = 3$ for the right one.

tions are called MINIMUM FIBER COST PATH MULTI-COLORING (MINFIBCOST-PMC) and MINIMUM WAVELENGTHS PATH MULTI-COLORING (MINWAV-PMC) respectively. Since any optimal routing must use simple paths these version make sense only in topologies where it is possible to route requests in more than one ways, e.g. ring, mesh, etc. In acyclic topologies there is a unique path between any two nodes, hence the problems MINFIBCOST-RPMC and MINFIBCOST-PMC coincide (as well as MINWAV-RPMC and MINWAV-PMC).

MINFIBCOST-RPMC in rings is NP-hard, since the problem with uniform costs, which is a special case, is NP-hard [9]; the same holds for MINFIBCOST-PMC in rings. MINWAV-RPMC in rings, stars and spiders is also NP-hard (since it is a generalization of the classical routing and path coloring problem which is NP-hard for such topologies [11]); this is also true for MINWAV-PMC in rings as well as for the directed version of both problems in rings.

We distinguish between two types of models: undirected and directed. The undirected model corresponds to the case where the communication for every request is two-way and signals in both directions must use the same set of links and the same wavelength (full-duplex communication). One-way communication can be modeled by using directed requests and paths; the corresponding problem variations have the same names, preceded by the word "DIRECTED". Note that in the directed case, color collisions may occur only between paths that pass through the same edge in the same direction.

In this paper we present constant-ratio approximation or exact algorithms for MINFIBCOST-RPMC in rings with or without pre-routed requests and for MINWAV-RPMC in chains, rings, stars and spiders. We also present appropriate adaptation of our algorithms for the directed versions of the problems. All the proposed algorithms run in polynomial time. A comprehensive table of the results is given in section 4.

1.1 Related work

The problem of minimizing the number of active fibers in multifiber networks with *uniform fiber costs* was introduced in [9], where polynomial-time solvability was shown for chains and 2-approximation algorithms were given for the undirected problem in ring and star networks. Their results for chains and stars extend to MINFIBCOST-RPMC. Moreover an exact algorithm for DIRECTED MINFIBCOST-RPMC in chains and stars is implicit. In [13] they also study the undirected problem with uniform fiber costs for chains and give a new polynomial-time algorithm for this class of graphs; they also define other variations and show them NP-hard. A 2-approximation algorithm for MinFibCost-RPMC in spiders is given in [8]; this algorithm yields an exact algorithm for the directed case.

The problem MinWav-RPMC was introduced in [6, 7] for the special case where $\mu(e)$ is the same for all edges of the network; the more general definition that we use here was first given in [5].

Multifiber tree networks have been studied only recently. For the problem MinFibCost-RPMC with uniform fiber costs two approximation algorithms, with ratios $1 + 4|E| \log |V|/OPT$ and 4, are presented in [5]; these results can be immediately extended to the case of non-uniform fiber costs. For MinWav-RPMC a 4-approximation algorithm is presented in [1].

A lot of work has been done on minimization and maximization routing and path coloring problems for single-fiber networks (see e.g. [11], [10] and references therein).

Other related work includes traffic grooming. In this approach we can combine low speed traffic components onto high speed channels in order to minimize the network cost. Traffic grooming for path, star and tree networks is studied in [4]; in [2] they consider the problem for ring networks.

1.2 Technical Preliminaries

A *chain* is a graph that consists of a single path, while a *ring* is a graph that consists of a single cycle. A *star* is a tree with one internal node. A *spider* is a star of chains, i.e. a star whose edges have been replaced by chains (also called *legs*).

Given a network $G = (V, E)$ and a set of requests $\mathcal{R}$ we denote by n the number of nodes, and by m the number of requests. A *routing* of the requests $\mathcal{R}$ is a set of paths $\mathcal{P}$, each connecting the endpoints of a request. For a set of paths $\mathcal{P}$ and an edge e we denote by $L(e, \mathcal{P})$ the *load* of edge e w.r.t. $\mathcal{P}$, i.e. the number of paths in $\mathcal{P}$ that pass through e.

Let a ring G consist of n nodes labeled clockwise from v_0 to v_{n-1}. We denote the path from u to v in clockwise direction by $\langle u, v\rangle$ and we say that it *begins* at u and it *ends* at v.

An algorithm A for a minimization problem Π is a ρ-approximation algorithm if for every instance I of Π, A runs in time polynomial in $|I|$ and delivers a solution with value $SOL \leq \rho \cdot OPT$, where OPT denotes the value of an optimal solution for I.

2 Minimizing Fiber Cost

In this section we deal with the problem of minimizing the cost of active fibers needed in order to satisfy all requests with a given number of wavelengths. We present approximation algorithms for ring networks. Recall that in rings we may consider two versions, depending on whether the routing is pre-determined (MinFibCost-PMC) or not (MinFibCost-RPMC).

Our algorithms make use of an algorithm for MinFibCost-RPMC in chains that gives optimal solutions in polynomial time. Such an algorithm was described in [9] for uniform fiber costs. That algorithm works for non-uniform costs too, as observed in [8].

Once a routing $\mathcal{P}$ is determined (or unique, or given in advance), each edge contributes at least cost $\lceil L(e_i, \mathcal{P})/w \rceil \cdot c(e_i)$, because at least $\lceil L(e_i, \mathcal{P})/w \rceil$ fiber units are needed for this edge. Summarizing over all edges in E we get $OPT \geq \sum_{e \in E} \lceil L(e_i, \mathcal{P})/w \rceil \cdot c(e_i)$. Note also that for the directed version the sum must be taken over both directions.

2.1 MinFibCost-PMC in Rings (pre-routed requests)

Without loss of generality we assume that all edges of G are used by some path (otherwise we would eliminate an unused edge and obtain a chain instance, which can be solved optimally using the algorithm for chains). Therefore, at least one fiber per edge is needed, thus the total cost of an optimal solution is at least $OPT \geq C = \sum_{e \in E} c(e)$.

We denote by $\mathcal{P}_v$ the set of paths in $\mathcal{P}$ that contain v as an internal node. Let $\mathcal{P}'_v$ be the set of paths that results from splitting paths in $\mathcal{P}_v$ at node v. Paths in $\mathcal{P}'_v$ are called v-*clockwise* if they contain edge (v, u), where u is the neighbor of v in clockwise direction; the remaining paths in $\mathcal{P}'_v$ are called v-*counterclockwise*. Consider the longest v-clockwise path and the longest v-counterclockwise path in $\mathcal{P}'_v$; let $p(v)$ be the one of the two using edges with minimum sum of costs. We define the *tare* $t(v)$ of v to be the sum of edge-costs of $p(v)$ and the *span* $s(v)$ of v to be the length of $p(v)$ (the number of its edges). If $\mathcal{P}_v$ is empty, then $t(v) = 0$ and $s(v) = 0$. Let v_0 be the node with minimum tare; let also $t = t(v_0)$ and $s = s(v_0)$. W.l.o.g. we may assume that $p(v_0)$ is v_0-clockwise (if not we may consider a 'mirror' instance instead). Our algorithm for MinFibCost-PMC in rings first selects node v_0 as above. The complete algorithm follows.

Algorithm for MinFibCost-PMC *in rings*

Input: $I = (G, c, \mathcal{P}, w)$; $G = (V, E)$ is a ring network, c is the edge-cost function,
  $\mathcal{P}$ is a set of paths and w is the number of colors.
Output: A multicoloring of paths in $\mathcal{P}$.

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1. Find node v_0 with minimum tare and reindex nodes accordingly.
2. Transform the given ring instance to a chain instance $(G', c, \mathcal{P}', w)$ as follows:
   a. The chain graph G' consists of $n + s + 1$ nodes, namely $v'_0, \ldots, v'_{n+s}$.
   Set $c(e'_i) = c(e'_{i+n}) = c(e_i)$.
   b. For each path $\langle v_i, v_j \rangle \in \mathcal{P}$, add a path to $\mathcal{P}'$:
      if $i < j$ add $\langle v'_i, v'_j \rangle$ to $\mathcal{P}'$, otherwise add $\langle v'_i, v'_{j+n} \rangle$ to $\mathcal{P}'$.
3. Call Algorithm for MinFibCost-PMC in chains [9] on instance $(G', c, \mathcal{P}', w)$.
4. Color each path in $\mathcal{P}$ with the color of the corresponding path in $\mathcal{P}'$.

Theorem 1. *The algorithm for* MinFibCost-PMC *in rings computes a multicoloring with cost at most* $OPT + t$.

Proof. Let us abbreviate edges in G by $e_i = (v_i, v_{(i+1) \bmod n}), (0 \le i \le n-1)$, and edges in G' by $e'_i = (v'_i, v'_{i+1}), (0 \le i \le n+s-1)$. It is easy to see that if a path in $\mathcal{P}$ uses edge e_i and $0 \le i \le s-1$, then the corresponding path of $\mathcal{P}'$ uses either edge e'_i or edge e'_{i+n}. Thus, for $0 \le i \le s-1$, the load of e_i in G is split into two parts in G': $L(e_i, \mathcal{P}) = L(e'_i, \mathcal{P}') + L(e'_{i+n}, \mathcal{P}')$. Notice that $L(e_i, \mathcal{P}) = L(e'_i, \mathcal{P}')$ for $s \le i \le n-1$. Due to the optimality of the chain algorithm, the number of repetitions of any color on an edge e'_i is at most $\mu(e'_i) = \lceil L(e'_i, \mathcal{P}')/w \rceil$.

Hence the cost of the solution computed by our algorithm is:

$$
\begin{aligned}
SOL &\le \sum_{i=0}^{n+s-1} \mu(e'_i) \cdot c(e'_i) = \sum_{i=0}^{n+s-1} \lceil \frac{L(e'_i, \mathcal{P}')}{w} \rceil \cdot c(e'_i) \\
&= \sum_{i=0}^{s-1} (\lceil \frac{L(e'_i, \mathcal{P}')}{w} \rceil + \lceil \frac{L(e'_{i+n}, \mathcal{P}')}{w} \rceil) \cdot c(e'_i) + \sum_{i=s}^{n-1} \lceil \frac{L(e'_i, \mathcal{P}')}{w} \rceil \cdot c(e'_i) \\
&= \sum_{i=0}^{n-1} \lceil \frac{L(e_i, \mathcal{P})}{w} \rceil \cdot c(e_i) + \sum_{i=0}^{s-1} c(e_i) \le OPT + t
\end{aligned}
$$
□

The approximation ratio is at most $1 + \frac{t}{OPT}$ which is smaller than 2 and gets close to 1 for instances with heavy communication traffic.

The computation of each tare and span and the transformation can be performed in $O(m+n)$ time. The complexity of algorithm for MinFibCost-PMC in rings is determined by that of the chain algorithm, which is $O((m + n \cdot w) \log w)$ [9].

2.2 MinFibCost-RPMC in Rings

We now propose an algorithm for MinFibCost-RPMC in rings, i.e. the routing is also sought. Our algorithm uses lightest-path routing: each request is routed along the path with minimum cost (sum of edge costs along path p) between the two alternative complementary paths.

Algorithm for MinFibCost-RPMC *in rings*
Input: $I = (G, c, \mathcal{R}, w)$; $G(V, E)$ is a ring network, c is the cost function, $\mathcal{R}$ is a set of requests and w is the number of colors.
Output: A routing $\mathcal{P}$ for $\mathcal{R}$ and a multicoloring of paths in $\mathcal{P}$.
1. Perform a lightest-path routing obtaining a set of paths $\mathcal{P}$. Call Algorithm MinFibCost-PMC in rings (pre-routed requests) on $(G, c, \mathcal{P}, w)$ to multicolor the set of paths $\mathcal{P}$.
2. For each edge $e \in E$: route all requests in $\mathcal{R}$ avoiding e obtaining set of paths $\mathcal{P}_e$. Call Algorithm for MinFibCost-PMC in chains on instance $(G, c, \mathcal{P}_e, w)$.
3. Choose the best solution among the one found in step 1 and those found in step 2.

The selection of lightest paths minimizes the quantity $\sum_{i=0}^{n-1} L(e_i) \cdot c(e_i)$, and decreases the upper bound for t to $t \le C/2$, where $C = \sum_{i=0}^{n-1} c(e_i)$. A bound for the cost of the solution computed by this algorithm is given by the following theorem:

Theorem 2. *The algorithm for* MinFibCost-RPMC *in rings computes a multicoloring with cost at most* $OPT + C + t$.

Proof. We prove the claim for the solution returned by step 1 of the algorithm (in fact step 2 is only needed for the case in which an optimal solution completely avoids an edge).

Let $\mathcal{P}$ be the set of paths selected by our algorithm for MinFibCost-RPMC in rings and $\mathcal{P}^*$ be the set of paths in an optimal solution. We denote by e_i the edge between nodes i and $(i+1) \bmod n, 0 \leq i \leq n-1$. Note that $OPT \geq \lceil L(e_i, \mathcal{P}^*)/w \rceil \cdot c(e_i)$. Since $\mathcal{P}$ consists of lightest paths it holds:

$$\sum_{i=0}^{n-1} L(e_i, \mathcal{P}) \cdot c(e_i) \leq \sum_{i=0}^{n-1} L(e_i, \mathcal{P}^*) \cdot c(e_i) \tag{1}$$

The following properties of ceilings hold for all $n \in \mathbb{N}^+, a_i \in \mathbb{R},\ 0 \leq i < n$:

$$\sum_{i=0}^{n-1} \lceil a_i \rceil \leq \lceil \sum_{i=0}^{n-1} a_i \rceil + n - 1 \quad \text{and} \quad \lceil a_i \rceil \cdot n - n + 1 \leq \lceil a_i \cdot n \rceil \tag{2}$$

From (1) and (2) we get:

$$\sum_{i=0}^{n-1} (\lceil \frac{L(e_i, \mathcal{P}) \cdot c(e_i)}{w} \rceil) \leq \sum_{i=0}^{n-1} \lceil \frac{L(e_i, \mathcal{P}^*) \cdot c(e_i)}{w} \rceil + n \Rightarrow$$
$$\sum_{i=0}^{n-1} (\lceil \frac{L(e_i, \mathcal{P})}{w} \rceil \cdot c(e_i)) - C \leq OPT$$

By an inequality used in the proof of Theorem 1 and the above inequality, the cost of the approximate solution returned by the algorithm is at most

$$SOL(I) \leq \sum_{i=0}^{n-1} (\lceil \frac{L(e_i, \mathcal{P})}{w} \rceil \cdot c(e_i)) \ + t \leq OPT + C + t$$ □

If $OPT \geq C$ then the algorithm for MinFibCost-RPMC in rings achieves approximation ratio 5/2, using the fact that $t \leq C/2$.

If $OPT < C$, then it must be the case that paths in the optimal solution do not pass through some edge, say e. In step 2, the algorithm considers, among others, the (unique) routing in which all requests avoid e. Algorithm MinFibCost-RPMC in rings then uses the Algorithm for MinFibCost-RPMC in chains, which returns an optimal solution for the corresponding chain instance. Hence, the solution returned is optimal.

As about the complexity, the most costly step is step 2, which employs n calls to algorithm for MinFibCost-PMC in chains. The overall cost is thus $O(n(m + nw) \log w)$.

2.3 Directed Fiber Cost Minimization

In the directed version the requests are directed, while the underlying graph is considered bidirected. We assume that the cost of an edge is the same in both directions.

- For DIRECTED MINFIBCOST-PMC in rings (pre-routed requests) we obtain the same approximation as for the undirected case, because we can split the instance into one instance of clockwise direction and one of counterclockwise direction and solve the two instances separately as undirected ones.
- For DIRECTED MINFIBCOST-RPMC in rings we obtain a 4-approximation algorithm by first performing lightest path routing and then applying the above algorithm for the problem with pre-routed requests.

3 Minimizing the Number of Wavelengths

In this section we present exact and approximate algorithms for the wavelength minimization problem in chains, rings, stars and spiders. In this problem, the multiplicity of fibers on each edge is given and the goal is to find a valid routing and path multicoloring using a minimum number of colors. This number is denoted by w_{opt}. Note that, once a routing $\mathcal{P}$ is determined (or unique, or given in advance), $w_{opt} \geq w_{lb} = \max_{e \in E} \lceil \frac{L(e,\mathcal{P})}{\mu(e)} \rceil$. In the directed version this maximum is taken over all edges in both directions.

We can solve MINWAV-PMC in chains using exactly w_{lb} colors, which is optimal. This can be done as follows: Call algorithm MINFIBCOST-PMC in chains for the same requests, unit edge cost everywhere, and w_{lb} available colors. As shown in [9] this call returns a multicoloring that uses exactly $\mu'(e) = \lceil L(e,\mathcal{P})/w_{lb} \rceil \leq \mu(e)$ fibers on each edge e. Hence, this is a valid path multicoloring.

3.1 MinWav-PMC in Rings (pre-routed requests)

For solving MINWAV-PMC in rings we observe that every instance of the problem falls in exactly one of the following three categories:

1. $\forall e \in E : \mu(e) \geq 2$.
2. There exists at least one edge $e_i \in E$ with $\mu(e_i) = 1$ and no edges of multiplicity 0 exist.
3. There exists at least one edge $e_i \in E$ with $\mu(e_i) = 0$.

Instances that fall in category 3 are actually chain instances and can be solved optimally. An algorithm that copes with instances in categories 1 and 2 is presented below.

Algorithm for MinWav-PMC *in rings*
Input: $I = (G, \mathcal{P}, \mu)$; $G(V, E)$ is a ring network, $\mathcal{P}$ is a set of paths   and $\mu : E \rightarrow \mathbb{N}$ is the edge multiplicity function Output: A valid multicoloring of paths in $\mathcal{P}$.

if $\forall e \in E : \mu(e) \geq 2$ **then** (*category 1*)
  Set $w = \max_{e \in E} \lceil \frac{L(e,\mathcal{P})}{\mu(e)-1} \rceil$
  Call Algorithm MinFibCost-PMC in rings on $(G, 1, \mathcal{P}, w)$
else (*category 2*)
  Choose an edge e_i with $\mu(e_i) = 1$. Set of paths $\mathcal{P}_i$: paths in $\mathcal{P}$ passing through e_i.
  Set $\mathcal{P}' = \mathcal{P} \setminus \mathcal{P}_i$. Remove edge e_i from G, let this graph be G'.
  Call Algorithm for MinWav-PMC in chains on instance $(G', \mathcal{P}', \mu)$.
  Color paths in $\mathcal{P}_i$ using $\lvert\mathcal{P}_i\rvert$ new colors.

Theorem 3. *Algorithm* MinWav-PMC *in rings is a 2-approximation algorithm.*

Proof. Instance in Category 1: $\forall e \in E: \ \mu(e) \geq 2 \Rightarrow \mu(e) - 1 \neq 0$ and $\mu(e) - 1 \geq \frac{\mu(e)}{2}$. Consider an edge e^* for which $\lceil \frac{L(e^*,\mathcal{P})}{\mu(e^*)-1} \rceil = \max_{e \in E} \lceil \frac{L(e,\mathcal{P})}{\mu(e)-1} \rceil$.

The number of colors (w) used by the algorithm is:

$$w = max_{e \in E} \lceil \frac{L(e,\mathcal{P})}{\mu(e)-1} \rceil = \lceil \frac{L(e^*,\mathcal{P})}{\mu(e^*)-1} \rceil \leq \lceil \frac{2 \cdot L(e^*,\mathcal{P})}{\mu(e^*)} \rceil \leq 2 \cdot \lceil \frac{L(e^*,\mathcal{P})}{\mu(e^*)} \rceil$$
$$\leq 2 \cdot max_{e \in E} \lceil \frac{L(e,\mathcal{P})}{\mu(e)} \rceil \leq 2 \cdot w_{opt}$$

Instance in Category 2: The algorithm uses $|\mathcal{P}_i|$ colors for the paths passing through e_i. It is $|\mathcal{P}_i| \leq w_{opt}$, because any optimal solution would need at least $|\mathcal{P}_i|$ colors for paths passing through edge e_i.
The algorithm multicolors the remaining paths in $\mathcal{P}'$ ($\mathcal{P}' = \mathcal{P} \setminus \mathcal{P}_i$). All paths in $\mathcal{P}'$ avoid edge e_i, thus we can remove e_i from G and get G', which is a chain network. Algorithm MinWav-PMC in chains returns a solution using a number of colors $w = \max_{e \in E} \lceil \frac{L(e,\mathcal{P}')}{\mu(e)} \rceil \leq w_{opt}$. Hence, we use $w + |\mathcal{P}_i| \leq 2 \cdot w_{opt}$ colors in total. □

3.2 MinWav-RPMC in Rings

We now turn to the problem in rings where the routing is also sought. Our algorithm is based on the idea of routing the requests in such a way that the edge with minimum number of available fibers is completely avoided.

Algorithm for MinWav-RPMC *in rings*
Input: $I = (G, \mathcal{R}, \mu)$; $G(V, E)$ is a ring network, $\mathcal{R}$ is a set of requests   and $\mu : E \rightarrow \mathbb{N}$ is the edge multiplicity function. Output: A routing $\mathcal{P}$ for $\mathcal{R}$ and a valid multicoloring of paths in $\mathcal{P}$.
1. Pick an edge e_0 with minimum fiber multiplicity $\mu(e_0)$. 2. Route all requests in $\mathcal{R}$ so that the corresponding paths avoid edge e_0.   Let $\mathcal{P}$ denote the resulting set of paths. Remove edge e_0 from G, call the new graph G'. 3. Call Algorithm for MinWav-PMC in chains on instance $(G', \mathcal{P}, \mu)$.

Theorem 4. *Algorithm* MinWav-RPMC *in rings is a 2-approximation algorithm.*

Proof. Let $\mathcal{P}_{opt}$ denote the set of paths in an optimal solution, that uses w_{opt} colors. Let also w_{sol} denote the number of colors used by our algorithm for MinWav-RPMC in rings.

First, we observe that $\mathcal{P}_{opt}$ can be seen as a transformation of $\mathcal{P}$ in which some paths have been replaced by their complementary paths (that necessarily use edge e_0). Therefore, for any edge $e \neq e_0$ it holds:

$$L(e, \mathcal{P}) \leq L(e, \mathcal{P}_{opt}) + L(e_0, \mathcal{P}_{opt})$$

Dividing by $\mu(e)$ and taking into account that $\mu(e) \geq \mu(e_0)$ we obtain:

$$\lceil \frac{L(e, \mathcal{P})}{\mu(e)} \rceil \leq \lceil \frac{L(e, \mathcal{P}_{opt})}{\mu(e)} \rceil + \lceil \frac{L(e_0, \mathcal{P}_{opt})}{\mu(e_0)} \rceil$$

Since the above holds for any edge e, it also holds for the edge e^* with maximum load/multiplicity ratio w.r.t. routing $\mathcal{P}$, which is equal to the number of colors used by Algorithm MinWav-PMC in chains when applied to $(G', \mathcal{P}, \mu)$. On the other hand, each of the two quantities on the right side of the above inequality is a lower bound for the number of colors used by the optimal solution. Altogether:

$$\begin{aligned} w_{sol} &= \max_{e \in E} \lceil \frac{L(e, \mathcal{P})}{\mu(e)} \rceil = \lceil \frac{L(e^*, \mathcal{P})}{\mu(e^*)} \rceil \leq \lceil \frac{L(e^*, \mathcal{P}_{opt})}{\mu(e^*)} \rceil + \lceil \frac{L(e_0, \mathcal{P}_{opt})}{\mu(e_0)} \rceil \\ &\leq 2 \cdot \max_{e \in E} \lceil \frac{L(e, \mathcal{P}_{opt})}{\mu(e)} \rceil \leq 2 \cdot w_{opt} \end{aligned}$$

□

3.3 MinWav-PMC in Stars and Spiders

We now propose a 3/2-approximation algorithm for MinWav-PMC in stars, which is based on a transformation of the problem to edge coloring of a multigraph H. For the sake of brevity, we only point out few details: each node of H corresponds to a group of at most w_{lb} paths that use the same edge in the original graph G. There is an edge in H for each path p in $\mathcal{P}$, connecting the two groups that contain p. Multigraph H can be edge-colored using at most $3/2 \cdot w_{lb} \leq 3/2 \cdot w_{opt}$ colors [12]; it is not hard to see that assigning to each path p in $\mathcal{P}$ the color of the corresponding edge in H, we obtain a valid path multicoloring. The above idea can be extended to spiders (generalized stars), at a cost of at most w_{lb} additional colors (for paths that do not pass through the center), giving a valid path multicoloring with at most $5/2 \cdot w_{lb} \leq 5/2 \cdot w_{opt}$ colors. Hence the following is true:

Theorem 5. MinWav-PMC *can be approximated within* 3/2 *in stars and* 5/2 *in spiders.*

3.4 Directed Wavelength Minimization

For the directed version of MinWav-PMC and MinWav-RPMC we assume that fiber multiplicity is symmetric, i.e. the number of fibers in two opposite edges (v_i, v_j) and (v_j, v_i) is the same. We briefly explain how to adapt the algorithm of this section to obtain algorithms for the version of Directed MinWav-RPMC and Directed MinWav-PMC.

- For chain networks we can use the minimum possible number of wavelengths, by computing an optimal solution for each direction independently; our solution is the maximum of the two solutions. This gives exactly the above lower bound.
- For Directed MinWav-PMC in rings (pre-routed requests) we can easily obtain the same approximation ratio (2 in the worst case) as for the undirected case (subsection 3.1); we can split the instance into one of clockwise direction and one of counterclockwise direction and solve them independently as undirected ones. Our solution is the maximum of the two solutions.
- For Directed MinWav-RPMC in rings we can obtain an algorithm by modifying Algorithm MinWav-RPMC in rings (subsection 3.2). This gives an approximation algorithm with ratio 2. In the analysis of the algorithm we use the fact that for the clockwise direction and for each edge $e \neq e_0$: $L(e, \mathcal{P}_+) \leq L(e, \mathcal{P}^*_+) + L(e_0, \mathcal{P}^*_-)$. A similar inequality holds for $L(e, \mathcal{P}_-)$. We use $\mathcal{P}_+$ ($\mathcal{P}_-$) to denote the set of paths of our solution that are oriented clockwise (counterclockwise respectively); $\mathcal{P}^*_+$ ($\mathcal{P}^*_-$) are defined analogously.
- For Directed MinWav-RPMC in stars our algorithm gives an optimal solution, due to the fact that the multigraph H is now bipartite and it is known that it can be edge-colored with exactly w_{lb} (degree of H) colors (see e.g. [3]). Similarly we obtain a 2-approximation algorithm for Directed MinWav-RPMC in spiders.

4 Summary of Results - Conclusions

We studied two up-to-date optimization problems: fiber cost minimization and wavelength minimization in multifiber WDM networks. Both problems deal with limited resources: in the former the number of wavelengths is given and the goal is to minimize the cost of fiber usage; in the latter it is the number of fibers that is given and we aim at minimizing the number of necessary wavelengths. We remark that for MinWav-RPMC we follow a very recently introduced model [5] under which the number of fibers may differ from link to link; previous models were based on the rather restrictive assumption that the number of fibers is uniform [6, 7]. We follow the same assumption for MinFibCost-RPMC.

We summarize our algorithms in the following table, where the approximation ratio of each of them is shown (algorithms giving optimal solutions are referred to as "exact" and the term "pre-rings" stands for "rings with pre-routed requests"). Note that our new results are shown in boldface; we also mention algorithms from [9] (MinFibCost-RPMC in chains and stars) and [8] (MinFibCost-RPMC in spiders) in order to obtain a complete picture.

	MinFibCost-RPMC		MinWav-RPMC	
Network	*Undirected*	*Directed*	*Undirected*	*Directed*
chains	exact	exact	**exact**	**exact**
pre-rings	**2-approx.**	**2-approx.**	**2-approx.**	**2-approx.**
rings	**5/2-approx.**	**4-approx.**	**2-approx.**	**2-approx.**
stars	2-approx.	exact	**3/2-approx.**	**exact**
spiders	2-approx.	exact	**5/2-approx.**	**2-approx.**

The proposed algorithms are easy to implement and we have proven for all of them a guaranteed approximation ratio. We anticipate that they will prove even better in practice. In particular, it can be shown that for heavily loaded instances the approximation ratio gets close to 1. This is due to the fact that the cost of our solutions differ from the cost of an optimal solution by an additive term only, which is usually very small.

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