Closed-Form Statistical Modeling of PDL-Induced SNR Margins for Reliable Optical Networks

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***Abstract*—We develop closed-form formulas for PDL-induced SNR margins using solutions based on central limit theorem. Experimental validations confirm accurate and conservative performance predictions, enabling precise quality of transmission assessment and margin-aware design in optical networks.**

***Index Terms*—PDL, statistical margin assessment, optical network digital twin, central limit theorem**

I. Introduction

Quality of transmission (QoT) assessment is a critical step in optical network design and operation, as it determines whether lightpaths can meet performance requirements with sufficient system margin. In modern networks, reconfigurable optical add-drop multiplexers (ROADMs), leveraging wavelength selective switches (WSSs), are widely deployed to provide flexibility and scalability; however, they also introduce polarization-dependent loss (PDL). PDL distorts signal quality and increases performance variability [1], reducing the available system margin. Margins are essential in optical networks to guarantee error-free operation throughout their lifetime, either at initial deployment (greenfield) or when adding new services or light paths (brownfield) [2]. However, margins come at a cost, as they lead to network over-dimensioning, so they should be minimized whenever possible. An accurate QoT assessment that accounts for PDL is therefore crucial to avoid overestimation of margins and optimize resource utilization. This margin-aware evaluation becomes a key capability when developing an optical network digital twin (ONDT) [3], as ONDTs rely on precise modeling of impairments and margins to allow realistic performance prediction and efficient management. In this context, the QoT metric adopted by the ONDT is the signal-to-noise ratio (SNR).

In previous works, SNR-based margin assessment in the presence of PDL was addressed using linear regression [4] and, more recently, through statistical evaluation via Monte Carlo simulation [5], [6]. The determination of the statistical margin based on a target probability of out of service (OOS) is analogous to the approach adopted for differential group delay (DGD) in the case of polarization mode dispersion (PMD) [7]. For PMD, the probability density function (PDF) of DGD was modeled as Maxwellian, which enables the corresponding margin to be computed in closed form. Moreover, computing the margin directly from the PDF is almost instantaneous, leading to substantial time savings compared to simulation. In the case of PDL, the SNR distribution is bell-shaped [8]. Although approximations have been proposed [9], to the best of our knowledge, a closed-form expression for determining the SNR margin directly from the PDF is still lacking.

The novelty of this work is the development of closed formulas, based on the central limit theorem (CLT) approximation, to compute the margin on the SNR in the presence of PDL. We account for ideal digital signal processing (DSP) equalization, and we perform an experimental validation, computing conservative and accurate margin.

II. Mathematical Modeling

The investigated optical link abstraction is shown in Fig. 1. Noise contributions and attenuations (due to PDL devices) are alternated, while the DSP fully recovers the transmitted signal. To derive the expression of the PDL-induced SNR distribution based on the CLT, we start from the statistical characterization of the attenuation on each polarization axis introduced by each PDL device, which was demonstrated to be a uniform random variable (RV) [10], both on X and Y polarizations, and is independent of the other random attenuations. The statistical characterization of such random variables is provided in (1), assuming a polarization average gain of each PDL device equal to unity [11]. We perform our statistical analysis referring to X polarization, identical steps can be followed to derive the expression of the PDL-induced SNR distirbution on the Y polarization. According to Fig. 1, we call A_i, with $i = 1, \ldots, N$, the N attenuations.

$$f_{A_i}(x) = \frac{1}{2\gamma_i}; \quad x \in [1-\gamma_i; 1+\gamma_i] \tag{1}$$

$$\gamma_i = \frac{PDL_{lin,i} - 1}{PDL_{lin,i} + 1} \in [0, 1]; \quad i = 1, \ldots, N \tag{2}$$

where $PDL_{lin,i} = 10^{PDL_{i,dB}/10}$ comes from the characterization of each device. The first step is to compute the SNR on a single polarization axis after the DSP action and invert it, obtaining the formula for SNR^{-1}:

$$SNR = \frac{p_s \prod_{j=1}^{N} A_j \cdot \prod_{k=1}^{N} A_k^{-1}}{\sum_{i=1}^{N+1} p_{n,i} \prod_{l=i}^{N} A_l \cdot \prod_{k=1}^{N} A_k^{-1}} \tag{3}$$

$$SNR^{-1} = \frac{p_{n,1}}{p_s} + \frac{1}{p_s}\sum_{i=1}^{N}\frac{p_{n,i+1}}{\prod_{k=1}^{i} A_k} \tag{4}$$

$$X_i = \frac{p_{n,i+1}}{\prod_{k=1}^{i} A_k}; \quad i = 1, \dots, N \tag{5}$$

We observe that SNR^{-1} is a function of N independent uniform RVs A_i. SNR^{-1} can also be expressed as a sum of N dependent RVs X_i, as in (4). To apply the CLT approximation in the case of the sum of dependent RVs [12], we computed the expected value $\mu_{SNR^{-1}}$ and the variance $\sigma^2_{SNR^{-1}}$ of SNR^{-1}. The evaluation of the expected value is straightforward:

$$\mu_{SNR^{-1}} = \frac{p_{n,1}}{p_s} + \frac{1}{p_s}\cdot\sum_{i=1}^{N}\mathbb{E}[X_i] \tag{6}$$

$$= \frac{p_{n,1}}{p_s} + \frac{1}{p_s}\cdot\left(\sum_{i=1}^{N} p_{n,i+1}\cdot\prod_{k=1}^{i}\frac{1}{2\gamma_k}\log\left(\frac{1+\gamma_k}{1-\gamma_k}\right)\right) \tag{7}$$

The expression of the variance of SNR^{-1} is more complex because it involves the computation of all possible cross-covariances among the N random variables X_i:

$$\sigma^2_{SNR^{-1}} = \frac{1}{p_s^2}\cdot\left(\sum_{i=1}^{N} Var(X_i) + 2\sum_{i<j} Cov(X_i, X_j)\right) \tag{8}$$

$$= \frac{1}{p_s^2}\sum_{i=1}^{N} p^2_{n,i+1}\beta_i + \frac{2}{p_s^2}\sum_{i<j\leq N} p_{n,i+1}p_{n,j+1}\beta_i \prod_{l=i+1}^{j}\frac{1}{2\gamma_l}\log\left(\frac{1+\gamma_l}{1-\gamma_l}\right) \tag{9}$$

where $\beta_i = \prod_{k=1}^{i}\frac{1}{1-\gamma_k^2} - \prod_{k=1}^{i}\frac{1}{4\gamma_k^2}\log^2\left(\frac{1+\gamma_k}{1-\gamma_k}\right)$. In addition, since we can compute in closed form the maximum and minimum value of SNR^{-1}, we can truncate the Gaussian distribution of the CLT approximation in correspondence of:

$$inf = \frac{p_{n,1}}{p_s} + \frac{1}{p_s}\sum_{i=1}^{N}\frac{p_{n,i+1}}{\prod_{k=1}^{i}(1+\gamma_k)} \tag{10}$$

$$sup = \frac{p_{n,1}}{p_s} + \frac{1}{p_s}\sum_{i=1}^{N}\frac{p_{n,i+1}}{\prod_{k=1}^{i}(1-\gamma_k)} \tag{11}$$

Assuming a Gaussian distribution approximation for SNR^{-1}, the final PDF of SNR is obtained by the well-known formula for the PDF of the inversion of a Gaussian RV, given its expected value and variance. The PDF has no singularities, since its support is strictly positive because both $1/sup > 0$ and $1/inf > 0$. The scaling factor $\delta_{norm} = \frac{1}{2}\left[erf\left(\frac{sup-\mu_{SNR^{-1}}}{\sigma_{SNR^{-1}}\sqrt{2}}\right) - erf\left(\frac{inf-\mu_{SNR^{-1}}}{\sigma_{SNR^{-1}}\sqrt{2}}\right)\right]$ normalizes the PDF so that its integral equals 1, and $x \in \left[\frac{1}{sup}; \frac{1}{inf}\right]$:

$$f_{SNR}(x) = \frac{1}{\delta_{norm}\cdot\sqrt{2\pi\sigma^2_{SNR^{-1}}}\cdot x^2}\exp\left(-\frac{\left(\frac{1}{x}-\mu_{SNR^{-1}}\right)^2}{2\sigma^2_{SNR^{-1}}}\right) \tag{12}$$

III. Experimental Validation

The three experimental setups adopted to validate the proposed mathematical model are shown in Fig. 2; from setup (a) to (c), the number of WSS that cause PDL increases. In all cases, the single-channel transmitted signal was generated by a commercial transceiver at a frequency of 191.5 THz. The modulation was DP-QPSK with a symbol rate of 32 GB and a roll-off of 25%. The bit error rate (BER) was measured in the two polarization axes separately. From BER the SNRs for both the x and the y polarization axes were obtained, giving similar results: we focused on the results for polarization x. Three WSSs were characterized in terms of the corresponding PDL introduced, for the frequency and

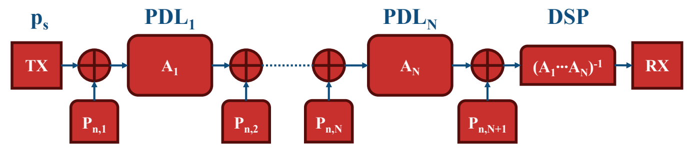

Fig. 1: System abstraction of PDL-impaired optical link on a single polarization.

f [THz]	EDFA [dB]	WSS_1 [dB]	WSS_2 [dB]	WSS_3 [dB]
191.5	0.11	0.20	0.31	0.28

TABLE I: Characterization of the devices in terms of PDL.

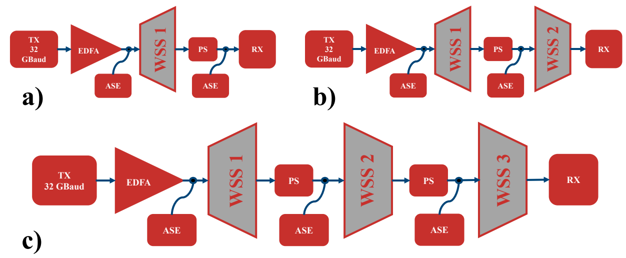

Fig. 2: Experimental setups. PS = Polarization Scramble, ASE = ASE noise sources

ports considered. Additionally, an erbium doped fiber amplifier is put right after the transmitter because of power budget requirements. The PDL introduced by the amplifier was characterized as well. Characterization is summarized in Tab. I. Polarization scramblers (PSs) change the PDL alignment between the devices, and ASE noise sources were responsible for noise injection. In setups (a) and (b) a single PS and two ASE sources were deployed, while in setup (c) two PSs and three ASE sources were deployed. ASE sources added noise according to two different strategies: distributing noise along the link (DISTR) or concentrating it in the last ASE source (RX). This choice was made since the DISTR strategy is closer to a real-case scenario, while the RX strategy gives the worst case [13]. The noise was calibrated to achieve an overall SNR at the receiver of 11.8 dB for setups (a) and (b) and 12.7 dB for setup (c), independently of the strategy.

In Fig. 3 the results of the experimental validation in all the setups considered and the noise strategies are provided. The experimental distributions and PDF of (12) are superposed. Moreover, the results of Monte Carlo simulations, performed with the same simulator validated and exploited in [5], [13], are included. Since the theoretical analysis applies to the SNR expressed in linear scale, the comparison of the distributions is provided in linear scale. In addition, the experimental and simulated distributions are shown with respect to their reference SNR, and so are the theoretical curves, whose reference is their expected value. The model shows very good agreement with both experiments and simulations in all the cases considered. As expected, the width of the distributions increases when the noise strategy is RX and when the number of WSSs deployed increases. The experimental and simulated distributions do not exceed the theoretical limit given by the PDF support (delimited by the green and blue vertical dashed lines in Fig. 3), suggesting a sufficient precision in

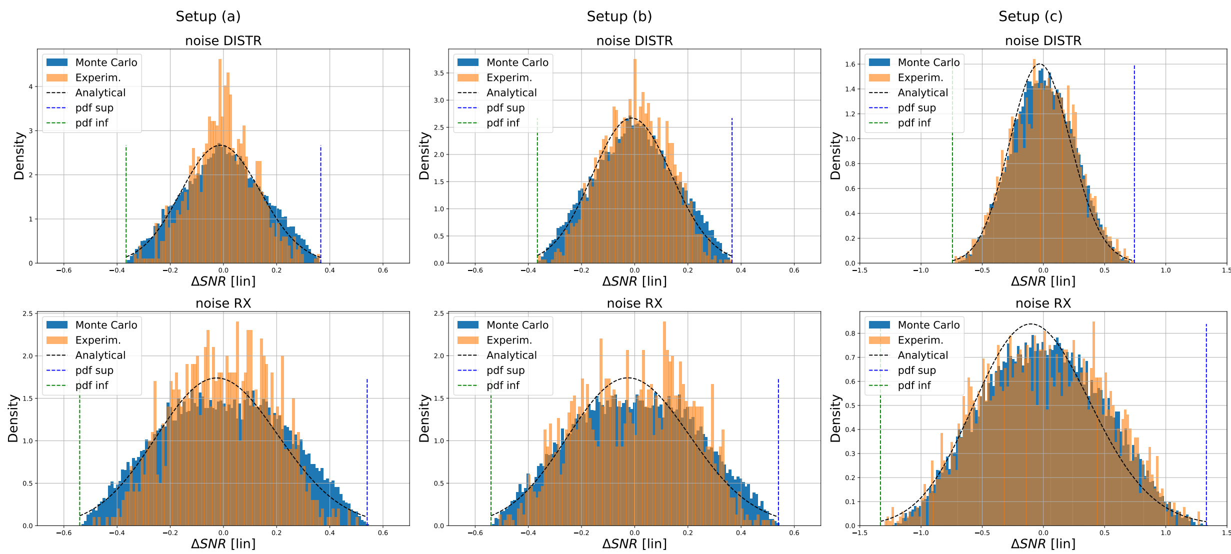

Fig. 3: SNR distributions in linear scale with respect to their reference SNR.

the characterization of the devices. Moreover, we point out that, even with a limited number of PDL devices (up to 4), the proposed approximation appears to be sufficiently close to both simulation and measurements. This result is important because, from the theoretical point of view, the CLT applied to SNR^{-1} is more precise with a large number of PDL devices. This explains why PDF fits better in the case of setup (c), but it does not appear as a limitation, since for setup (a) and (b) the CLT truncation leads to a conservative distribution evaluation. Finally, the SNR margin based on PDL-induced distributions was computed both from experimental data and exploiting the PDF of (12). The results are compared against the corresponding OpenROADM guidelines for 32 GB [14]. In our setups, the total accumulated PDL is less than 1 dB (obtained by summing the PDL values reported in Tab. I). According to the OpenROADM specifications, this corresponds to a margin of 0.5 dB [14]. In contrast, the margin obtained from the mathematical model is derived in two steps. First, the threshold ξ_{th} is determined from the target outage probability $p_{OOS} = 10^{-5}$, which corresponds to an OOS of 5 minutes per year and guarantees "five nines" availability (99.999%). The threshold is obtained from the integral $\int_{-\infty}^{\xi_{th}} f_{SNR}(x)dx = p_{OOS}$, where $f_{SNR}(x)$ is given in (12). Then, the margin in dB is computed as $m_{dB} = 10\log_{10}\left(\frac{\overline{SNR}}{\xi_{th}}\right)$, with $\overline{SNR} = \mathbb{E}[SNR]$. With the same p_{OOS} we extracted the margin also from the experimental distributions shown in Fig. 3, and the results are summarized in Tab. II. It can be observed that the proposed mathematical model yields accurate yet conservative margins compared to the experiment. Moreover, the analytical margins are consistently lower than those recommended by the OpenROADM specifications across all setups and noise strategies, thereby confirming the validity of the proposed approach and highlighting its strong potential.

IV. Conclusions

In this work, we introduced closed-form formulas to model polarization-dependent loss induced signal to noise ratio distributions, leveraging the central limit theorem. Experimental validation demonstrated strong agreement with theoretical predictions, confirming the accuracy and conservativeness of the proposed approach. Compared with existing specifications, the proposed model enables tighter margins while maintaining reliability, thus improving resource utilization and supporting precise quality of transmission assessments. This methodology represents a valuable step forward for margin-aware design and digital twin applications in modern optical networks.

Setup	Noise strategy	Experiment [dB]	Analytical [dB]	OpenROADM [dB]
(a)	DISTR	0.098	0.106	0.5
	RX	0.144	0.156	0.5
(b)	DISTR	0.105	0.106	0.5
	RX	0.150	0.156	0.5
(c)	DISTR	0.173	0.176	0.5
	RX	0.314	0.314	0.5

TABLE II: SNR margin in [dB] computed for the considered configurations and $p_{OOS} = 10^{-5}$.

References

[1] L. E. Nelson et al., "Statistics of polarization dependent loss in an installed long-haul WDM system", Opt. Express, 2011.
[2] Y. Pointurier, "Design of Low-Margin Optical Networks," JOCN, 2017.
[3] V. Curri, "GNPy model of the physical layer for open and disaggregated optical networking" JOCN, 2022.
[4] J. Girard-Jollet et al., "Estimating Network Components PDL Using Performance Statistical Measurements" JLT, 2022.
[5] E. Miotto et al., "Statistical Assessment od System Margin in Metro Networks Impaired by PDL", OFC, 2025.
[6] A. Rosso et al., "Is the Gaussian Channel Model Suitable for Converged Metro-Access Optical Networks?", ICTON, 2025.
[7] C. T. Allen et al., "Measured Temporal and Spectral PMD Characteristics and Their Implications for Network-Level Mitigation Approaches", JLT, 2003.
[8] A. Mecozzi and M. Shtaif, "Signal-to-noise-ratio degradation caused by polarization-dependent loss and the effect of dynamic gain equalization" JLT, 2004.
[9] A. D'Amico et al., "Disaggregated Statistical Characterization of the PDL Impairment on the Optical Signal-to-Noise Ratio", PSC, 2026.
[10] S. Zarkosvky and M. Shtaif, "Statistical distribution of polarization-dependent loss in systems characterized by the hinge model," Optics Letters, 2020.
[11] Mark Shtaif, "Performance degradation in coherent polarization multiplexed systems as a result of polarization dependent loss," Opt. Express, 2008.
[12] C.S. Withers, "Central Limit Theorems for dependent variables" I. Z. Wahrscheinlichkeitstheorie verw Gebiete 57, 1981.
[13] A. D'Amico et al., "Experimental Probing and Modeling of the PDL Impact on the Optical Signal-to-Noise Ratio", OFC, 2023.
[14] OpenROADM Optical Specifications, https://github.com/OpenROADM/OpenROADM_MSA_Public/wiki/files/20250327_open-roadm_msa_specification_ver8.0.1.xlsx