Analytical Model for Filtering Impact Estimation of Coherent Systems with Finite-Length Equalizer

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***Abstract*—We present an analytical model to estimate the performance penalty due to filtering in coherent systems using finite-length equalizers, reporting a high model accuracy against numerical simulations with a maximum error of 0.1 dB.**

***Index Terms*—Modelling, performance, fiber optics, filter**

I. Introduction

The increase in the capacity of optical networks has required the use of higher symbol rates per channel with the corresponding spectral broadening of the transmitted signals. This has made the transmission more sensitive to frequency-dependent impairments, such as those caused by filtering and colored noise [1], [2]. The most used quality of transmission (QoT) estimation tools [3] for network design, planning, dimensioning and control, normally do not consider the impact of filtering and colored noise. Previous analytical models including these features have been presented, and experimentally verified, for single-carrier [4], [5] and multi-carrier [6] transmission. All of them are based on the minimum mean square error equalizer (MMSEE) theory and assume ideal equalization with infinite number of taps.

In this work, we show an analytical model accounting for the more realistic case of equalization with a finite number of taps to evaluate the performance degradation due to filtering. We verify the model against error-counting-based time-domain numerical simulations, under different filtering conditions, obtaining a high accuracy with a maximum error of 0.1 dB in the estimation of the Signal-to-Noise-Ratio (SNR) at the output of the equalizer (SNR_{EQ}).

II. Simulation Setup

We show in Fig. 1(a) the setup used for time-domain numerical simulations. We emulate a net 400 Gbps band-limited transmission using a single-carrier coherent transceiver (TRX). The symbol rate is R_S = 64 GBaud, the modulation format is DP-16-QAM and the pulse-shaping is a Root Raised Cosine (RRC) with roll-off factor $\rho = 0.1$. The optical signal traverses $M-$stages of filtering plus noise addition prior to reception. We use super-Gaussian filters (SGFs) of order η

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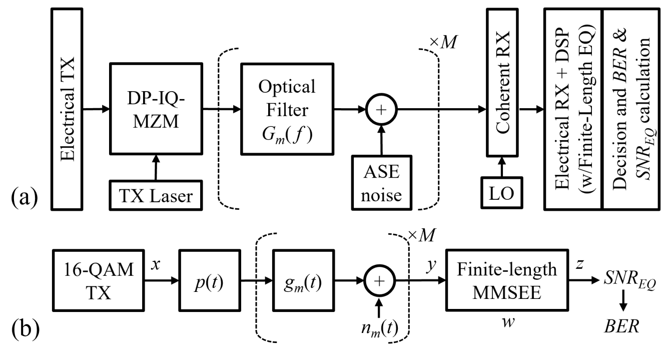

Fig. 1: a) Simulation setup and b) model abstraction.

and -3 dB band-pass frequency equal to B to study the degradation due to filtering. The frequency field response of each individual SGF, $G_m(f)$, is given by Eq. 1.

$$G_m(f) = \exp\left(-\ln(\sqrt{2}) \cdot (2f/B)^{2\eta}\right) \quad (1)$$

After each filter, Amplified Spontaneous Emission (ASE) noise is loaded. Besides filtering and noise, other impairments, such as IQ imbalance, fiber nonlinearities, laser phase noise, and chromatic dispersion, are neglected in the simulation, being outside the scope of this work. After ideal coherent detection, the received signal is resampled at $L = 2$ samples per symbol and passed through a training-symbol-aided decision-directed least mean square (DD-LMS) algorithm for equalization. The equalizer is implemented in a FIR structure with length N_f symbol periods, i.e. $L \cdot N_f$ taps. A total of 2024 symbols are used to train the equalizer. Finally, after the decision, the Bit Error Ratio (BER) is calculated by error counting and used to compute the corresponding SNR_{EQ} [2].

III. Analytical model

We display in Fig. 1(b) the block-diagram we use in the analytical modeling. We abstract a linear optical coherent system, performing ideal optical-to-electrical conversion, and affected by $M-$stages of filtering plus ASE noise.

The model is defined in the time domain, based on the finite-length MMSEE theory presented in [7], [8]. In the following we summarize the main equations, for the case where $M = 1$,

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and then $g(t) = g_1(t)$, $G(f) = G_1(f)$, and $n(t) = n_1(t)$. At the end of this section we generalize for $M > 1$ stages.

The receiver digitize the channel output signal $y(t)$ taking L samples per symbol. The sampled channel output is:

$$y(kT - \frac{iT}{L}) = \sum_{j=-\infty}^{\infty} x_j \cdot h(kT - \frac{iT}{L} - jT) + n(kT - \frac{iT}{L}) \quad (2)$$

where T is the symbol period, $i = 0, ..., L-1$, x_j is the j-th transmitted symbol, $h(t)$ is the channel impulse response equal to $p(t) * g(t)$, $p(t)$ is the pulse-shaping response, $g(t)$ is the SGF response (obtained by applying the inverse Fourier transform to $G(f)$), and $n(t)$ is the noise signal, having a per-dimension sampled variance equal to $L \cdot N_0/2$ (if white).

Given N_f succesive $L-$tuples of samples of $y(t)$, and a discrete channel response $h = [h_0, h_1, ..., h_v]$ with channel-memory of v, the oversampled vector representation of the channel at the equalizer input is $\mathbf{Y}_k = \mathbf{H} \cdot \mathbf{X}_k + \mathbf{N}_k$, where:

$$\mathbf{X}_k \triangleq \left[x_k, x_{k-1}, ..., x_{k-N_f-v+1}\right]^T \quad (3)$$

$$\mathbf{Y}_k \triangleq \left[y_k, y_{k-1}, ..., y_{k-N_f+1}\right]^T \quad (4)$$

$$\mathbf{N}_k \triangleq \left[n_k, n_{k-1}, ..., n_{k-N_f+1}\right]^T \quad (5)$$

$$\mathbf{H} \triangleq \begin{bmatrix} h_0 & h_1 & \dots & h_v & 0 & \dots & 0 \\ 0 & h_0 & h_1 & \dots & h_v & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & h_0 & h_1 & \dots & h_v \end{bmatrix} \quad (6)$$

The MMSEE processes each $\mathbf{Y}_k$ vector, to output the k-th equalized symbol z_k, as $z_k = w \cdot \mathbf{Y}_k$, where w is the vector containing the FIR MMSEE coefficients. The equalizer output error, considering a properly chosen value of Δ for causal implementation, is then: $e_k = x_{k-\Delta} - z_k$.

In this work, we neglect polarization-dependent impairments thus assuming identical performance for both X and Y polarizations. In the following, we show only the expressions for the X-polarization. The SNR at the output of the finite-length MMSEE is evaluated as:

$$SNR_{EQ} = \frac{\overline{\mathcal{E}}_x}{\sigma^2_{MMSEE}} - 1 \quad (7)$$

The term $\overline{\mathcal{E}}_x$ is the average energy of the signal and σ^2_{MMSEE} is the mean square error (MSE) of the equalizer output, evaluated as follows [7]:

$$\sigma^2_{MMSEE} = \mathbb{E}\{|e_k|^2\} = \overline{\mathcal{E}}_x - w \cdot R_{YY} \cdot w^* \quad (8)$$

For a sampling rate L/T, delay Δ, and length N_f symbol periods, w is evaluated as in Eq. 9 [8], which provides the minimum MSE since in this condition e_k and $\mathbf{Y}^*_k$ are uncorrelated.

$$w = R_{xY} \cdot R^{-1}_{YY} \quad (9)$$

The terms R_{xY} and R_{YY} are the FIR MMSEE cross-correlation and autocorrelation matrix, respectively, calculated as follows [8]:

$$R_{xY} = \mathbb{E}\{x_{k-\Delta} \cdot \mathbf{Y}^*_k\} = \left[0 \; ... \; 0 \quad \overline{\mathcal{E}}_x \quad 0 \; ... \; 0\right] \cdot \mathbf{H}^* \quad (10)$$

$$R_{YY} = \mathbb{E}\{\mathbf{Y}_k \cdot \mathbf{Y}^*_k\} = \overline{\mathcal{E}}_x \cdot \mathbf{H} \cdot \mathbf{H}^* + R_{NN} \quad (11)$$

where R_{NN} is the fractionally spaced noise autocorrelation matrix and, when the noise is white, is $R_{NN} = L \cdot N_0 \cdot I/2$.

For the general case of $M > 1$, with distributed filtering and noise addition, we calculate the equivalent filter $g(t)$ and total colored-noise $n(t)$ from the sets of $g_m(t)$ and $n_m(t)$, following Eq. 2 in [9] (applying the proper frequency-time conversions). Using these values we compute $\mathbf{H}$ and $\mathbf{N}_k$, and perform the procedure described before to estimate SNR_{EQ}.

IV. Analytical model versus numerical simulation results

A. Full noise loading after filtering scenario

By means of simulation, we test the performance of the system shown in Fig. 1(a) for $M = 1$, using as a metric the SNR_{EQ}. By using the analytical model described in the previous section, we estimate the SNR_{EQ} for the tested cases.

We present the results in terms of SNR penalty (ΔSNR), calculated as ΔSNR, dB $= SNR_{Ref}$, dB $- SNR_{EQ}$, dB, where SNR_{Ref} is the baseline SNR obtained when filtering is absent and the equalizer length tends to infinite. We set the proper noise variance to get an $SNR_{Ref} = 20$ dB.

In Fig. 2, we show the comparison of the simulation and analytical approaches. The calculated SNR penalty is plotted as a function of the number of taps of the FIR MMSEE, which is $2 \cdot N_f$ since a half-space equalizer is used. We tested different cases in terms of filter bandwidth B and filter orders. The filter bandwidth is reported normalized to the symbol rate, as B/R_S. In all cases, the analytical results (in markers only) overlap with the simulation ones (in solid lines only), thus showing a very good accuracy of the analytical model. The maximum estimation error is less than 0.05 dB.

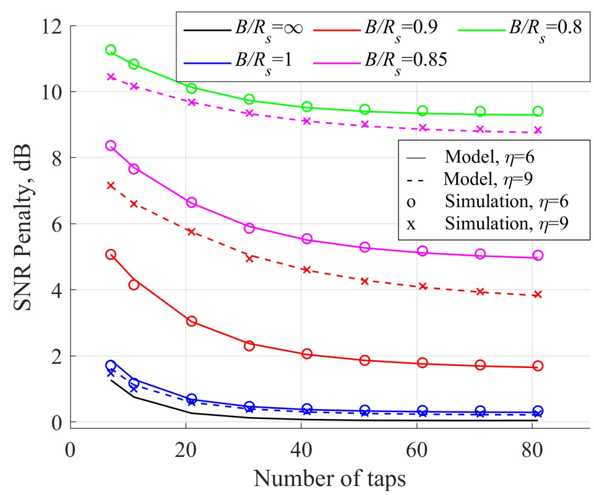

Fig. 2: Simulated equalizer SNR penalty (markers only), and its analytical estimation (lines only), as a function of the number of taps of the equalizer, for different filter bandwidths using a super-Gaussian filter $G(f)$ of order $\eta = 6$ (solid and circles) and $\eta = 9$ (dashed and crosses). SNR penalty referred to a baseline SNR = 20 dB.

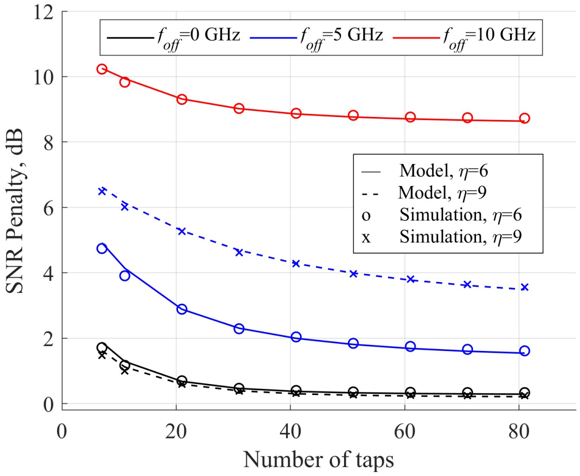

Fig. 3: Simulated equalizer SNR penalty (markers only), and its analytical estimation (lines only), as a function of the number of taps of the equalizer, for different filter orders and signal-to-filter central frequency offsets, considering a super-Gaussian filter $G(f)$ with -3 dB bandwidth equal to the symbol rate. The SNR penalty is referred to a baseline SNR = 20 dB.

Next, we test the model accuracy when introducing a frequency offset between the central frequency of the signal and the central frequency of the filter, termed here as f_{off}. This analysis is useful to study the impact of laser drifting, and also to explore the possibilities of using the model in multi-carrier systems [2], for future work extensions. The SNR penalty for different values of f_{off} is shown in Fig. 3 as a function of the number of taps. An SGF with bandwidth equal to the symbol rate is used. We observe an increase in the SNR penalty when increasing the f_{off}, which is accurately captured by the analytical model, for both filter orders.

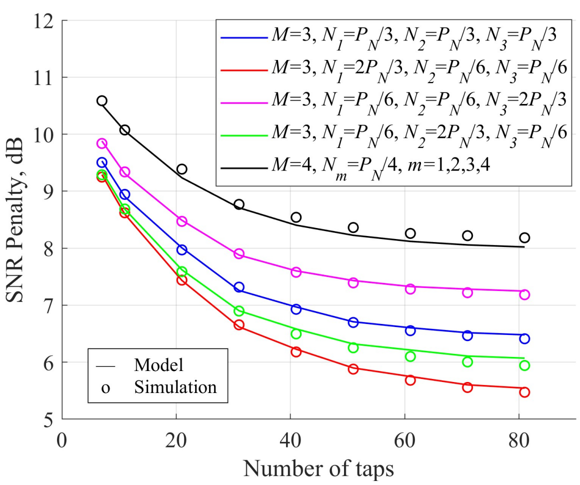

Fig. 4: Simulated equalizer SNR penalty (markers only), and its analytical estimation (lines only), as a function of the number of taps ($L \cdot N_f$) of the equalizer, for different distributed cases using SGFs having $B = 0.9 \cdot R_S$ and $\eta = 6$. The SNR penalty is referred to a baseline SNR = 20 dB, corresponding to a noise power equal to $P_N = \sum N_m$.

B. Distributed noise loading scenario

Finally, in Fig. 4, we compare the model estimations and the simulation results for the distributed filtering and noise scenario, for $M = 3$ and $M = 4$. In the first case, we tested different distributions of the power of each noise stage (N_m): (a) uniformly distributed, (b) more noise in the first stage, (c) more noise in the last stage, and (d) more noise in the middle stage. We corroborate that cases (b) and (c) are the best- and worst-case scenarios for noise loading, respectively, as reported in [4]. For $M = 4$, we tested only uniformly distributed noise. In all the cases shown in Fig. 4, the agreement between the model and the simulations is accurate, with a maximum SNR estimation error of 0.1 dB.

V. Conclusions

We verified against numerical simulations a semi-analytical model to assess the performance penalty due to filtering in finite-length equalizer-based coherent systems performing ideal optical-to-electrical conversion. We test the model accuracy for different filtering conditions when the noise is added after the filter. The model achieves a good accuracy with a maximum estimation error of 0.1 dB. We verify that the analytical approach can reduce computational time by at least one order of magnitude compared to error-counting-based numerical simulations. In this work, we did not consider the impact of Chromatic Dispersion (CD), since this is DSP-compensated by a specific block placed before the adaptive equalization. Further extension of this work should address the impact of the amount of residual CD that is not exactly compensated by the CD compensation algorithm. Furthermore, we use here SGFs without group delay, whose impact is negligible using an infinitely-long equalizer, but can introduce a performance penalty using a finite-length equalizer, whose estimation is an open topic for future research extensions.

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