THE PERFORMANCE ANALYSIS OF A MULTI-OBJECTIVE IMMUNE GENETIC ALGORITHM FOR FLEXIBLE JOB SHOP SCHEDULING

XiuLi Wu , ShuDong Sun, GangGang Niu, YinNi Zhai
Departerment of Mechanical and Electronic Engineering, Northwestern Polytechnical University, Xi'An, 710072, China; Email: henanwxl@hotmail.com

Abstract: First, a multi-objective immune genetic algorithm integrating immune algorithm and genetic algorithm for flexible job shop scheduling is designed. Second, Markov chain is used to analyze quantitatively its convergence. Third, a simulation experiment of the flexible job shop scheduling is carried out. Running results show that the proposed algorithm can converge to the Pareto frontier quickly and distribute evenly along the Pareto frontier.

Key words: flexible job shop scheduling, multi-objective genetic algorithm, convergence, Markov chain.

1. INTRODUCTION

Flexible job shop multi-objective scheduling problem (FJSP) is typical NP hard, which is not easy to solve with a polynomial algorithm. Therefore, evolutionary algorithms, such as genetic algorithm (GA) and immune algorithm (IA), are the efficient procedures. Many literatures presented different multi-objective optimization methods. But the theoretical analysis of the algorithm itself is sparse. Those focusing on the subject mostly concentrate on GA with binary encoding [1], while research on GA with decimal encoding is little. Rudolph's research [2, 3, 4], who is the most contributing scientist, mainly focused on the effect of the elitist strategy on the convergence. The convergence of the hybrid GA taking into more factors, such as the immune technology, has not been discussed in the publicly published literatures. Therefore, a multi-objective immune genetic algorithm (MOIGA) for FJSP is presented and its performance is analyzed.

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2. A MULTI-OBJECTIVE IMMUNE GENETIC ALGORITHM FOR SCHEDULING

Let < **P(t),** $\prec$ > be the non-dominated set of the set P(t), P(t) the set of the population, Q(t) the Pareto set of P(t), and S_j the sharing degree of the i^{th} chromosome when processed by the niche technology. The procedure of MOIGA is as follows.

P(0) is randomly generated;
Q(0) is the set for Pareto set of P(0);
t=1;
repeat
  if t>=T then end repeat;
  $f(X_j)= f(X_j)/S_j$;
  get bacterin from P(t-1);
  P(t) = generate(P(t-1));
  crossover(P(t));
  mutation(P(t));
  vaccination(P(t));
  If $\exists a \in Q(t-1)$ and $\exists x \in P(t)$ and $f(x) \prec f(a)$ then
  Q(t) $\leftarrow$ x, P(t) $\leftarrow$ a;
  else if $\langle P(t), \prec \rangle \subseteq Q(t-1)$ then
  $Q(t)=Q(t-1)$;
  t=t+1;
end.

3. ANALYSIS OF THE CONVERGENCE

Now let us describe the proposed MOIGA in terms of Markov Chain. Let $\boldsymbol{\Omega}$ be the finite state space and X ($X \in \boldsymbol{\Omega}$) the individual. All populations cover the whole state space $\boldsymbol{G}$, in which a population represents a state. In the operation-based encoding, encoding character set $\boldsymbol{\lambda}$ is composed of the job numbers. The number of the element C in the set $\boldsymbol{\lambda}$ equals the total number N of the jobs, i.e. $C = N$. Let the length of each chromosome be L, then $L = N \times \max\{n_j\}$. If the population size P is given and finite, then the dimension of $\boldsymbol{\Omega}$ and $\boldsymbol{G}$ is finite and $|\boldsymbol{G}| = |\boldsymbol{\Omega}|^P$. The evolutionary procedure of GA is repeating, i.e. the population transfers from one state to another state after the selection, crossover, mutation and vaccination operation. According to [6], we know the process is a homogeneous Markov Chain with a finite state space.

1） The crossover operation

MOIGA adopts linear order crossover operation. Let the transition matrix determined by crossover operation be $\boldsymbol{C}=(c_{ij})_{|G|\times|G|}$, and c_{ij} the probability from the state i to j after the crossover operation. Because one state is transferred to another state at all events, there exists $\sum_{j=1}^{|G|} c_{ij}=1$. According to [6], we know the matrix $\boldsymbol{C}$ is random.

2） The mutation operation

The mutation of MOIGA adopts exchanging operation method. Let the transition matrix determined by the mutation operation be $\boldsymbol{T}=(t_{ij})_{|G|\times|G|}$ and t_{ij} the probability from the state i to j after the mutation operation. Let the hamming distance between the state i and j be H_k , k=1, 2..., P. For example, in a FJSP problem with 3 jobs and 2 operations for each, its two states, i.e. the state i and j, are as follows:

s_i= {(1 2 3 3 2 1), (2 1 3 1 2 3), (3 2 1 1 2 3)}

s_i= {(2 1 3 3 2 1), (2 1 2 1 3 3), (3 2 1 3 2 1)}

Then the hamming distance between them is $H_1=H_2=...=H_P=2$.

Each gene of a chromosome has the equal mutation probability p_m ($0<p_m<1$). Therefore

$$t_{ij}=\prod_{i=1}^{P}\left(\frac{p_m}{C-1}\right)^{H_i}(1-p_m)^{L-H_i}=\left(\frac{1}{N-1}\right)^{2P}p_m^{2P}(1-p_m)^{NP\times\max\{n_j\}-2P}>0$$

According to [6], the matrix $\boldsymbol{T}$ is positive and random.

3） The vaccination operation

The vaccination of MOIGA is another mutation operation. Let the transition matrix determined by the vaccination operation be $\boldsymbol{T}=(t_{ij})_{|G|\times|G|}$ and t_{ij} the probability from the state i to j after the mutation operation. Each chromosome has the equal vaccination probability p_h ($0<p_h<1$). Therefore

$$h_{ij}=\prod_{i=1}^{P}\left(\frac{p_h}{C-1}\right)^{H_i}(1-p_h)^{L-H_i}=\prod_{i=1}^{P}\left(\frac{p_h}{N-1}\right)^{H_i}(1-p_h)^{N\times\max\{n_j\}-H_i}>0$$

According to [6], the matrix $\boldsymbol{T}$ is positive and random. Therefore, $\boldsymbol{Z}=\boldsymbol{TH}$ is strictly positive.

4） The selection operation

Before selection, MOIGA adopts the elitist strategy, i.e. the Pareto optimization solutions will enter into the next generation directly. Recall the probability transition matrix of this operation as $\boldsymbol{E}=(e_{ij})_{|G|\times|G|}$ and the k^{th} chromosome as π_k . Put the Pareto set of the current population in the front

of the next generation population and recall them as $\pi_1, \pi_2, ..., \pi_r$. Therefore, the state space size extends to $|\boldsymbol{\Omega}|^r$ times, i.e. $|\boldsymbol{G}| = |\boldsymbol{\Omega}|^{P+r}$. If there is at least a chromosome in the set $P(t+1)$ which can dominate one (or some) in the set $Q(t)$, then replace it (or them). Therefore, the transition probability from the state i to j is:

$$e_{ij} = \begin{cases} 1 & \text{if } \exists a \in Q(t) \text{ and } \exists x \in P(t+1) \text{ and } f(x) \prec f(a) \\ 1 & \text{if } \langle P(t+1), \prec \rangle \subseteq Q(t) \\ 0 & \text{else} \end{cases}$$

Order the states with the same Pareto set as the original status and order other states according to the order of the Pareto set. Therefore, the matrix $\boldsymbol{E}$ is under-triangle matrix, the element in the matrix $\boldsymbol{E}$ is zero or one, there is only a one in each row, and $\boldsymbol{E}_{11}$ is an identity matrix.

$$E = \begin{bmatrix} E_{11} & & & \\ E_{21} & E_{22} & & \\ \vdots & \vdots & \ddots & \\ E_{|\Omega|^r 1} & E_{|\Omega|^r 2} & \cdots & E_{|\Omega|^r |\Omega|^r} \end{bmatrix}$$

According to [6], we know that the matrix $\boldsymbol{E}$ is a random matrix.

After pre-selection, roulette selection is used to select the outstanding chromosomes to the next generation from the set $P(t)$. Recall the transition matrix of the selection operation as $\boldsymbol{S} = (s_{ij})_{|G|\times|G|}$ and s_{ij} is the probability from the state i to j. In the roulette selection, there exists

$$s_{ij} = \prod_{j=1}^{P} \left(\frac{f(X_j) / S_j}{\sum_{k=1}^{P} f(X_k) / S_k} \right) > 0$$

According to [6], we know that $\boldsymbol{S} = (s_{ij})_{|G|\times|G|}$ is a column allowable random matrix and that $\boldsymbol{CZS} > \boldsymbol{0}$. After the elitist strategy, the state space extends to $|\boldsymbol{\Omega}|^r$ times, the transition matrices of the crossover, the mutation and the selection change into the matrices $\boldsymbol{C}^+$, $\boldsymbol{T}^+$ and $\boldsymbol{S}^+$, individually.

$$\boldsymbol{C}^+ = \begin{bmatrix} \boldsymbol{C} & & \\ & \boldsymbol{C} & \\ & & \ddots \\ & & & \boldsymbol{C} \end{bmatrix}, \quad \boldsymbol{Z}^+ = \begin{bmatrix} \boldsymbol{Z} & & \\ & \boldsymbol{Z} & \\ & & \ddots \\ & & & \boldsymbol{Z} \end{bmatrix}, \quad \boldsymbol{S}^+ = \begin{bmatrix} \boldsymbol{S} & & \\ & \boldsymbol{S} & \\ & & \ddots \\ & & & \boldsymbol{S} \end{bmatrix}$$

Therefore, the transition probability matrix is as follows according to the evolutionary process of MOIGA and the Chapman-Kolmogorov equation.

$$P^+ = C^+ Z^+ E S^+$$

$$= \begin{bmatrix} C & & \\ & C & \\ & & \ddots \\ & & & C \end{bmatrix} \begin{bmatrix} Z & & \\ & Z & \\ & & \ddots \\ & & & Z \end{bmatrix} \begin{bmatrix} E_{11} & & & & \\ E_{21} & E_{22} & & & \\ \vdots & \vdots & \ddots & & \\ E_{|\Omega|^r 1} & E_{|\Omega|^r 2} & \cdots & E_{|\Omega|^r|\Omega|^r} \end{bmatrix} \begin{bmatrix} S & & \\ & S & \\ & & \ddots \\ & & & S \end{bmatrix}$$

$$= \begin{bmatrix} CZE_{11}S & & & & \\ CZE_{21}S & CZE_{22}S & & & \\ \vdots & \vdots & \ddots & & \\ CZE_{i1}S & \cdots & CZE_{ij}S & \cdots & CZE_{ii}S \\ \vdots & & \vdots & & \vdots \\ CZE_{|\Omega|^r 1}S & CZE_{|\Omega|^r 2}S & \cdots & & CZE_{|\Omega|^r|\Omega|^r}S \end{bmatrix}$$

Let

$$R = \begin{bmatrix} CZE_{21}S \\ \vdots \\ CZE_{i1}S \\ \vdots \\ CZE_{|\Omega|^r 1}S \end{bmatrix} \quad W = \begin{bmatrix} CZE_{22}S & & & \\ \vdots & \ddots & & \\ CZE_{i2}S \cdots & CZE_{ik}S \cdots & CZE_{ii}S & \\ \vdots & \vdots & \vdots & \ddots \\ CZE_{|\Omega|^r 2}S & \cdots & & CZE_{|\Omega|^r|\Omega|^r}S \end{bmatrix}$$

Because the product of CZS is positive, and E_{11} is an identity matrix, it can be concluded that $P = CTE_{11}S$ is positive with the state space size of $|\Omega|^r \times |\Omega|^r$. Then $P^+ = \begin{bmatrix} P & O \\ R & W \end{bmatrix}$.

Hereinto, there is at least a nonzero in each row of the matrix R. According to [6], those states without the inclusion of optimal chromosome have zero probability in the limited distribution of Markov Chain; while the limited distribution sum of those states with the optimal chromosome equals to 1. Thus it can be seen that MOIGA can converge to the true Pareto optimal set with the probability 1.

4. AN EXPERIMENT

To prove the performance of the proposed MOIGA, a practical scheduling problem in an aeronautic company with two objectives to be optimized, i.e. production time and production cost. The encoding method is operation-based way. The experiment result is as follows: the two objectives converge to the stable distribution with the evolutionary process (Fig 1). In

figure 1, the Pareto frontier is shown with red asterisks and the convergence and diversity are assured perfectly.

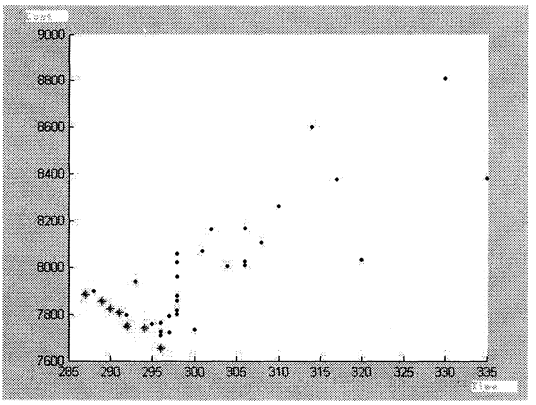

Figure 1 The printed Pareto frontier plot

5. CONCLUSION

The paper analyzed a multi-objective immune genetic algorithm and proofed that it can converge to the Pareto optimization set with the probability one. The running result of a practical problem shows the effectiveness and efficiency of the proposed algorithm.

6. REFERENCES

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