On the Low SNR Capacity of MIMO Fading Channels with Imperfect Channel State Information

Fatma Benkhelifa*, Abdoulaye Tall**, Zouheir Rezki*, and Mohamed-Slim Alouini*
* Computer, Electrical and Mathematical Science and Engineering (CEMSE) Division
King Abdullah University of Science and Technology (KAUST)
Thuwal, Makkah Province, Saudi Arabia
{fatma.benkhelifa, zouheir.rezki, slim.alouini}@kaust.edu.sa
** Orange Labs, France
abdoulaye.tall@orange.com

***Abstract*—[1] The capacity of Multiple Input Multiple Output (MIMO) Rayleigh fading channels with full knowledge of channel state information (CSI) at both the transmitter and the receiver (CSI-TR) has been shown recently to scale at low Signal-to-Noise Ratio (SNR) essentially as SNR $\log(1/\mathbf{SNR})$, independently of the number of transmit and receive antennas. In this paper, we investigate the ergodic capacity of MIMO Rayleigh fading channel with estimated channel state information at the transmitter (CSI-T) and possibly imperfect channel state information at the receiver (CSI-R). Our framework can be seen as a generalization of previous works as it can capture the perfect CSI-TR as a special case when the estimation error variance goes to zero. In our work, we mainly focus on the low SNR regime and we show that the capacity scales as $(1-\alpha)$ SNR $\log(1/\mathbf{SNR})$, where α is the estimation error variance. This characterization shows the loss of performance due to error estimation over the perfect channel state information at both the transmitter and the receiver. As a by-product of our new analysis, we show that our framework can also be extended to characterize the capacity of MIMO Rician fading channels at low SNR with possibly imperfect CSI-T and CSI-R.**

***Index Terms*—Ergodic capacity, MIMO, CSI-T, CSI-R, channel estimation, Rayleigh fading channel, Rician fading channel, low SNR.**

I. Introduction

Multiple-Input Multiple-Output (MIMO) systems have captured in the last two decades great interest as they have been shown to provide an increased spectral efficiency compared to the conventional wireless systems [1]–[11]. For an independent and identically distributed (i.i.d.) Rayleigh fading channel, the capacity scales linearly $\min(N_r, N_t)$ times compared to Single-Input Single-Output (SISO), where N_t is the number of transmit antennas and N_r is the number of receive antennas with perfect Channel State Information (CSI) at high signal-to-noise ratio (SNR) [1]–[3]. When perfect CSI is available at both the transmitter and at the receiver (CSI-TR), the capacity of MIMO channels is well known and has been widely studied in [1], [12].

In a realistic scenario, the estimates of the channel gains are not perfectly known at the transmitter side and/or the receiver side. Indeed, the CSI at the receiver (CSI-R) is generally obtained using the transmission of training sequences. The CSI at the transmitter (CSI-T) is obtained via a feedback channel or from previous measurements via the channel reciprocity property [13]. While CSI-R can be assumed to be perfect or not, depending on the practical cases, CSI-T is far from being perfectly known in realistic models. Thus, much more work has been conducted in the literature in order to characterize the MIMO capacity with more realistic assumptions than perfect CSI at the transmitter (CSI-T). When only the CSI is available at the receiver (CSI-R), the capacity of MIMO channels has been derived in [1], [3]. The capacity of MIMO channels with partial CSI-T or statistical CSI-T and perfect CSI-R was also derived in [1], [8], [10]. The capacity of some channels with imperfect CSI-T and perfect CSI-R was derived in [14]. In [4], lower and upper bounds on the capacity of MIMO channels subject to an average power constraint were derived under channel estimation error at both the transmitter and the receiver. The optimal transmitter power allocation was also determined with and without estimated CSI-T in [4]. In [5], [6], the capacity of MIMO Rician channels was derived when perfect CSI-R is assumed but the transmitter has neither instantaneous nor statistical CSI-T. In [6], the capacity of MIMO Rician channels was approximated by that of correlated MIMO Rayleigh channels. These works show that the availability of the channel state information at the transmitter has a considerable impact on the channel capacity. This influence vanishes at high SNR , but at low SNR the ratio of the capacity with CSI-T over the capacity without CSI-T goes to infinity according to [15]. Hence, it is interesting to investigate how does the capacity scale at low SNR with imperfect CSI-T. Besides to the assumption of imperfect CSI-T, we also investigate the effect of channel estimation at the receiver on the capacity at low SNR.

Moreover, our motivation to focus on the low SNR regime is not only dictated by the high energy efficiency of communication at this regime, but also by the nature of wireless communication in some specific but important situations. Indeed, many wireless systems now operate at low SNR. For instance, in wideband communication, we have a huge bandwidth and we can operate at low power regime and still achieve high capacity [16]. In sensor networks, there

[1] An enhanced version of this paper has been submitted to IEEE Transactions on Communications for a possible publication and is now under review.

is a detrimental request for power savings [17], [18]. More generally, the low power regime applies to any communication system where the bandwidth and the power are fixed, but the system degrees of freedom are large enough such that the power per degree of freedom is very low [15]. Hence, analyzing the capacity as a performance metric at low power regime is of practical and theoretical interests.

In [16], Borade and Zheng have shown that the capacity of the SISO Rayleigh flat fading channel at low SNR essentially scales as SNR $\log(1/\text{SNR})$ nats/symbol. In [19], the low-SNR capacity of M-ary phase-shift keying (PSK) over both the additive white Gaussian noise (AWGN) and fading channels is studied when a hard-decision detection is employed at the receiver end. In [9], [10], the low SNR regime of the MIMO Rician capacity has been investigated with no CSI-T or mean CSI-T and perfect CSI-R. In [10], the low SNR capacity of MIMO Rician channels with statistical CSI-T and perfect CSI-R was investigated. In [11], Tall *et al.* have characterized the ergodic capacity of MIMO Rayleigh fading channels with perfect channel state information at both the receiver and the transmitter (CSI-TR) for any number of transmit and receive antennas at asymptotically low SNR and they proposed an on off power scheme to be capacity achieving. However, the channel may not be perfectly known at the transmitter and/or the receiver. Several techniques have been proposed in order to estimate the channel gains at the transmitter or/and the receiver. Motivated by these results, we propose in this paper to investigate the effect of the channel estimation on the ergodic capacity of a MIMO Rayleigh fading channel with an estimated CSI-T and possibly imperfect CSI-R, and provide the asymptotically low-SNR characterization of the capacity. We then deduce the ergodic capacity of a MIMO Rician fading channel with possibly imperfect CSI at both the transmitter and the receiver at low SNR.

II. System Model

We consider a MIMO Gaussian fading channel with N_t transmit antennas and N_r receive antennas given by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{x}$ is the $N_t \times 1$ complex random vector that represents the channel input, $\boldsymbol{y}$ is the $N_r \times 1$ complex random vector that represents the channel output, $\boldsymbol{n}$ is the $N_r \times 1$ complex random vector that represents the AWGN noise with zero mean and covariance matrix $\mathbb{E}(\boldsymbol{n}\boldsymbol{n}^H) = I_{N_r}$ and $\boldsymbol{H}$ is the $N_r \times N_t$ random complex Gaussian channel matrix with i.i.d. entries that are complex circularly symmetric Gaussian with zero mean and $1/2$ variance per dimension, i.e. $\mathcal{CN}(0,1)$, where $\mathbb{E}[\cdot]$ denotes the expectation operation.

Let us denote by $m = \min(N_r, N_t)$ and $n = \max(N_r, N_t)$. The transmitted signal $\boldsymbol{x}$ is subject to an average power constraint P_{avg}:

$$\mathbb{E}[\boldsymbol{x}^H\boldsymbol{x}] \leq P_{avg}. \tag{2}$$

Since the noise has normalized covariance, P_{avg} can be simply designated as the transmit SNR, i.e., $P_{avg} = \text{SNR}$.

When the channel state information is available at the transmitter and the receiver, the expression of the capacity is given by [2], [3]

$$C = \mathbb{E}_{\boldsymbol{H}}\left[\sum_{i=1}^{m}\left(\log\left(\mu\lambda_i\right)\right)^+\right], \tag{3}$$

where $\lambda_i, i = 1, ..., m$ are the eigenvalues of the matrix $\boldsymbol{H}\boldsymbol{H}^H$ and μ is chosen via the water-filling policy to satisfy the average power constraint with equality as

$$\text{SNR} = \mathbb{E}_{\boldsymbol{H}}\left[\sum_{i=1}^{m}(\mu - \frac{1}{\lambda_i})^+\right], \tag{4}$$

where $(\cdot)^+$ is defined as $(x)^+ = max(x, 0)$. However, when the transmitter does not have full knowledge of the channel matrix $\boldsymbol{H}$ and only an estimated version of the channel is available at the transmitter, to the best of our knowledge, no closed form expression of the capacity is available in the literature. Only upper and lower bounds on the capacity were derived in [4] when the channel is estimated at both the transmitter and the receiver. In this paper, we derive an asymptotic expression of the ergodic capacity at low SNR and characterize the loss in capacity due to channel estimation error at the transmitter or the receiver, in the low power regime. In the remainder of this paper, we define $f(x) \cong g(x)$ if and only if $\lim_{x\to 0}\frac{f(x)}{g(x)} = 1$. Note that $f\tilde{\leq}g$, and $f\tilde{\geq}g$ are defined analogously.

III. Low SNR capacity of MIMO Rayleigh Fading Channel

We assume that CSI-T is always imperfect. However, CSI-R is possibly assumed to be perfect or imperfect. The case when CSI-T is imperfect and CSI-R is perfect (Case I) can be associated to the practical scenario when we have perfect channel estimation at the receiver and a noisy channel feedback between the receiver and the transmitter [20]. The noisy feedback channel is basically the cause of the mismatch between the estimated channel at the receiver and the transmitter. This case also can be justified by the fact that the true channel evolves over time by the addition of an independent matrix with $\mathcal{C}(0,1)$ entries where α is a forgetting factor which describes the rate of evolution of the channel matrix. However, the case when CSI-T and CSI-R are both imperfect (Case II) can be associated to the practical scenario when the channel estimation at the receiver is not perfect, while the feedback channel is perfect and instantaneous [4], [20]. We suppose that the channel matrix $\boldsymbol{H}$ in both cases can be expressed as [4], [20]

$$\boldsymbol{H} = \sqrt{1-\alpha}\hat{\boldsymbol{H}} + \sqrt{\alpha}\tilde{\boldsymbol{H}}, \tag{5}$$

where $\boldsymbol{H}$ is the channel matrix described in (1), $\hat{\boldsymbol{H}}$ is the estimate of $\boldsymbol{H}$ where the entries are assumed to be $\mathcal{CN}(0,1)$, $\tilde{\boldsymbol{H}}$ is the error matrix independent of $\hat{\boldsymbol{H}}$ where the entries are assumed to be $\mathcal{CN}(0,1)$, and $\alpha \in [0,1]$ is the estimation error variance due to the noisy feedback channel (Case I) or the channel estimation at the receiver (Case II). Note that in Case II, we consider the case when we have the same channel estimate at the transmitter and the receiver. Different channel

estimates at the transmitter and the receiver can be the object of further works.

A. Case I: Imperfect CSI-T and Perfect CSI-R

In this subsection, we investigate the case when CSI-R is perfectly available at the receiver; whereas only an estimated version of $\boldsymbol{H}$ is available at the transmitter as given in (5). The effect of the CSI-T estimation on the low SNR capacity of the MIMO Rayleigh channel is stated as follows.

Theorem 1: The capacity of MIMO Gaussian channels with Rayleigh fading at low SNR when only an MMSE estimator of the channel is available at the transmitter and when the channel is perfectly known at the receiver is given by

$$C_{\alpha,0} \approx \begin{cases} -(1-\alpha)\ L\ SNR\ W_0((SNR)^{\frac{1}{L}}), \text{if } L<0, \\ -(1-\alpha)\ SNR\ \log(SNR), \text{if } L=0, \\ -(1-\alpha)\ L\ SNR\ W_{-1}(-(SNR)^{\frac{1}{L}}), \text{if } L>0, \end{cases} \quad (6)$$

$$\approx -(1-\alpha)\ SNR\ \log(SNR), \quad (7)$$

where $L = n+m-4$, $W_0(.)$ and $W_{-1}(.)$ are the principal branch and the lower branch of the Lambert function, respectively. In other words, Theorem 1 states that the asymptotic capacity loss due to CSI-T estimation is proportional to α.

Proof: The proof is in Appendix 1. ∎

Note that Theorem 1 captures the perfect CSI-TR scenario as a special case by setting $\alpha = 0$. Naturally, the SISO ($n = m = 1$) Rayleigh fading with perfect CSI-TR is also a special case of Theorem 1. Therefore, our result in Theorem 1 may be regarded as a non-trivial generalization of previous works [11], [16].

In this section, we have assumed that the receiver has perfect CSI-R and we have studied the impact of imperfect CSI-T. This relies on the fact that the receiver is able to track perfectly the channel using pilots, for instance, and then feeds back this channel gain to the transmitter via a noisy channel. However, in the low-SNR regime, channel estimation becomes very challenging and thus perfect CSI-R does not seem realistic. Whether the result in Theorem 1 is robust against the perfect CSI-R assumption or not is addressed next.

B. Case II: Imperfect CSI-T and Imperfect CSI-R

In this subsection, we investigate the case when only an estimated version of $\boldsymbol{H}$ is available at both the transmitter and the receiver as given in (5). To the best of our knowledge, there is no closed form expression of the capacity. Upper and lower bounds of the capacity have been derived in e.g. [4]. Likewise in [4], we assume that there is a perfect and instantaneous feedback from the receiver to the transmitter, so that whatever CSI-R the receiver has, is also available at the transmitter. Hence, the estimated channel $\hat{\boldsymbol{H}}$ is the same at the transmitter and at the receiver and the channel estimation error α is perfectly known to both the transmitter and the receiver. Different estimated channels and channel estimation errors at the transmitter and the receiver might be the object of further research work. The effect of the CSI-T and CSI-R estimation on the low SNR capacity of the MIMO Rayleigh channel is stated as follows.

Proposition 1: The capacity of MIMO Gaussian channels with Rayleigh fading at low SNR when only an MMSE estimator of the channel is available at both the transmitter and the receiver is given by

$$C_{\alpha,\alpha} \cong C_{\alpha,0}$$

$$\cong \begin{cases} -(1-\alpha)\ L\ \text{SNR}\ W_0((\text{SNR})^{\frac{1}{L}}), \text{if } L<0, \\ -(1-\alpha)\ \text{SNR}\ \log(\text{SNR}), \text{if } L=0, \\ -(1-\alpha)\ L\ \text{SNR}\ W_{-1}(-(\text{SNR})^{\frac{1}{L}}), \text{if } L>0, \end{cases} \quad (8)$$

$$\cong -(1-\alpha)\ \text{SNR}\ \log(\text{SNR}). \quad (9)$$

Proof: The proof is in Appendix 2. ∎

Proposition 1 states together with Theorem 1 that the channel estimation at the receiver is not crucial at low signal-to-noise ratio (SNR) and only channel state information (CSI) at the transmitter (CSI-T) matters for the asymptotic capacity loss. Note that CSI at the receiver (CSI-R) is not crucial as long as the later has better or equal CSI than the one at the transmitter.

IV. Low SNR capacity of MIMO Rician Fading

In this section, we consider a MIMO Rician fading channel where the channel matrix in (1) is modeled by [7]

$$\boldsymbol{H} = \sqrt{\frac{K}{K+1}}\overline{\boldsymbol{H}} + \sqrt{\frac{1}{K+1}}\boldsymbol{H}_\omega, \quad (10)$$

where $K > 0$ is the Rician factor, $\overline{\boldsymbol{H}} \in C^{N_r \times N_t}$ is a normalized deterministic complex matrix that represents the line of sight component in $\boldsymbol{H}$, $\boldsymbol{H}_\omega \in C^{N_r \times N_t}$ is a random complex matrix with i.i.d. CCSG entries with zero mean and variance $1/2$ per dimension. For this type of fading, we consider the case when we have perfect CSI-TR, and the case when we have an estimated CSI-T and possibly imperfect CSI-R.

A. Case I: Perfect CSI-TR

When we have perfect CSI-TR, the capacity of MIMO Rician fading channels at low power regime can be derived as follows.

Proposition 2: The capacity of MIMO Gaussian channel with Rician fading at low SNR with perfect CSI at both the transmitter and the receiver is given by

$$C_{0,0} \cong \begin{cases} -\frac{1}{K+1}\ L\ \text{SNR}\ W_0((\text{SNR})^{\frac{1}{L}}), \text{if } L<0, \\ -\frac{1}{K+1}\ \text{SNR}\ \log(\text{SNR}), \text{if } L=0, \\ -\frac{1}{K+1}\ L\ \text{SNR}\ W_{-1}(-(\text{SNR})^{\frac{1}{L}}), \text{if } L>0, \end{cases} \quad (11)$$

$$\cong -\frac{1}{K+1}\ \text{SNR}\ \log(\text{SNR}). \quad (12)$$

Proof: To prove the result in Proposition 2, we will derive an upper bound and a lower bound in a similar way as in the proof of the Theorem 1.

- Upper bound on $C_{0,0}$

  If we take into account (10), the channel model in (1)

can be seen as the combination of the outputs of two-MIMO receivers that observes $\boldsymbol{y}_1 = \sqrt{\frac{K}{K+1}}\overline{\boldsymbol{H}}\boldsymbol{x} + \sqrt{\epsilon}\boldsymbol{n}_1$ and $\boldsymbol{y}_2 = \sqrt{\frac{1}{K+1}}\boldsymbol{H}_\omega\boldsymbol{x} + \sqrt{1-\epsilon}\boldsymbol{n}_2$, where $0 < \epsilon < 1$, $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ are complex symmetric Gaussian vector with zero mean and covariance matrix I_{N_r}. The capacity of the channel in (1) is upper bounded by the capacity of the channel with outputs $\boldsymbol{y}_1$ and $(\boldsymbol{y}_2, \boldsymbol{H}_\omega)$, respectively. Indeed, the capacity of the channel with the output $\boldsymbol{y}_1$ can be seen as the capacity of a MIMO AWGN channel (since $\overline{\boldsymbol{H}}$ is deterministic) and is proportional to $\log\left(1 + \frac{K}{K+1}\frac{\mathrm{SNR}}{\epsilon}\right)$. However, the capacity of the channel with the output $(\boldsymbol{y}_2, \boldsymbol{H}_\omega)$ is exactly the capacity of a MIMO channel that encounters a Rayleigh fading channel as derived in Theorem 1 but with an average power equal to $\left(\frac{1}{K+1}\frac{\mathrm{SNR}}{1-\epsilon}\right)$. As we are operating at low SNR, the capacity of the channel with the output $\boldsymbol{y}_1$ is asymptotically vanishing when compared to the capacity of the channel with the output $\boldsymbol{y}_2$. Then we can easily obtain the result in Proposition 2 by using the facts [21]

$$\lim_{x\to\infty}\frac{\boldsymbol{W}_0(\beta x)}{\boldsymbol{W}_0(x)} = 1,\ \lim_{x\to\infty}\frac{W_0(x)}{\log(x)} = 1,\ \lim_{y\to 0^-}\frac{\boldsymbol{W}_{-1}(\beta y)}{\boldsymbol{W}_{-1}(y)} = 1,$$
$$\lim_{y\to 0^-}\frac{W_{-1}(y)}{\log(-y)} = 1,\ \forall \beta > 0, x > 0, y < 0,$$

and letting $\epsilon \to 0$.

- Lower bound on $C_{0,0}$
  The lower bound on $C_{0,0}$ can be obtained in a similar way as in the proof of Theorem 1 using an on-off power control scheme applied to $\boldsymbol{H}$. The proof is the same as in the Theorem 1, except that the average power is now $\frac{1}{K+1}\mathrm{SNR}$ instead of SNR.

■

B. Case II: Imperfect CSI-T and Possibly Imperfect CSI-R

The low SNR capacity of the MIMO Rician channel with estimated CSI-T and possibly imperfect CSI-R can also be derived in a similar way as in Theorem 1 and Proposition 1 and it is given in the following Corollary. Note that likewise in III-B, we assume that we have a perfect and instantaneous feedback from the receiver to the transmitter [4]. Hence, the estimated channel $\hat{\boldsymbol{H}}$ is the same at the transmitter and at the receiver and the channel estimation error α is perfectly known to both the transmitter and the receiver.

Corollary 1: The capacity of MIMO Gaussian channel with Rician fading at low SNR with the same MMSE estimated CSI at the transmitter and at the receiver is given by

$$C_{\alpha,\alpha} \cong C_{\alpha,0} \cong (1-\alpha)\, C_{0,0}, \tag{13}$$

where α is the estimation error variance and $C_{0,0}$ is the capacity of MIMO Gaussian channel with Rician fading with perfect CSI at both the transmitter and the receiver stated in Proposition 2.

Proof: When the channel matrix $\boldsymbol{H}$ in (10) is not fully known by the transmitter and only an MMSE estimate of the channel is available, the low SNR capacity of the MIMO Rician channel with noisy CSI-T and perfect CSI-R can be derived in a similar way as in the proof of Theorem 1 and it is straightforward to see that in this case the capacity scales asymptotically as $(1-\alpha)\, C_{0,0}$. Then, we can deduce the capacity with estimated CSI-T and estimated CSI-R in a similar way as in Proposition 1, provided that the channel estimates at the transmitter and the receiver are the same. ■

Note that this result captures the case of SIMO channel when we have only one transmit antenna and multiple receive antennas that we have studied in previous works [22], [23]. Again, Corollary 1 can be seen as a generalization of our previous works. Note that in [23], we have only considered the perfect CSI-TR [2].

V. Numerical Results

In this section, selected numerical results are provided to show the accuracy of our asymptotic characterization in Proposition 1 (or equivalently Theorem 1), Proposition 2 and Corollary 1. In Figs. 1, 2 and 3, we have plotted the MIMO Rayleigh fading channel capacity at low SNR in nats per channel use (npcu) versus SNR in dB. The exact capacity plotted in the latter figures is computed for perfect CSI-TR as in [11]. We have used standard root finding algorithms to find the water filling level μ and then we have numerically computed the capacity using [11, (3)]. The no CSI-T capacity has been obtained using the Monte Carlo simulation. To the best of our knowledge, the exact capacity of the MIMO Rayleigh channel with an estimated CSI-T that is not a deterministic function of the estimated CSI-R is still not known. To validate our asymptotic results, we compare them to the upper bound that was previously derived by Yoo *et al.* in [4]. In fact, the upper bound derived by Yoo *et al.* is plotted using [4, (13)]. Details on how to numerically compute this bound can be found in [4].

In Figs. 1, and 2, we have plotted the MIMO Rayleigh fading channel capacity at low SNR for 2-transmit antennas and 2-receive antennas. The estimation error variance α in Figs. 1, and 2 has been chosen equal to 0.2, and 0.5, respectively. In Figs. 1 and 2, the expressions (6) and (7) are the same since $L = 0$ in this case. Moreover, our asymptotic results in Theorem 1 claim that the expression of the low SNR capacity when $L = 0$ is independent of the spatial diversity (number of transmit antennas and receive antennas). The expression of the low SNR capacity only depends on the estimation error variance, along with the value of SNR. As we increase α, we decrease the capacity compared to the perfect CSI-T case. In Fig. 1, the asymptotic capacity is below the upper bound derived in [4] which implies that our characterization and thus our upper bound is more refined than the one in [4] at low SNR values. This observation holds for different channel estimation quality as shown in Fig. 2. Note also that the asymptotic capacity loss due to CSI-T estimation has increased from 0.2 in Fig. 1 to 0.5 in 2. In Fig. 2, the asymptotic capacity with estimated CSI-T is about half the ergodic capacity with perfect CSI-TR for all SNR values below -10 dB. As SNR increases above -10 dB, the gap to the later curve increases slightly.

[2]In [23], the Rician factor represents the number of paths L times the per branch K factor so that the results here and in [23] are consistent.

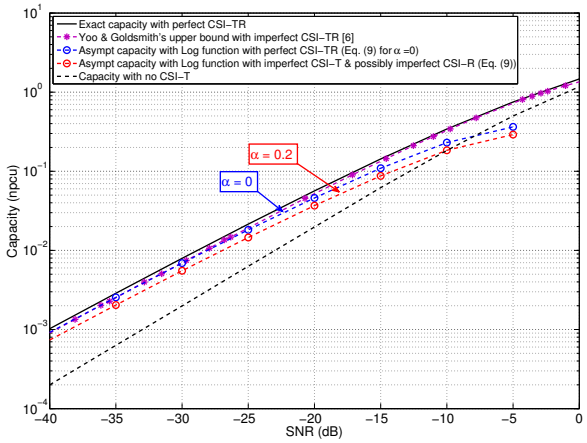

Figure 1. Capacity in nats per channel use (npcu) of a Rayleigh fading channel with 2-transmit and 2-receive antennas versus SNR, for $\alpha = 0.2$.

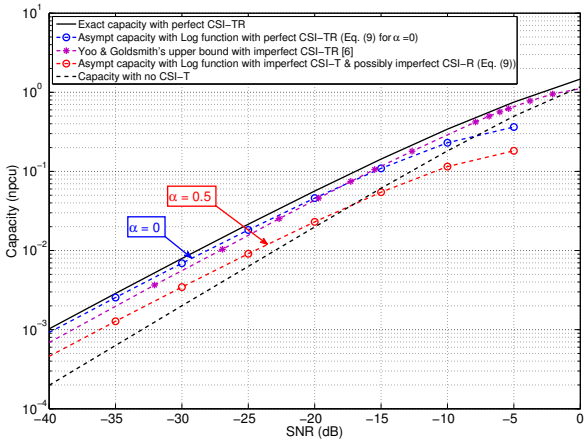

Figure 2. Capacity in nats per channel use (npcu) of a Rayleigh fading channel with 2-transmit and 2-receive antennas versus SNR, for $\alpha = 0.5$.

In Fig. 3, we have plotted the MIMO Rayleigh fading channel capacity at low SNR for 3-transmit antennas and 2-receive antennas. The estimation error variance α has been chosen equal to 0.6. In Fig. 3, we have considered the case when $L > 0$. In this case, we observe that the asymptotic capacity with Lambert function in (6) is lower bounded by the asymptotic capacity with Log function in (7) as depicted in Fig. 3. This claims that the spatial diversity when $L > 0$ improves the capacity loss due to CSI-T channel estimation error. If we compare Fig. 2 and Fig. 3, we can see that even though we have increased α from 0.5 to 0.6, the asymptotic capacity with Lambert function in Fig. 3 is very close to the asymptotic capacity with Log function in Fig. 2.

On the other hand, we have plotted the MIMO Rician fading channel capacity at low SNR versus SNR in dB in Figs. 4, and 5. The exact capacity with perfect CSI-TR defined in (3) was computed using Monte Carlo simulations. There is no framework (to the best of our knowledge) that deals with Rician capacity with imperfect CSI-T and possibly imperfect

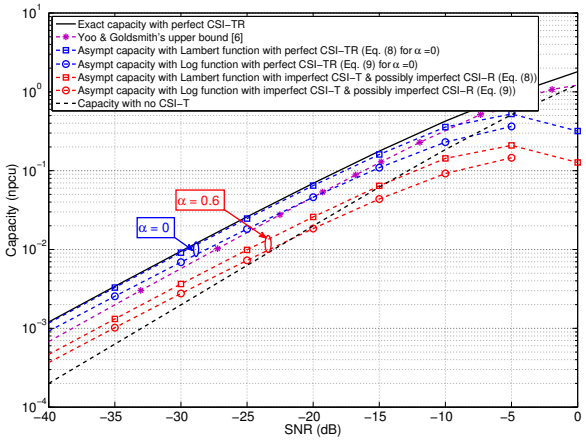

Figure 3. Capacity in nats per channel use (npcu) of a Rayleigh fading channel with 3-transmit and 2-receive antennas versus SNR, for $\alpha = 0.6$.

CSI-R. The asymptotic capacities with perfect CSI-TR at low SNR were computed using the expressions in (11) and (12). The asymptotic capacities with estimated CSI-T and possibly imperfect CSI-R at low SNR were computed using (13) combined with the expressions in (11) and (12).

In Fig. 4, we have plotted the low SNR Rician fading channel capacity of a MISO channel with 2-transmit antennas and 1-receive antennas for $K = 0.5$, which captures the case studied in [23]. In this case, the asymptotic capacity with Lambert function in (11) is upper bounded by the asymptotic capacity with Log function in (12) since we have $L < 0$ (or equivalently $t < 3$) [21, (23)]. We can also see that the asymptotic capacities with estimated CSI-T ($\alpha = 0.5$) is about half the asymptotic capacities with perfect CSI-TR ($\alpha = 0$) for all SNR values.

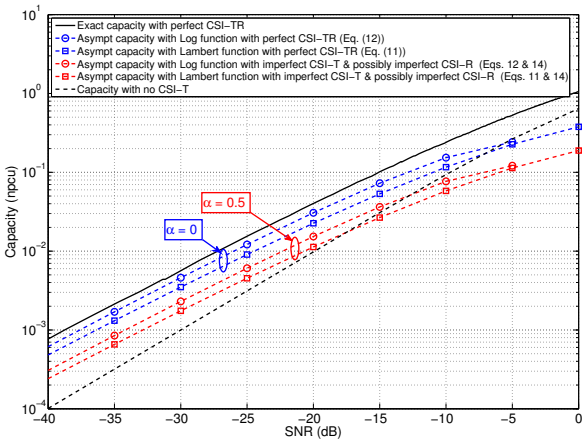

Figure 4. Capacity in nats per channel use (npcu) of a Rician fading channel with 2-transmit and 1-receive antennas versus SNR, for $K = 0.5$.

In Fig. 5, we have plotted the low SNR Rician fading channel capacity with 4-transmit antennas and 2-receive antennas for $K = 0.1$. In this case, the asymptotic capacity with Lambert function in (11) is lower bounded by the asymptotic capacity

with Log function in (12) since we have $L > 0$.

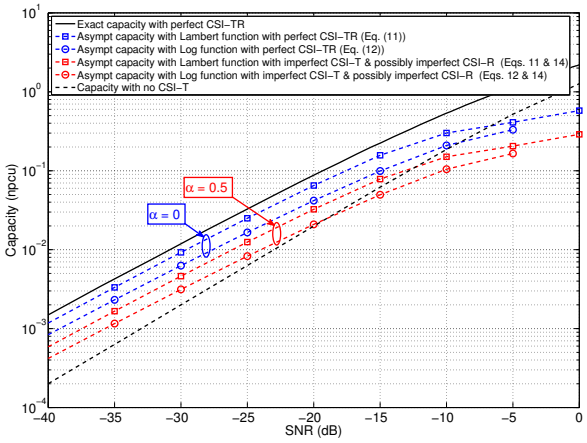

Figure 5. Capacity in nats per channel use (npcu) of a Rician fading channel with 4-transmit and 2-receive antennas versus SNR, for $K = 0.1$.

VI. Conclusion

In this paper, we have analyzed the capacity of MIMO Gaussian channels undergoing Rayleigh fading at low SNR with estimated CSI at the transmitter and possibly imperfect CSI at the receiver. We have shown that the capacity scales asymptotically as $(1-\alpha)\ SNR\ \log(1/SNR)$ regardless of the number of the transmit antennas and the number of receive antennas with α as the estimation error variance. We have deduced that the CSI-T estimation induces an asymptotic capacity loss proportional to the estimation error, α. We have also deduced that the CSI-R estimation is not crucial at low SNR. As a by-product of our analysis, we have also obtained the low SNR capacity of MIMO Rician fading channels with perfect CSI-TR and shown that it scales as $\frac{1}{K+1} SNR \log(1/SNR)$, where K is the Rician factor. We have also deduced that the low SNR capacity of MIMO Rician fading channels with estimated CSI-T and possibly imperfect CSI-R scales as $(1-\alpha)$ times the low SNR capacity with perfect CSI-TR.

Appendix 1

In this Appendix, we will derive an upper bound and a lower bound on the capacity to get the result in Theorem 1.

- Upper bound on $C_{\alpha,0}$
  The channel model described by (1) when taking into account (5) becomes

$$\boldsymbol{y} = (\sqrt{1-\alpha}\hat{\boldsymbol{H}} + \sqrt{\alpha}\tilde{\boldsymbol{H}})\ \boldsymbol{x} + \boldsymbol{n}. \tag{14}$$

  One can think of (14) as a specific combining of two receivers with N_r antennas each. Each receiver observes an output

$$\boldsymbol{y}_1 = \sqrt{g_1}\hat{\boldsymbol{H}}\boldsymbol{x} + \sqrt{\epsilon_1}\boldsymbol{n}_1, \tag{15}$$
$$\boldsymbol{y}_2 = \sqrt{g_2}\tilde{\boldsymbol{H}}\boldsymbol{x} + \sqrt{\epsilon_2}\boldsymbol{n}_2, \tag{16}$$

  where $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ are complex symmetric Gaussian vector with zero mean and covariance matrix I_{N_r}, $g_1 = 1-\alpha$, $g_2 = \alpha$,$\epsilon_1 = 1-\epsilon$, $\epsilon_2 = \epsilon$, and $0 < \epsilon < 1$. Clearly, this specific combining may not be optimal. Consequently, we have

$$I(\boldsymbol{x};\boldsymbol{y}|\boldsymbol{H}) \le I(\boldsymbol{x};\boldsymbol{y}_1,\boldsymbol{y}_2|\hat{\boldsymbol{H}},\tilde{\boldsymbol{H}}) \tag{17}$$
$$\le I(\boldsymbol{x};\boldsymbol{y}_1|\hat{\boldsymbol{H}},\tilde{\boldsymbol{H}}) + I(\boldsymbol{x};\boldsymbol{y}_2|\hat{\boldsymbol{H}},\tilde{\boldsymbol{H}}) \tag{18}$$
$$= I(\boldsymbol{x};\boldsymbol{y}_1|\hat{\boldsymbol{H}}) + I(\boldsymbol{x};\boldsymbol{y}_2|\tilde{\boldsymbol{H}}). \tag{19}$$

  Maximizing both sides of (19) over all conditional inputs $p(x|\hat{\boldsymbol{H}})$, we can see that the capacity of the channel described in (1) is upper bounded by the sum of the capacity of the channel with input x and output $(y_1,\hat{\boldsymbol{H}})$, and the capacity of the channel with input x and $(y_2,\tilde{\boldsymbol{H}})$. The capacity of the channel with input x and output $(y_1,\hat{\boldsymbol{H}})$, which is the first term on the right hand side of the (19), corresponds to the capacity where the transmitter and the receiver perfectly know the channel matrix $\hat{\boldsymbol{H}}$ (perfect CSI-TR), which is asymptotically equal to [11]

$$I(\boldsymbol{x};\boldsymbol{y}_1|\hat{\boldsymbol{H}}) \le C_1 = \max_{p(x|\hat{\boldsymbol{H}})} I(\boldsymbol{x};\boldsymbol{y}_1|\hat{\boldsymbol{H}}) \tag{20}$$
$$\tilde{\le} \begin{cases} -L\ \frac{1-\alpha}{\epsilon_1}\ \mathrm{SNR}\ W_0\left(\left(\frac{1-\alpha}{\epsilon_1}\mathrm{SNR}\right)^{\frac{1}{L}}\right), & \text{if } L < 0, \\ -\frac{1-\alpha}{\epsilon_1}\ \mathrm{SNR}\ \log\left(\frac{1-\alpha}{\epsilon_1}\mathrm{SNR}\right), & \text{if } L = 0, \\ -L\ \frac{1-\alpha}{\epsilon_1}\ \mathrm{SNR}\ W_{-1}\left(-\left(\frac{1-\alpha}{\epsilon_1}\mathrm{SNR}\right)^{\frac{1}{L}}\right), & \text{if } L > 0. \end{cases}$$

  However, the capacity of the channel with input x and $(y_2,\tilde{\boldsymbol{H}})$, which is the second term on the right hand side of the (19), corresponds to the capacity where the transmitter has absolutely no knowledge about $\tilde{\boldsymbol{H}}$ and the receiver knows perfectly $\tilde{\boldsymbol{H}}$ (no CSI-T and perfect CSI-R) [3]. Then, it is asymptotically equal to

$$I(\boldsymbol{x};\boldsymbol{y}_2|\tilde{\boldsymbol{H}}) \le C_2 = \max_{p(x|\tilde{\boldsymbol{H}})} I(\boldsymbol{x};\boldsymbol{y}_2|\tilde{\boldsymbol{H}}) \tilde{\le} N_r \frac{\alpha}{\epsilon_2}\mathrm{SNR}. \tag{22}$$

  At asymptotically low SNR, we can neglect the second term on the right hand side of (19) when compared to the first term in the expression above. Then, using the fact [21]

$$\lim_{x\to\infty} \frac{W_0(\beta x)}{W_0(x)} = 1,\ \lim_{y\to 0^-} \frac{W_{-1}(\beta y)}{W_{-1}(y)} = 1,\ \forall \beta > 0, x > 0, y < 0,$$

  and letting $\epsilon_2 \to 0$, we get

$$C_{\alpha,0} \tilde{<} \begin{cases} -L\ (1-\alpha)\ \mathrm{SNR}\ W_0((\mathrm{SNR})^{\frac{1}{L}}), & \text{if } L < 0, \\ -(1-\alpha)\ \mathrm{SNR}\ \log(\mathrm{SNR}), & \text{if } L = 0, \\ -L\ (1-\alpha)\ \mathrm{SNR}\ W_{-1}(-(\mathrm{SNR})^{\frac{1}{L}}), & \text{if } L > 0, \end{cases} \tag{23}$$
$$\cong -(1-\alpha)\ \mathrm{SNR}\ \log(\mathrm{SNR}). \tag{24}$$

  Note that (24) has been obtained from (23) using the fact that we can approximate the Lambert function by a familiar function (log(.)) since $\lim_{x\to\infty} \frac{W_0(x)}{\log(x)} = 1$ and $\lim_{y\to 0^-} \frac{W_{-1}(y)}{\log(-y)} = 1$, for $x > 0$ and $y < 0$, respectively [21].
- Lower bound on $C_{\alpha,0}$
  Let us consider an on-off power control scheme [11], [21], [22] that allows transmission only when the estimated channel is very good. Since we are operating at low SNR, we are expecting that the spatial diversity will

not improve much the spectral efficiency of our channel. Therefore, we only exploit one transmit antenna at the transmitter while turning off all the other transmit antennas. Then, the equivalent channel is a Single-Input Multiple-Output (SIMO) channel. We define the selected channel as the one that corresponds to the best link with respect to the best transmit antenna selected as

$$\hat{\boldsymbol{h}}_{max}^H \hat{\boldsymbol{h}}_{max} = \max_{1\le j\le N_t} \hat{\boldsymbol{h}}_j^H \hat{\boldsymbol{h}}_j, \tag{25}$$

where $\hat{\boldsymbol{h}}_j$ are the columns of the complex channel matrix $\hat{\boldsymbol{H}}$. As we can see, the selected transmit antenna corresponds to the best instantaneous channel gain. Note that we are doing our selection with respect to the estimate and not the true channel. After selecting the transmit antenna, we perform the optimal Maximum Ratio Combining (MRC) technique [24] at the receiver to maximize the output SNR. Let $g_{max} = \boldsymbol{h}_{max}^H \boldsymbol{h}_{max}$, $\hat{g}_{max} = \hat{\boldsymbol{h}}_{max}^H \hat{\boldsymbol{h}}_{max}$, and $\tilde{g}_{max} = \tilde{\boldsymbol{h}}_{max}^H \tilde{\boldsymbol{h}}_{max}$. Note that $\boldsymbol{h}_{max}$ and $\tilde{\boldsymbol{h}}_{max}$ are the corresponding columns of $\hat{\boldsymbol{h}}_{max}$ and that they don't necessarily satisfy (25). Hence, the on-off power control scheme is given by

$$P(\hat{g}_{max}) = \begin{cases} P_0, & \text{if } \hat{g}_{max} \ge 1/\mu \\ 0, & \text{otherwise.} \end{cases} \tag{26}$$

In [11], it has been shown that μ satisfies:

$$\text{SNR} \cong \frac{1}{(m-1)!(n-1)!} \mu^{-(n+m-4)} e^{-\frac{1}{\mu}}, \tag{27}$$

and is thus given by

$$1/\mu \cong \begin{cases} -L\, W_0\left((\text{SNR})^{\frac{1}{L}}\right), & \text{if } L < 0, \\ -\log(\text{SNR}), & \text{if } L = 0, \\ -L\, W_{-1}\left(-(\text{SNR})^{\frac{1}{L}}\right), & \text{if } L > 0. \end{cases} \tag{28}$$

P_0 in (26) satisfies the average power constraint as

$$P_0 = \frac{\text{SNR}}{1 - F_{\hat{g}_{max}}(1/\mu)}, \tag{29}$$

where $F_{\hat{g}_{max}}(\cdot)$ is the cumulative distribution function (CDF) of $\hat{g}_{max}$. Obviously, $F_{\hat{g}_{max}}(\cdot)$ is the product of the CDFs of $\hat{\boldsymbol{h}}_j^H \hat{\boldsymbol{h}}_j$, for $j = 1, \ldots, N_t$ that are i.i.d and chi-square distributed with $2N_r$ degrees of freedom. In fact, we have

$$\begin{aligned} F_{\hat{g}_{max}}(x) &= \text{Prob}_{\hat{g}_{max}}(\hat{g}_{max} \le x) && (30)\\ &= \prod_{j=1}^{N_t} \text{Prob}_{\hat{\boldsymbol{h}}_j^H \hat{\boldsymbol{h}}_j}\left(\hat{\boldsymbol{h}}_j^H \hat{\boldsymbol{h}}_j \le x\right) && (31)\\ &= \left(1 - \frac{\Gamma\left(N_r, \frac{x}{\Omega}\right)}{(N_r-1)!}\right)^{N_t} && (32)\\ &\underset{x\to\infty}{\cong} \left(1 - \frac{\left(\frac{x}{\Omega}\right)^{N_r-1} e^{-\frac{x}{\Omega}}}{(N_r-1)!}\right)^{N_t}, && (33) \end{aligned}$$

where $\Omega = \mathbb{E}[\hat{g}_{max}]/N_r$. The achievable rate of the above on-off scheme is given by

$$\begin{aligned} R_{\alpha,0} &= \mathop{\mathbb{E}}_{\tilde{g}_{max},\hat{g}_{max}} \left[\log\left(1 + P(\hat{g}_{max})\, g_{max}\right)\right] && (34)\\ &= \mathop{\mathbb{E}}_{\tilde{g}_{max},\hat{g}_{max}\ge 1/\mu} \left[\log\left(1 + P_0\, g_{max}\right)\right] && (35)\\ &= \mathop{\mathbb{E}}_{\tilde{g}_{max},\hat{g}_{max}\ge 1/\mu} \Bigg[\log\Bigg(1 \\ &\quad + P_0 \left\| \sqrt{1-\alpha}\, \hat{\boldsymbol{h}}_{max} + \sqrt{\alpha}\, \tilde{\boldsymbol{h}}_{max} \right\|^2 \Bigg)\Bigg] && (36)\\ &\ge \mathop{\mathbb{E}}_{\tilde{g}_{max},\hat{g}_{max}\ge 1/\mu} \Bigg[\log\Bigg(1 \\ &\quad + P_0 \left| \sqrt{(1-\alpha)\, \hat{g}_{max}} - \sqrt{\alpha\, \tilde{g}_{max}} \right|^2 \Bigg)\Bigg] && (37)\\ &\ge \mathop{\mathbb{E}}_{\tilde{g}_{max}\le a,\hat{g}_{max}\ge 1/\mu} \Bigg[\log\Bigg(1 \\ &\quad + P_0 \left| \sqrt{(1-\alpha)\, \hat{g}_{max}} - \sqrt{\alpha\, \tilde{g}_{max}} \right|^2 \Bigg)\Bigg] && (38)\\ &\ge F_{\tilde{g}_{max}}(a)\left(1 - F_{\hat{g}_{max}}(1/\mu)\right) \log\Bigg(1 \\ &\quad + P_0 \left(\sqrt{(1-\alpha)1/\mu} - \sqrt{\alpha\, a} \right)^2 \Bigg), && (39) \end{aligned}$$

where (**??**) is obtained from (36) using the fact that $\|z_1 + z_2\| \ge |\, \|z_1\| - \|z_2\|\, |$, where z_1 and z_2 are two complex vectors, where $F_{\tilde{g}_{max}}(\cdot)$ is the cumulative function of $\tilde{g}_{max}$, where a in (38) is an arbitrary positive real number. Now, choosing a in (39) equal to $a = \frac{1-\alpha}{\alpha} 1/\mu^{1/3}$, we have $\left|\sqrt{(1-\alpha)1/\mu} - \sqrt{\alpha\, a}\right| \cong \sqrt{(1-\alpha)1/\mu}$ and as $\text{SNR} \to 0$, $a \to \infty$ and $F_{\tilde{\boldsymbol{g}}_{max}}(a) \cong 1$. Hence,

$$R_{\alpha,0} \tilde{>} \left(1 - F_{\hat{\boldsymbol{g}}_{max}}(1/\mu)\right) \log(1 + P_0(1-\alpha)1/\mu), \tag{40}$$

Then, using (29) along with (27), we have:

$$\begin{aligned} P_0 \frac{1}{\mu} &= \frac{\text{SNR}\ \frac{1}{\mu}}{1 - F_{\hat{g}_{max}}(1/\mu)}, && (41)\\ &\cong \frac{\frac{1}{(m-1)!(n-1)!}\mu^{-(n+m-3)} e^{-\frac{1}{\mu}}}{1 - \left(1 - \frac{\left(\frac{1}{\mu\Omega}\right)^{N_r-1} e^{-\frac{1}{\mu\Omega}}}{(N_r-1)!}\right)^{N_t}}, && (42)\\ &\cong \frac{\frac{1}{(m-1)!(n-1)!}\mu^{-(n+m-3)} e^{-\frac{1}{\mu}}}{N_t \frac{\left(\frac{1}{\mu\Omega}\right)^{N_r-1} e^{-\frac{1}{\mu\Omega}}}{(N_r-1)!}}. && (43) \end{aligned}$$

Note that as SNR tends toward zero, μ also converges to zero due to (27) and so does the right hand side of (43). Finally, using the fact that $\log(1+x) \cong x$ for $x \to 0$, the achievable rate $R_{\alpha,0}$ in (40) is asymptotically lower-bounded as follows:

$$\begin{aligned} R_{\alpha,0} &\tilde{>} \left(1 - F_{\hat{\boldsymbol{g}}_{max}}(1/\mu)\right)\ P_0\ (1-\alpha)\ 1/\mu, && (44)\\ &= (1-\alpha)\ 1/\mu\ \text{SNR}, && (45) \end{aligned}$$

where (45) follows from (41). By substituting (28) in (45), we obtain the result in Theorem 1.

Appendix 2

In order to get the result in Proposition 1, we follow the same sketch of proof as in Theorem 1. We derive an upper and lower bound on $C_{\alpha,\alpha}$ as follows.

- Upper bound on $C_{\alpha,\alpha}$
  In order to derive the upper bound, we decompose the channel in a similar way as in (15) in the proof of Theorem 1. Maximizing both sides of (19) over all conditional inputs $p(x|\hat{\boldsymbol{H}})$, we can see that the first term on the right hand side of the (19) corresponds to the capacity where the transmitter perfectly knows the channel matrix $\hat{\boldsymbol{H}}$ (perfect CSI-TR) and is given in a similar way as in (20). However, the second term on the right hand side corresponds to the capacity where the transmitter and the receiver have absolutely no knowledge about $\tilde{\boldsymbol{H}}$ (no CSI-TR). Then, it is upper bounded by the capacity C_2 where only the transmitter has no knowledge about $\tilde{\boldsymbol{H}}$ (no CSI-T and perfect CSI-R) as given in (22). Hence, the same upper bound given in the proof of Theorem 1 still holds true.
- Lower bound on $C_{\alpha,\alpha}$
  Now let us derive the lower bound on the capacity. We again consider an on-off power control scheme as in (26). The achievable rate of the on-off scheme when only $\hat{g}_{max}$ is known at both the transmitter and the receiver is given by

$$R_{\alpha,\alpha} = \mathop{\mathbb{E}}_{\hat{g}_{max}\geq 1/\mu}\left[\log\left(1+P_0\ \frac{\hat{g}_{max}(1-\alpha)}{1+\alpha\mathrm{SNR}}\right)\right] \tag{46}$$

$$\geq \left(1-F_{\hat{g}_{max}}(1/\mu)\right) \times \log\left(1+P_0\ \frac{1/\mu(1-\alpha)}{1+\alpha\mathrm{SNR}}\right). \tag{47}$$

  Note that (46) is a direct consequence of the lower bound in [4, (6)] with an on-off power policy. Then, using (43) along with (27), we have

$$\frac{P_0\ \frac{1}{\mu}(1-\alpha)}{1+\alpha\mathrm{SNR}} \cong \frac{\frac{1}{(m-1)!(n-1)!}\mu^{-(n+m-3)}e^{-\frac{1}{\mu}}(1-\alpha)}{N_t\frac{\left(\frac{1}{\mu\Omega}\right)^{N_r-1}e^{-\frac{1}{\mu\Omega}}}{(N_r-1)!}+\alpha\frac{1}{(m-1)!(n-1)!}\mu^{-(n+m-4)}e^{-\frac{1}{\mu}}} \underset{\mu\to 0}{\to} 0 \tag{48}$$

  Then, we can write (47) as

$$R_{\alpha,\alpha} \tilde{>} \left(1-F_{\hat{g}_{max}}(1/\mu)\right)\ \frac{P_0\ 1/\mu(1-\alpha)}{1+\alpha\mathrm{SNR}}, \tag{49}$$

$$= \frac{\mathrm{SNR}\ 1/\mu(1-\alpha)}{1+\alpha\mathrm{SNR}}, \tag{50}$$

$$\cong (1-\alpha)\ 1/\mu\ \mathrm{SNR}, \tag{51}$$

  and hence, by substituting (28) in (51), we obtain the result in Proposition 1.

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