Asymmetric–valued Spectrum Auction and Competition in Wireless Broadband Services

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***Abstract*—We study bidding and pricing competition between two spiteful mobile network operators (MNOs) with considering their existing spectrum holdings. Given asymmetric-valued spectrum blocks are auctioned off to them via a first-price sealed-bid auction, we investigate the interactions between two spiteful MNOs and users as a three-stage dynamic game and characterize the dynamic game's equilibria. We show an asymmetric pricing structure and different market share between two spiteful MNOs. Perhaps counter-intuitively, our results show that the MNO who acquires the less-valued spectrum block always lowers his service price despite providing double-speed LTE service to users. We also show that the MNO who acquires the high-valued spectrum block, despite charing a higher price, still achieves more market share than the other MNO. We further show that the competition between two MNOs leads to some loss of their revenues. By investigating a cross-over point at which the MNOs' profits are switched, it serves as the benchmark of practical auction designs.**

I. Introduction

Due to the exploding popularity of all things wireless, the demand for wireless data traffic increases dramatically. According to a Cisco report, global mobile data traffic will increase 13-fold between 2012 and 2017 [1]. This dramatic demand puts on pressure on mobile network operators (MNOs) to purchase more spectrum. However, wireless spectrum is a scarce resource for mobile services. Even if the continued innovations in technological progress relax this constraint as it provides more capacity and higher quality of service (QoS), the shortage of spectrum is still the bottleneck when the mobile telecommunications industry is moving toward wireless broadband services [2].

To achieve a dominant position for future wireless services, thus, it is significant how new spectrum is allocated to MNOs. Since the spectrum is statically and infrequently allocated to an MNO, there has been an ongoing fight over access to the spectrum. In South Korea, for example, the Korea Communications Commission (KCC) planed to auction off additional spectrum in both 1.8 GHz and 2.6 GHz bands. The main issue was whether Korea Telecom (KT) acquires the contiguous spectrum block or not. Due to the KT's existing holding downlink 10 MHz in the 1.8 GHz band, it could immediately double the existing Long Term Evolution (LTE) network capacity in the 1.8 GHz band at little or no cost. This is due to the support of the downlink up to 20 MHz contiguous bandwidth by LTE Release 8/9. To the user side, there is no need for upgrading their handsets. LTE Release 10 (LTE-A) can support up to 100 MHz bandwidth but this requires the carrier aggregation (CA) technique, for which both infrastructure and handsets should be upgraded. If KT leases the spectrum block in the 1.8 GHz band, KT might achieve a dominant position in the market. On the other hand, other MNOs expect to make heavy investments as well as some deployment time to double their existing LTE network capacities compared to KT [3]. Thus, the other MNOs requested the government to exclude KT from bidding on the contiguous spectrum block to ensure market competitiveness. Although we consider the example of South Korea, this interesting but challenging issue on spectrum allocation is not limited to South Korea but to most countries when asymmetric-valued spectrum blocks are auctioned off to MNOs.

Spectrum auctions are widely used by governments to allocate spectrum for wireless communications. Most of the existing auction literatures assume that each bidder (i.e., an MNO) only cares about his own profit: what spectrum block he gets and how much he has to pay [4]. Given spectrum constraints, however, there is some evidence that a bidder considers not only to maximize his own profit in the event that he wins the auction but to minimize the weighted difference of his competitor's profit and his own profit in the event that he loses the auction [5]. This strategic concern can be interpreted as a *spite motive*, which is the preference to make competitors worse off. Since it might increase the MNO's relative position in the market, such concern has been observed in spectrum auctions [6].

In this paper, we study bidding and pricing competition between two competing/spiteful MNOs with considering their existing spectrum holdings. Given that asymmetric-valued spectrum blocks are auctioned off to them, we develop an analytical framework to investigate the interactions between two MNOs and users as a three-stage dynamic game. In Stage I, two spiteful MNOs compete in a first-price sealed-bid auction. Departing from the standard auction framework, we address the bidding behavior of the spiteful MNO. In Stage II, two competing MNOs optimally set their service prices to maximize their revenues with the newly allocated spectrum. In Stage III, users decide whether to stay in their current MNO

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or to switch to the other MNO for utility maximization.

Our results are summarized as follows:

- *Asymmetric pricing structure*: We show that two MNOs announce different equilibrium prices to the users, even providing the same quality in services to the users.
- *Different market share*: We show that the market share leader, despite charging a higher price, still achieve more market share.
- *Impact of competition*: We show that the competition between two MNOs leads to some loss of their revenues.
- *Cross-over point between two MNO's profits*: We show that two MNOs' profits are switched.

The rest of the paper is organized as follows: Related works are discussed in Section II. The system model and three-stage dynamic game are described in Section III. Using backward induction, we analyze user responses and pricing competition in Sections VI and V, and bidding competition in Section VI. We conclude in Section VII together with some future research directions.

II. Related Work

In wireless communications, the competition among MNOs have been addressed by many researchers [7]–[12]. Yu and Kim [7] studied price dynamics among MNOs. They also suggested a simple regulation that guarantees a Pareto optimal equilibrium point to avoid instability and inefficiency. Niyato and Hossain [8] proposed a pricing model among MNOs providing different services to users. However, these works did not consider the spectrum allocation issue. More closely related to our paper are some recent works [9]–[12]. The paper [9] studied bandwidth and price competition (i.e., Bertrand competition) among MNOs. By taking into account MNOs' heterogeneity in leasing costs and users' heterogeneity in transmission power and channel conditions, Duan *et al*. presented a comprehensive analytical study of MNOs' spectrum leasing and pricing strategies in [10]. In [11], a new allocation scheme is suggested by jointly considering MNOs' revenues and social welfare. X. Feng *et al.* [12] suggested a truthful double auction scheme for heterogeneous spectrum allocation. None of the prior results considered MNOs' existing spectrum holdings even if the value of spectrum could be varied depending on MNOs' existing spectrum holdings.

III. System Model and Game Formulation

We consider two MNOs ($i, j \in \{1, 2\}$ and $i \neq j$) compete in a first-price sealed-bid auction[1], where two spectrum blocks A and B are auctioned off to them as shown in Fig. 1. Note that A and B are the same amount of spectrum (i.e., 10 MHz spectrum block). Without loss of generality, we consider only the downlink throughput the paper. Note that both MNOs operate Frequency Division Duplex LTE (FDD LTE) in the same area.

[1]It is a form of auction where two MNOs submit one bid in a concealed fashion. The MNO with the highest bid wins and pays his bid for the spectrum block.

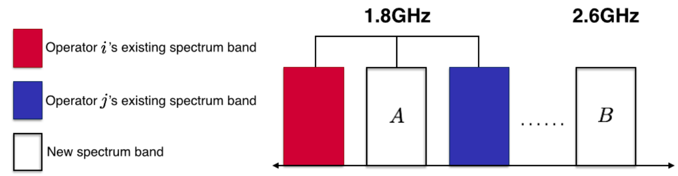

Fig. 1. System model for spectrum auction. Without loss of generality, we consider only the downlink throughput the paper.

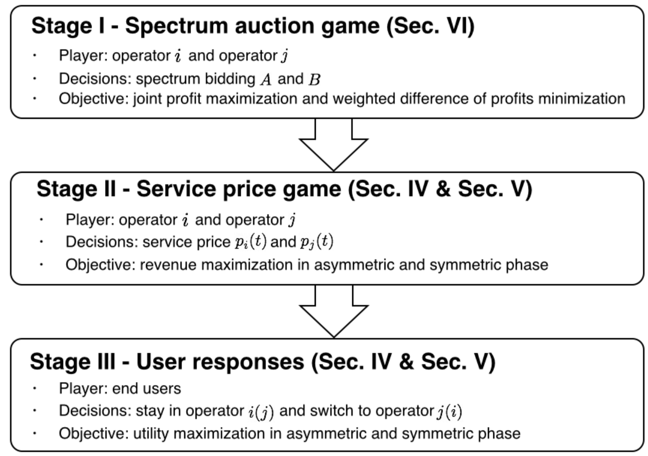

Fig. 2. Three stages of the dynamic game.

Due to the MNOs' existing spectrum holdings (i.e., each MNO secures 10 MHz downlink spectrum in the 1.8 GHz band), the MNOs put values on spectrum blocks A and B asymmetrically. If MNO i leases A, twice (2x) improvements in capacity over his existing LTE network capacity are directly supported to users. In Third Generation Partnership Project (3GPP) LTE Release 8/9, LTE carriers can support a maximum bandwidth of 20 MHz for both in uplink and downlink, thereby allowing for MNO i to provide double-speed LTE service to users without making many changes to the physical layer structure of LTE systems [13]. On the other hand, MNO j who leases B should make a huge investment to double the capacity after some deployment time T_1. Without loss of generality, we assume that MNO i leases A.

To illustrate user responses, we define the following terms as follows.

Definition 1. (Asymmetric phase) *Assume that MNO j launches double-speed LTE service at time T_1. When $0 \leq t \leq T_1$, we call this period asymmetric phase due to the different services provided by MNOs i and j.*

Definition 2. (Symmetric phase) *Assume that T_2 denotes the expiration time for the MNOs' new spectrum rights. When $T_1 < t \leq T_2$, we call this period symmetric phase because of the same services offered by MNOs i and j.*

We investigate the interactions between two MNOs and users as a three-stage dynamic game as shown in Fig. 2. In Stage I, two spiteful MNOs compete in a first-price sealed-bid auction where asymmetric-valued spectrum blocks A and B are auctioned off to them. The objective of each MNO is maximizing his own profit when A is assigned to him, as well as minimizing the weighted difference of his competitor's profit and his own profit when B is allocated to him. In Stage II, two competing MNOs optimally announce their service prices to maximize their revenues given the result of Stage I. The analysis is divided into two phases: asymmetric phase and symmetric phase. In Stage III, users determine whether to stay in their current MNO or to switch to the new MNO for utility maximization. To predict the effect of spectrum allocation, we solve this three-stage dynamic game by applying the concept of backward induction, from Stage III to Stage I.

IV. User Responses and Pricing Competition in Asymmetric Phase

A. User Responses in Stage III in Asymmetric Phase

Each user subscribes to one of the MNOs based on his or her MNO preference. Let us assume that MNOs i and j provide same quality in services to the users so they have the same reserve utility u_o before spectrum auction. Each MNO initially has 50% market share and the total user population is normalized to 1.

In asymmetric phase, the users in MNOs i and j obtain different utilities, i.e.,

$$u_i(t) = (1+\eta)u_o,\ u_j(t) = u_o,\ 0 \le t \le T_1. \tag{1}$$

where $\eta \in (0,1)$ is a user sensitivity parameter to the double-speed LTE service than existing one. It means that users care more about the data rate as η increases. The users in MNO j have more incentive to switch to MNO i as η increases. When they decide to change MNO i, however, they face switching costs, the disutility that a user experiences from switching MNOs. In the case of higher switching costs, the users in MNO j have less incentive to switch. The switching cost varies among users and discounts over time. To model such users' time-dependent heterogeneity, we assume that the switching cost is heterogeneous across users and uniformly distributed in the interval $[0, e^{-\lambda t}]$ at $t \ge 0$, where λ denotes the discount rate [15]. This is due to the fact that the pays for the penalty of terminating contract with operators decrease as time passes.

Now let us focus on how users churn in asymmetric phase. A user k in MNO j, with switching cost, $s_k(t)$, observes the prices charged by MNOs i and j ($p_i(t)$ and $p_j(t)$). A user k in MNO j will switch to MNO i if and only if

$$u_j(t) - p_j(t) \le u_i(t) - p_i(t) - s_k(t),\ 0 \le t \le T_1. \tag{2}$$

Thus the mass of switching users from MNO j to i is

$$Q_{j\to i}(t)=\frac{1}{2}\int_0^{\eta u_o+p_j(t)-p_i(t)} e^{\lambda t} ds=\frac{e^{\lambda t}\left(\eta u_o+p_j(t)-p_i(t)\right)}{2}, \tag{3}$$

where s is a uniform $(0,1)$ random variable and $\frac{1}{2}$ denotes the initial market share.

Since the market size is normalized to one, each MNO's market share in asymmetric phase is as follows:

$$Q_i(t)=\frac{1}{2}+Q_{j\to i}(t),\ Q_j(t)=\frac{1}{2}-Q_{j\to i}(t). \tag{4}$$

B. Pricing Competition in Stage II in Asymmetric Phase

Given users' responses (4), MNOs i and j set their service prices $p_i^*(t)$ and $p_j^*(t)$ to maximize their revenues, respectively, i.e.,

$$p_i^*(t)=\arg\max_{p_i(t)}\ p_i(t)Q_i(t),\ i,j\in\{1,2\} \text{ and } i\neq j. \tag{5}$$

The Nash equilibrium in this pricing game is described in the following proposition.

Proposition 1. *When $0\le t\le T_1$ and $\eta u_o<e^{-\lambda t}$, there exists a unique Nash equilibrium, i.e.,*

$$p_i^*(t)=e^{-\lambda t}+\frac{1}{3}\eta u_o,\ p_j^*(t)=e^{-\lambda t}-\frac{1}{3}\eta u_o. \tag{6}$$

Proof. In asymmetric phase, two competing MNOs try to maximize their revenues $r_i(t)$ and $r_j(t)$, respectively, given users' responses, i.e.,

$$\max_{p_i(t)} r_i(t)=p_i(t)Q_i(t),\ i,j\in\{1,2\} \text{ and } i\neq j.$$

A Nash equilibrium exists by satisfying and solving the following first order conditions with respect to $p_i(t)$ and $p_j(t)$, i.e.,

$$\frac{\partial r_i(t)}{\partial p_i(t)}=\frac{1+\left(\eta u_o+p_j(t)-2p_i(t)\right)e^{\lambda t}}{2}=0,$$

$$\frac{\partial r_j(t)}{\partial p_j(t)}=\frac{1-\left(\eta u_o-p_i(t)+2p_j(t)\right)e^{\lambda t}}{2}=0.$$

■

Proposition 1 shows two MNOs' equilibrium prices in asymmetric phase. Intuitively, $p_i^*(t)$ increases as η increases. With larger η, users care more about the data rate. Thus, MNO i increases his service price to obtain more revenue. On the other hand, $p_j^*(t)$ decreases as η increases. It means that MNO j tries to sustain the revenue margin by lowering the service price and holding onto market share. An interesting observation is that both MNOs decrease their service prices as t increases. Due to the discount factor (λ), the users in MNO j are not locked-in and tries to maximize their utilities by churning to MNO i as switching costs decrease over time. Therefore, MNO i lowers his service price to maximize his revenue, which forces MNO j to decrease the service price. This phenomenon is consistent with the previous results [7], [15] in that the reduction of switching costs intensifies the price-down competition between two MNOs. If $\eta u_o>e^{-\lambda t}$, then all users in MNO j churn to MNO i. However, it is an unrealistic feature of the mobile telecommunication industry so we add the constraint $\eta u_o<e^{-\lambda t}$.

Next we will show how each MNO's market share changes in asymmetric phase. Inserting the equilibrium prices (6) into (4), each MNO's market share can be calculated as follows:

$$Q_i(t)=\frac{3+\eta u_o e^{\lambda t}}{6},\ Q_j(t)=\frac{3-\eta u_o e^{\lambda t}}{6},\ 0\le t\le T_1. \quad (7)$$

Intuitively, MNO i takes MNO j's market share more as t increases or η increases. To hold onto or take MNO i's market share, the time to launch double-speed LTE service T_1 is of great importance to MNO j.

When MNO j launches double-speed LTE service at time T_1, each MNO's total revenue in asymmetric phase is given by

$$r_i(T_1)=\int_0^{T_1} r_i(t)dt=\frac{1-e^{-\lambda T_1}}{2\lambda}-\frac{1-e^{\lambda T_1}}{18\lambda}(\eta u_0)^2+\frac{1}{3}\eta u_0 T_1,$$

$$r_j(T_1)=\int_0^{T_1} r_j(t)dt=\frac{1-e^{-\lambda T_1}}{2\lambda}-\frac{1-e^{\lambda T_1}}{18\lambda}(\eta u_0)^2-\frac{1}{3}\eta u_0 T_1. \quad (8)$$

Similar to the analysis of market share, Equation (8) shows that MNO j should launch double-speed LTE service as quickly as possible to narrow the revenue gap between MNO i and MNO j (see the last term of the revenues (8)).

V. User Responses and Pricing Competition in Symmetric Phase

A. User Responses in Stage III in Symmetric Phase

Since MNO j launches double-speed LTE service in symmetric phase, we assume that the users in MNOs i and j obtain same utility, i.e.,

$$u_i(t)=u_j(t)=(1+\eta)u_o, T_1<t\le T_2. \quad (9)$$

For better understanding of user responses in symmetric phase, we first discuss the effect of switching costs on market competition. Given the same services offered by two MNOs, an MNO's current market share plays an important role in determining its price strategy. Each MNO faces a trade-off between a low price to increase market share, and a high price to harvest profits by exploiting users' switching costs. The following Lemma examines this trade-off and characterizes each MNO's price strategy, which is directly related to user responses in symmetric phase.

Lemma 1. *In a competitive market with switching costs, the market share leader (i.e., MNO i) charges a high price to exploit its current locked-in users while the marker share followers (i.e., MNO j) charge low prices to increase market share for revenue maximization, respectively, given the same services offered by them.*

Proof. We prove Lemma 1 by contradiction. Suppose that MNO j charges a higher price than MNO i (i.e., $p_i(t)<p_j(t), T_1<t\le T_2$). The mass of switching users from MNO j to i is

$$Q_{j\to i}(t)=Q_j(T_1)\int_0^{p_j(t)-p_i(t)} e^{\lambda t}ds=(p_j(t)-p_i(t))Q_j(T_1)e^{\lambda t}, \quad (10)$$

where $Q_j(T_1)=\frac{3-\eta u_o e^{\lambda T_1}}{6}$ is the market share of MNO j at the end of asymmetric phase. Then, each MNO's market share is given by

$$Q_i(t)=Q_i(T_1)-(p_j(t)-p_i(t))Q_j(T_1)e^{\lambda t},$$

$$Q_j(t)=Q_j(T_1)\big(1-(p_j(t)-p_i(t))e^{\lambda t}\big), \quad (11)$$

where $Q_i(T_1)=\frac{3+\eta u_o e^{\lambda T_1}}{6}$ is the market share of MNO i at the end of asymmetric phase. Following the same steps of the Proposition 1, we can find the Nash equilibrium by satisfying and solving the following first order conditions with respect to $p_i(t)$ and $p_j(t)$, i.e.,

$$\frac{\partial r_i(t)}{\partial p_i(t)}=Q_i(T_1)+(p_j(t)-2p_i(t))Q_j(T_1)e^{\lambda t}=0,$$

$$\frac{\partial r_j(t)}{\partial p_j(t)}=Q_j(T_1)\big[1-(2p_j(t)-p_i(t))e^{\lambda t}\big]=0,$$

which yields the solution given as follows

$$p_i^*(t)=\left(\frac{9+\eta u_0 e^{\lambda T_1}}{9-3\eta u_0 e^{\lambda T_1}}\right)e^{-\lambda t}, p_j^*(t)=\left(\frac{9-\eta u_0 e^{\lambda T_1}}{9-3\eta u_0 e^{\lambda T_1}}\right)e^{-\lambda t}. \quad (12)$$

Thus, this contradicts to our assumption, completing the proof. ■

With Lemma 1, let us illustrate the process of user churn in symmetric phase. The mass of switching users from MNO i to j is

$$Q_{i\to j}(t)=Q_1(T_1)\int_0^{p_i(t)-p_j(t)} e^{\lambda t}ds=(p_i(t)-p_j(t))Q_i(T_1)e^{\lambda t}, \quad (13)$$

where $Q_i(T_1)=\frac{3+\eta u_o e^{\lambda T_1}}{6}$ is the market share of MNO i at the end of asymmetric phase. Then each MNO's market share in symmetric phase is given by

$$Q_i(t)=Q_i(T_1)\big(1-(p_i(t)-p_j(t))e^{\lambda t}\big),$$

$$Q_j(t)=Q_j(T_1)+Q_i(T_1)(p_i(t)-p_j(t))e^{\lambda t}, \quad (14)$$

where $Q_j(T_1)=\frac{3-\eta u_o e^{\lambda T_1}}{6}$ is the market share of MNO j at the end of asymmetric phase.

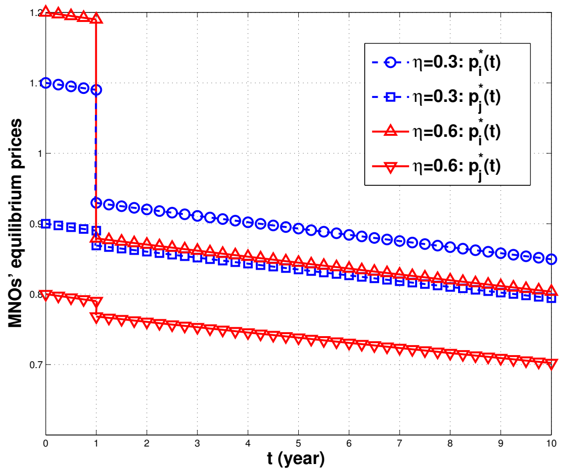

Fig. 3. MNOs' equilibrium prices in asymmetric and symmetric phase under two different user sensitivities (η=0.3, η=0.6). Other parameters are u_o=1, λ=0.01, T_1=1, and T_2=10.

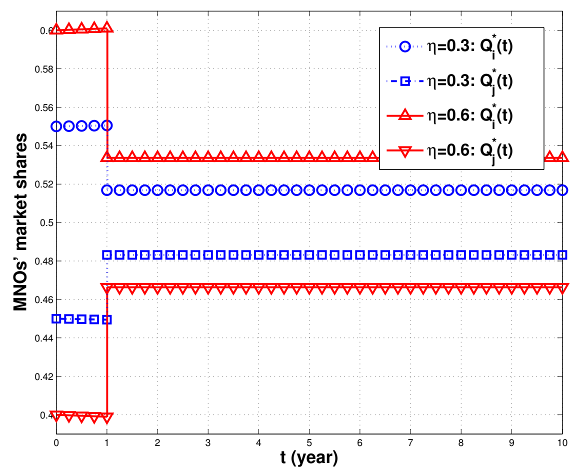

Fig. 4. User responses in asymmetric and symmetric phase under two different user sensitivities (η=0.3, η=0.6). Other parameters are u_o=1, λ=0.01, T_1= 1 and T_2=10.

B. Pricing Competition in Symmetric Phase in Stage II

As noted in Lemma 1, MNO j charges a lower price than MNO i in symmetric phase. Following the same procedure (5), the Nash equilibrium is described in the following proposition.

Proposition 2. *When $T_1<t\leq T_2$, there exists a unique Nash equilibrium, i.e.,*

$$p_i^*(t)=\left(\frac{9+\eta u_0 e^{\lambda T_1}}{9+3\eta u_0 e^{\lambda T_1}}\right)e^{-\lambda t}, p_j^*(t)=\left(\frac{9-\eta u_0 e^{\lambda T_1}}{9+3\eta u_0 e^{\lambda T_1}}\right)e^{-\lambda t}. \tag{15}$$

Proof. Following the same steps of the Proposition 1, a Nash equilibrium exists by satisfying and solving the following first order conditions with respect to $p_i(t)$ and $p_j(t)$, i.e.,

$$\frac{\partial r_i(t)}{\partial p_i(t)}=Q_i(T_1)\left[1-(2p_i(t)-p_j(t))e^{\lambda t}\right]=0,$$

$$\frac{\partial r_j(t)}{\partial p_j(t)}=Q_j(T_1)-(2p_j(t)-p_i(t))Q_i(T_1)e^{\lambda t}=0.$$

■

Proposition 2 states the MNOs' equilibrium prices in symmetric phase. As described in Lemma 1, MNO i, the market share leader announces a higher service price up to $\frac{2\eta u_o e^{\lambda T_1}}{9+3\eta u_o e^{\lambda T_1}}e^{-\lambda t}$ than MNO j.

To further investigate the effect of competition under the same quality in services, let us calculate each MNO's falling price level in the neighborhood of the point T_1. From (6) and (15), each MNO's falling price level (i.e., $\varphi_i(T_1)$ and $\varphi_j(T_1)$) is

$$\varphi_i(T_1)=\lim_{\varepsilon\to 0}(p_i^*(T_1-\varepsilon)-p_i^*(T_1+\varepsilon))=\frac{\eta u_0\left(5+\eta u_0 e^{\lambda T_1}\right)}{9+3\eta u_0 e^{\lambda T_1}},$$

$$\varphi_j(T_1)=\lim_{\varepsilon\to 0}\left(p_j^*(T_1-\varepsilon)-p_j^*(T_1+\varepsilon)\right)=\frac{\eta u_0\left(1-\eta u_0 e^{\lambda T_1}\right)}{9+3\eta u_0 e^{\lambda T_1}}. \tag{16}$$

Because $0<\eta u_o<e^{-\lambda T_1}$, MNOs i and j always decrease their prices up to $\varphi_i(T_1)$ and $\varphi_j(T_1)$ at the starting point of the symmetric phase, respectively. Perhaps counter-intuitively, it shows that MNO j always lowers his price despite launching double-speed LTE service at the starting point of the symmetric phase. It can be interpreted as follows. Since MNO j loses his market share in asymmetric phase, MNO j attempts to maximize his revenue by lowering his service price and increasing his market share, which forces MNO i to drop the service price at the same time. This means that the MNOs' competition under the same quality in services lead to some loss of their revenues, which, known as a *price war*, is consistent with our previous work [7]. Fig. 3 shows $p_i^*(t)$ and $p_j^*(t)$ as a function of t under two different user sensitivities (η=0.3, η=0.6). Note that MNO i's falling price level is more sensitive to η.

Next we show that how each MNO's market share varies in symmetric phase. From (14) and (15), each MNO's market share is

$$Q_i(t)=\frac{1}{2}+\frac{\eta u_0 e^{\lambda T_1}}{18}, Q_j(t)=\frac{1}{2}-\frac{\eta u_0 e^{\lambda T_1}}{18}, \ T_1<t\leq T_2. \tag{17}$$

Unlike the asymmetric phase, each MNO's market share only depends on the deployment time of carrier aggregation T_1 in symmetric phase. An interesting observation is that the market share leader (i.e., MNO i), despite charging a higher price, still achieves more market share up to $\frac{1}{9}\eta u_0 e^{\lambda T_1}$ than MNO j. In terms of market share, MNO i always gains a competitive advantage over MNO j if MNO j was forced to lease less-valued spectrum block. This explains how critical new spectrum is allocated to the MNOs, and how struggling they are over access to the spectrum for improving market competitiveness for future wireless services. Fig. 4 shows user responses as a function of t under two different user sensitivities (η=0.3, η=0.6).

If the new spectrum rights expire at $t{=}T_2$, each MNO's total revenue in symmetric phase is

$$r_i(T_1,T_2)=\int_{T_1}^{T_2} r_i(t)dt=\frac{(9+\eta u_0 e^{\lambda T_1})^2}{54(3+\eta u_0 e^{\lambda T_1})}\left(\frac{e^{-\lambda T_1}-e^{-\lambda T_2}}{\lambda}\right),$$

$$r_j(T_1,T_2)=\int_{T_1}^{T_2} r_j(t)dt=\frac{(9-\eta u_0 e^{\lambda T_1})^2}{54(3+\eta u_0 e^{\lambda T_1})}\left(\frac{e^{-\lambda T_1}-e^{-\lambda T_2}}{\lambda}\right). \quad (18)$$

Using (8) and (18), we examine the two MNOs' aggregate revenues when MNO i leases A and MNO j leases B. Each MNO's aggregate revenue at $t{=}T_2$ is given in (19).

When MNO j decides to launch double-speed LTE service, the optimal deployment time of the carrier aggregation T_1^* should be studied. The following Lemma describes the MNO j's optimal deployment time.

Lemma 2. *The market share followers (i.e., MNO j) should launch double-speed LTE service as quickly as possible not only for maximizing their own revenues but also for minimizing the market leader's revenue.*

Proof. By taking the derivative of the two MNO's aggregate revenues $r^A(T_1,T_2)$ and $r^B(T_1,T_2)$ with respect to T_1, respectively, it can be checked that $\frac{\partial r^A(T_1,T_2)}{\partial T_1}>0$ and $\frac{\partial r^B(T_1,T_2)}{\partial T_1}<0$. We omit the details of the derivations here. ■

Lemma 2 states that the revenue of MNO j is strictly decreasing over T_1 while the reverse is for MNO i. To gain more insight into the effect of the allocation of asymmetric-valued spectrum blocks, let us define the revenue gain as follows:

$$r_{gain}=\frac{r^A(T_1,T_2)}{r^B(T_1,T_2)}. \quad (20)$$

Fig. 5 shows the revenue gain as a function of η under two different deployment times ($T_1{=}1$, $T_1{=}2$). As expected, the revenue gain is strictly increasing over T_1 and η. In terms of η, it can be checked directly by following the same steps of the Lemma 2. Such result explains why each MNO should spitefully bid in a first-price sealed-bid auction to achieve a dominant position or compensate the revenue gap, which we will discuss these points in the next section.

VI. Bidding Competition in Stage I

In Stage I, two spiteful MNOs i and j compete in a first-price sealed-bid auction where asymmetric-valued spectrum blocks A and B are auctioned off to them. For fair competition, each MNO is constrained to lease only one spectrum block (i.e., A or B). We assume that the governments set the reserve prices c_A and c_B to A and B, respectively. Note that the reserve price is the minimum price to get the spectrum block. Since A is the high-valued spectrum block, we further assume that two spiteful MNOs are only competing on A to enjoy a dominant position in the market. MNOs i and j bid A independently as b_i and b_j, respectively. In this case,

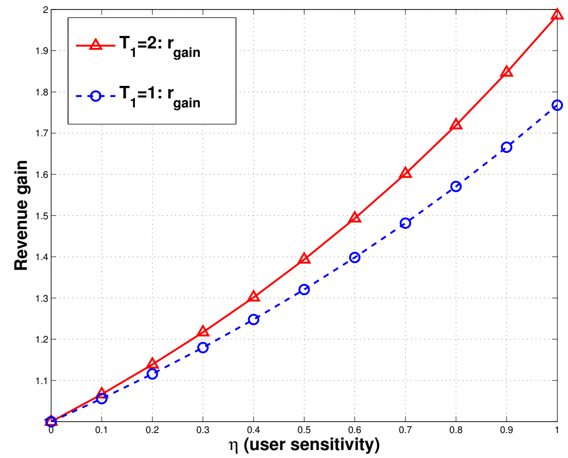

Fig. 5. Revenue gain as a function of η under two different times ($T_1{=}1$, $T_1{=}2$). Other parameters are $u_o{=}1$, $\lambda{=}0.01$ and $T_2{=}10$.

B is assigned to the MNO who loses in the auction as the reserve price c_B. Because the MNO who leases B should make huge investments to double the existing LTE network capacity compared to the other MNO, we also assume the only MNO who leases B incurs the investment cost c_{BS}.

When asymmetric-valued spectrum blocks are allocated to the MNOs, there is a trade-off between self-interest and spite. To illustrate this trade-off, we first restrict ourselves to the case where spite is not present. If MNO i is *self-interested*, his objective function is as follows.

$$\Pi_i(b_i,b_j)=\left[r^A(T_1,T_2)-b_i\right]\cdot I_{b_i\geqslant b_j}+\pi^B(T_1,T_2)\cdot I_{b_i<b_j}, \quad (21)$$

where I is the indicator function and $\pi^B(T_1,T_2)=r^B(T_1,T_2)-c_B-c_{BS}$ is the profit when leasing B. This case is the standard auction framework in that MNO i maximizes his own profit without considering the other MNO's profit.

In the real world, however, there is some evidence that some MNOs are *completely malicious*. The German third generation (3G) spectrum license auction in 2000 is a good example [16]. German Telekom kept raising his bid to prevent his competitors from leasing spectrum. If MNO i is completely malicious, his objective function can be changed as follows.

$$\Pi_i(b_i,b_j)=\left[r^A(T_1,T_2)-b_i\right]\cdot I_{b_i\geqslant b_j}-\left[r^A(T_1,T_2)-b_j\right]\cdot I_{b_i<b_j}. \quad (22)$$

It means that MNO i gets disutility as much as the profit of MNO j when he loses the auction. The minus term in (22) implies this factor.

To reflect this strategic concern, our model departs from the standard auction framework in that each spiteful MNO concerns about maximizing his own profit when he leases A, as well as minimizing the weighted difference of his competitor's profit and his own profit when he leases B. Combining (21) and (22), we define each MNO's objective function as follows.

$$r^A(T_1,T_2)=r_i(T_1)+r_i(T_1,T_2)=\frac{1-e^{-\lambda T_1}}{2\lambda}-\left(\frac{1-e^{\lambda T_1}}{18\lambda}\right)(\eta u_0)^2+\frac{1}{3}\eta u_0 T_1+\frac{\left(9+\eta u_0 e^{\lambda T_1}\right)^2}{54(3+\eta u_0 e^{\lambda T_1})}\left(\frac{e^{-\lambda T_1}-e^{-\lambda T_2}}{\lambda}\right),$$

$$r^B(T_1,T_2)=r_j(T_1)+r_j(T_1,T_2)=\frac{1-e^{-\lambda T_1}}{2\lambda}-\left(\frac{1-e^{\lambda T_1}}{18\lambda}\right)(\eta u_0)^2-\frac{1}{3}\eta u_0 T_1+\frac{\left(9-\eta u_0 e^{\lambda T_1}\right)^2}{54(3+\eta u_0 e^{\lambda T_1})}\left(\frac{e^{-\lambda T_1}-e^{-\lambda T_2}}{\lambda}\right). \quad (19)$$

Definition 3. *Assume that two spiteful MNOs (i.e., $i,j\in\{1,2\}$ and $i\neq j$) compete in a first-price sealed-bid auction. The objective function that each MNO tries to maximize is given by:*

$$\Pi_i(b_i,b_j)= \left[r^A(T_1,T_2)-b_i\right]\cdot I_{b_i\geqslant b_j} +\left[(1-\alpha_i)\pi^B(T_1,T_2)-\alpha_i(r^A(T_1,T_2)-b_j)\right]\cdot I_{b_i<b_j} \quad (23)$$

where I is the indicator function, $\pi^B(T_1,T_2)=r^B(T_1,T_2)-c_B-c_{BS}$ is the MNO's profit when leasing B, and $\alpha_i\in[0,1]$ is a parameter called the spite (or competition) coefficient.

As noted, MNO i is self-interested and only tries to maximize his own profit when α_i=0. When α_i=1, MNO i is completely malicious and only attempts to obtain more market share by forcing MNO j to lease the less-valued spectrum block. For given $\alpha_i\in[0,1]$ and $\alpha_j\in[0,1]$, we can derive the optimal bidding strategies that maximize the objective function in Definition 3 as follows.

Proposition 3. *In a first-price sealed-bid auction, the optimal bidding strategy for a spiteful MNO $i,j\in\{1,2\}$ and $i\neq j$ is:*

$$b_i^*=\frac{(1+\alpha_i)r^A(T_1,T_2)-(1-\alpha_i)\pi^B(T_1,T_2)+c_A}{2+\alpha_i},$$

$$b_j^*=\frac{(1+\alpha_j)r^A(T_1,T_2)-(1-\alpha_j)\pi^B(T_1,T_2)+c_A}{2+\alpha_j}. \quad (24)$$

Proof. Without loss of generality, suppose that MNO i knows his bid b_i. Further, we assume that MNO i infer that the bidding strategy of MNO j on A is drawn uniformly and independently from $\left[c_A,r^A(T_1,T_2)\right]$. The MNO i's optimization problem is to choose b_i to maximize the expectation of

$$E_{b_i}(\Pi_i)=\int_{c_A}^{b_i}\left[r^A(T_1,T_2)-b_i\right]f(b_j)db_j + \int_{b_i}^{r^A(T_1,T_2)}\left[(1-\alpha_i)(\pi^B(T_1,T_2))-\alpha_i(r^A(T_1,T_2)-b_j)\right]f(b_j)db_j. \quad (25)$$

Differentiating Equation (25) with respect to b_i, setting the result to zero and multiplying by $r_A^*-c_A$ give

$$\frac{\partial E_{b_i}(\Pi_i)}{\partial b_i} = (1+\alpha_i)r^A(T_1,T_2)-(1-\alpha_i)\pi^B(T_1,T_2) +c_A-(2+\alpha_i)b_i=0.$$

Applying the same way to MNO j completes the proof. ■

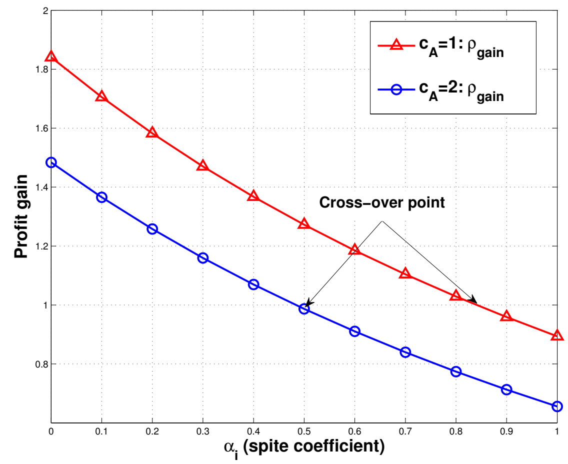

Fig. 6. Profit gain as a function of α_i under two different costs ($c_A=1$, $c_A=2$). Other parameters are $u_o=1$, λ=0.01, η=0.6, $T_1=1$, $T_2=10$, $c_B=1$, and $c_{BS}=1$.

Proposition 3 states that the MNOs' equilibrium bidding strategies. Intuitively, the more spiteful the MNO is, the more aggressively the MNO tends to bid. For consistency, we assume that $\alpha_i>\alpha_j$. Then we can now calculate MNO i's profit and MNO j's profit as follows

$$\pi_i^*=\frac{r^A(T_1,T_2)-(1-\alpha_i)\pi^B(T_1,T_2)+c_A}{2+\alpha_i},\ \pi_j^*=\pi^B(T_1,T_2) \quad (26)$$

where π_i^* is calculated by substraction of the bidding price b_i^* of (24) from $r^A(T_1,T_2)$ of (19).

To get some insight into the properties of the MNOs' equilibrium profits, let us define $\rho_{gain}=\frac{\pi_i^*}{\pi_j^*}$[2], which can be interpreted as the profit gain from A relative B. When $\rho_{gain}>1$, the profit of MNO i is higher than that of MNO j. It implies that MNO i could gain a competitive advantage over MNO j in both market share and profit. When $\rho<1$, the situation is reversed. MNO j could take the lead in the profit despite losing some market share to MNO i.

If the role of the government is to ensure fairness in two MNOs' profits, the government may devise two different schemes: setting appropriate reserve prices and imposing limits on the timing of the double-speed LTE services. According to the Ofcom report, setting the reserve prices closer to market value might be appropriate [17]. It indicates that the government set c_A and c_B by estimating the value asymmetries be-

[2]The gain ρ_{gain} is different from r_{gain} of (19) where $r_{gain}(T_1,T_2)$ is the revenue gain from A relative to B without considering any cost.

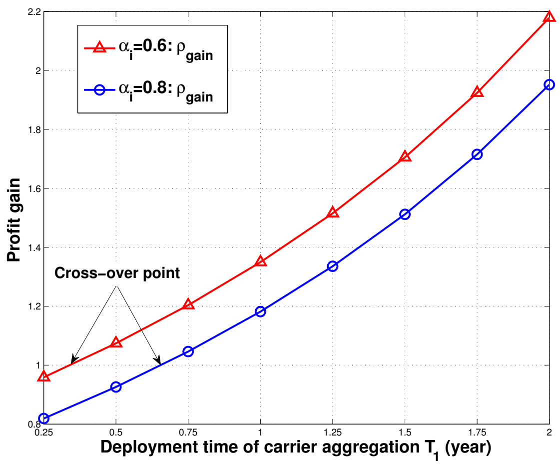

Fig. 7. Profit gain as a function of T_1 under two different spite coefficients ($\alpha_i = 0.6$, $\alpha_i = 0.8$). Other parameters are $u_o = 1$,$\lambda = 0.01$, $\eta = 0.6$, $T_2 = 10$, $c_A = 2$, $c_B = 1$, and $c_{BS} = 1$.

tween spectrum blocks A and B (i.e., $r^A(T_1, T_2) - r^B(T_1, T_2)$) and the spite coefficient α_i. Fig. 6 shows the profit gain as a function of α_i under two different reserve prices for A (i.e., $c_A = 1$, $c_A = 2$). For example, if $\alpha_i = 0.5$, the government should set the reserve prices $c_A = 2$, $c_B = 1$. On the other hand, the government should set the reserve prices $c_A = 1$, $c_B = 1$ when $\alpha_i = 0.85$.

Besides setting appropriate reserve prices, the government can impose limits on the timing of the double-speed LTE service. In South Korea, for instance, Korea Telecom (KT) who acquired the continuous spectrum is allowed to start its double-speed LTE service on metropolitan areas immediately in September 2013, other major cities starting next March, and nation-wide coverage starting next July [18]. This scheme implies to reduces T_1 by limiting the timing of the double-speed LTE service to the MNO who acquires spectrum block A. Fig. 7 shows the profit gain as a function of T_1 under two different spite coefficients (i.e., $\alpha_i = 0.6$, $\alpha_i = 0.8$).

VII. Conclusion

In this paper, we study bidding and pricing competition between two spiteful MNOs with considering their existing spectrum holdings. We develop an analytical framework to investigate the interactions between two MNOs and users as a three-stage dynamic game. Using backward induction, we characterize the dynamic game's equilibria. From this, we show the asymmetric pricing structure and different market share between two MNO. Perhaps counter-intuitively, our results show that the MNO who acquires the less-valued spectrum block always lowers his price despite providing double-speed LTE service to users. We also show that the MNO who acquires the high-valued spectrum block, despite charging a higher price, still achieves more market share than the other MNO. We further show that the competition between two MNOs leads to some loss of their revenues. With the example of South Korea, we investigate the cross-over point at which two MNOs' profits are switched, which serves as the benchmark of practical auction designs.

Results of this paper can be extended in several directions. Extending this work, it would be useful to propose some methodologies for setting reserve prices [19], [20]. Second, we could consider an oligopoly market where multiple MNOs initially have different market share before spectrum allocation, where our current research is heading.

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