UAV's Visit Scheduling for Age-of-Synchronization Minimization with Random Update Sensors

Jie Gong, Yunchao Liu, Yi Chai

Guangdong Key Laboratory of Information Security Technology
School of Computer Science and Engineering, Sun Yat-Sen University, Guangzhou, 510006, China
Email: gongj26@mail.sysu.edu.cn

***Abstract*—This paper considers an unmanned aerial vehicle (UAV) collecting status updates from remote sensors to a control center for environmental status monitoring. A fundamental problem is how to schedule the UAV's visits to the sensors according to their status update intensities. We consider a probabilistic scheduling policy by optimizing UAV's visit probability to each sensor. The objective is to minimize average Age-of-Synchronization (AoS), which is derived as a closed-form function of the sensors' update intensities and the UAV's visit probabilities. Our main analytical result is that when a sensor's update intensity is small, the UAV's visit probability to this sensor is approximately proportional to a cubic root function of the sensor's update intensity and the UAV's energy consumption. Based on this, a cubic root approximation algorithm is proposed, which can perform close to the optimal one. The algorithm is extended to the case that the UAV can visit multiple sensors in each round trip. It is found that visiting as many sensors per round trip as possible can perform the same as the optimal one. Numerical results also demonstrate the good performance of the extended cubic root approximation algorithm to multi-sensor per-visit case.**

***Index Terms*—Unmanned aerial vehicle (UAV), status update, Age-of-Synchronization (AoS), convex optimization**

I. Introduction

In recent years, real time communication for status update is drawing more and more attention in 6G communications [1]. Fresh status information is expected to be timely delivered to a control center for data synchronization and action taking. However, this may not be achieved for remote areas due to lack of connectivity. Unmanned aerial vehicle (UAV), which can provide flexible coverage with low deployment and operational costs, is a candidate solution to support connectivity for remote sensors or emergency communication for disaster-stricken areas without infrastructure [2]. In this paper, we consider UAV-assisted fresh data collection in status update systems.

To measure the data freshness, Age-of-Information (AoI) was proposed in [3] as a candidate performance metric. Recently, many works have explored the field of UAV data collection for AoI optimization. In [4], a multitask transfer deep reinforcement learning method was proposed to jointly optimize the UAV' s flight trajectory, transmission scheduling, and battery recharging. In [5], a safe-TD3 algorithm was proposed to minimize the average AoI while considering the safety issue. In [6], the trajectories and data collection scheduling of multiple UAVs were jointly optimized to minimize the average AoI. In [7], an algorithm based on Traveling Salesman Problem (TSP) was proposed to determine the optimal travel set for UAVs concerning the average AoI. In [8], a new mean field hybrid proximal policy with a long short term memory (LSTM) was proposed to minimize the average AoI. However, there is a drawback on AoI-oriented optimization, i.e., it overlooks the nature of status update itself but focuses only on the timestamps of received packets. In fact, a new update needs to be generated and delivered only when the status actually changes.

To deal with this issue, Age-of-Synchronization (AoS), defined the time elapsed since the source and the receiver are out of synchronization, was proposed in [9]. It only penalizes the time duration when the status of the source changes but the receiver is not informed. Therefore, it is more suitable for random update case compared with AoI. The AoS can also be viewed as a special case of Age-of-Incorrect-Information (AoII) [10] with infinite number of states and binary error function. In the literature, there were extensive studies on AoS such as [11]–[13], but limited pieces of work on UAV-assisted AoS optimization. Among them, ref. [14] jointly optimized the UAV trajectory and sensor node scheduling to minimize the AoS. Our previous work [15] introduced an AoI-assisted UAV trajectory planning to jointly minimize energy consumption and system AoS. Most of the existing works solve the trajectory design and scheduling problem via reinforcement learning algorithms. However, there is still an unsolved fundamental problem: given the sensors' status update intensities, how to schedule the UAV's visits to the sensors to minimize AoS. Although the learning based algorithms usually perform well, the inherent relation between UAV's visit scheduling and the sensors' update intensities is not explicitly illustrated.

This paper studies scheduling UAV's visit to the remote sensors. The problem is related to the multi-user scheduling for status update, which has been extensively studied in the literature. In [16], four low-complexity transmission scheduling policies that attempt to minimize AoI subject to minimum throughput requirements were developed and analyzed. The optimality of round robin scheduling policy

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with one-packet was proved in [17] for random update. For generate-at-will case, a simple threshold policy proposed in [18] was proved asymptotically optimal with many users. The scheduling problem was extended to computation intensive tasks [19]. However, the proposed index-based or threshold-based policies for AoI optimization cannot be directly applied for AoS minimization problem. One of the key challenges is that the AoS of a sensor is unknown to the receiver until it is visited. Therefore, it is necessary to redesign the scheduling policy.

In this paper, we consider random update sensors and focus on the random scheduling policy, i.e., the UAV visits the sensors randomly according to a probability distribution which needs to be optimized. The total visit probability is subject to the UAV's energy consumption and recharging power. We first consider the case that the UAV only visits a single sensor in each time slot. Then, the results are extended to the case that multiple sensors can be visited in each slot. Our main objective is to derive the theoretical relation between sensors' update intensities and the UAV's visit probabilities. Luckily, an approximate closed-form relation is found, which is shown to perform well in numerical results. The main contributions of this work are summarized as follows.

- We derived the closed-form expression of average AoS with random update sensors and random UAV's visit scheduling policy. Based on this, the average AoS minimization problem under the UAV's energy constraint is formulated, which is shown to be a convex optimization problem.
- In the case that the UAV visits a single sensor in each slot, based on duality analysis, we show that the UAV's visit probability is approximately proportional to a cubic root function of the sensor's update intensity and the UAV's energy consumption. Accordingly, a simple visit probability allocation algorithm is proposed, which is shown via simulation to perform close to the optimal.
- In the case that the UAV can visit multiple sensors in each slot, we show that the UAV should visit as many sensors per-slot as possible with sufficient energy. Interestingly, applying this conclusion to insufficient energy case performs the same to the optimal as well. In addition, we propose an algorithm to allocate the UAV's visit probability to each candidate sensor set according to the cubic root approximation, which also attains close-to-optimal performance.

II. System Model

Consider a status update system that consists of N remote sensors, a UAV and a control center as shown in Fig. 1. We focus on a slotted system with slot length T. At the beginning of each slot t, each sensor $n \in \mathcal{N} = \{1, 2, \cdots, N\}$ randomly generates status data with probability p_n, and the UAV decides to either visit a sensor to collect the potentially updated data or stay at the control center to recharge its onboard battery. The objective is to synchronize the data between the sensors and the control center. Ideally, the synchronization is guaranteed

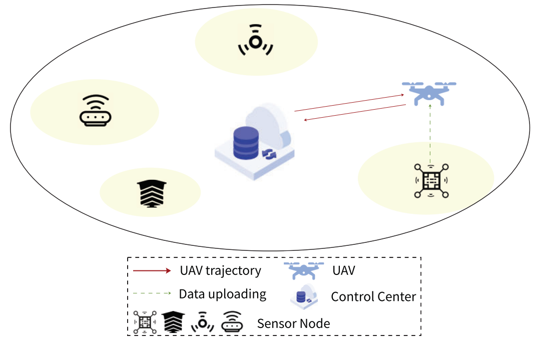


Fig. 1. System model.

if a sensor is visited immediately when it generates a new data. However, since the UAV does not know when the sensor updates, the decision should be made based on the historical observations. First of all, we describe the energy model and the performance metric.

A. Energy Model

We adopt the UAV's power consumption model in [20] as follows

$$P(v) = P_0\left(1 + \frac{3v^2}{U_{\text{tip}}^2}\right) + \frac{P_i v_0}{v} + \frac{1}{2} d_0 \rho s A v^3, \tag{1}$$

where v is the speed of UAV, U_{tip} is tip speed of the rotor blade, v_0 is the mean rotor induced velocity, d_0, ρ, s and A are fuselage drag ratio, air density, rotor solidity and rotor disc area, respectively. δ, Ω, R, W are profile drag coefficient, blade angular velocity, rotor radius and aircraft weight, respectively. Furthermore, we have $P_0 = \frac{\delta}{8}\rho s A \Omega^3 R^3$ and $P_i = 1.1\frac{W^{3/2}}{\sqrt{2\rho A}}$.

Assume the UAV flies between the control center and sensor n with a constant speed v_n, which should guarantee that a round-trip visit is accomplished within a slot. Moreover, if the round-trip time is strictly less than the slot length T, the UAV can recharge in the rest of the slot. Thus, the speed v_n can be optimized by minimizing the total energy reduction in a slot,

$$E_n \triangleq \min_{v_n} \left[P(v_n)\frac{2d_n}{v_n} - P_e \cdot \left(T - \frac{2d_n}{v_n}\right)\right] \tag{2a}$$

$$\text{s.t. } v_{\min} \leq v_n \leq v_{\max}, \tag{2b}$$

$$\frac{2d_n}{v_n} \leq T, \tag{2c}$$

where d_n is the distance between sensor n and the control center, P_e is the recharging power at the control center, $v_{\min}$ and $v_{\max}$ are the UAV's speed limitations. It is remarkable that E_n might be negative if the sensor n is close to the control center or the recharging power is large. In this case, the onboard energy increases by $|E_n|$.

In the above problem, we assume the data transmission time from a sensor to the UAV is ignored. This is reasonable as the transmission can be accomplished during the UAV's flight [21]. We also assume that the problem is feasible, i.e.,

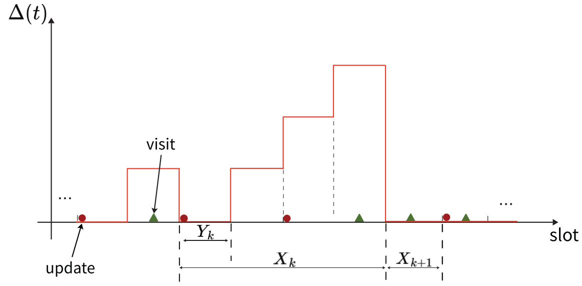


Fig. 2. AoS curve of a single sensor. Red dots refer to status updates at the sensor, and green triangles refer to UAV's visit to this sensor.

sensor n can be visited in one time slot. Thus, all the sensors should be within a given service range, i.e., $d_n \leq d_{\max} = \frac{1}{2} v_{\max} T, \forall n \in \mathcal{N}$.

The problem (2) can be directly solved by a line search algorithm. In addition, if the UAV decides to recharge in slot t, its onboard energy will increase by $E_0 = P_e T$.

B. Data Freshness Metric

We adopt AoS [9] as the data freshness metric, and modify the original definition to fit the slotted system. In particular, denote $U_n(t)$ as the generation time of the latest update of sensor n up to slot t, $V_n(t)$ as the time of the UAV's latest visit to sensor n up to t. The control center and sensor n becomes asynchronous if the UAV does not visit sensor n when it generates a new update. Thus, the AoS of sensor n can be calculated as

$$\Delta_n(t+1) = \begin{cases} 0, & \text{if } U_n(t) \leq V_n(t), \\ \Delta_n(t) + 1, & \text{else.} \end{cases} \tag{3}$$

An example of the AoS curve for one sensor is depicted in Fig. 2. Different from AoI [3] which grows linearly until a new visit, the AoS remains zero after a visit until a new update is generated. Since a sensor's update event is not known by the control center until it is visited, the AoS is only partially available to the control center.

III. Problem Formulation

We focus on the randomized visit policy with given update probabilities, i.e., the UAV selects the sensor n to visit with probability q_n in each slot, where $q_n > 0$ and $\sum_{n=1}^{N} q_n \leq 1$. The UAV will stay at the control center to recharge if its onboard energy is insufficient. On average, the UAV's energy consumption should not exceed its total charging energy. Thus statistically, the visit probabilities $q_n, n \in \mathcal{N}$ should satisfy

$$\sum_{n=1}^{N} q_n E_n \leq \left(1 - \sum_{n=1}^{N} q_n\right) E_0. \tag{4}$$

Based on the random policy, we can calculate the closed-form average AoS for sensor n. Denote X_k as the number of time slots between the k-th and the $(k+1)$-th visits to the sensor n, Y_k as the number of time slots between the k-th visit to the sensor n and the first status update of sensor n since then. It can be found that X_k follows the geometric distribution with parameter q_n, and Y_k also follows the geometric distribution with parameter p_n. According to Fig. 2, the time average AoS of sensor n can be calculated as

$$\begin{aligned} \bar{\Delta}_n &= \lim_{K\to\infty} \frac{\sum_{k=1}^{K} (1 + 2 + \cdots + (X_k - Y_k)^+)}{\sum_{k=1}^{K} X_k} && (5) \\ &= \lim_{K\to\infty} \frac{\frac{1}{K}\sum_{k=1}^{K} \left(((X_k - Y_k)^+)^2 + (X_k - Y_k)^+\right)}{2\frac{1}{K}\sum_{k=1}^{K} X_k} && (6) \\ &= \frac{\mathrm{E}((X_k - Y_k)^+)^2 + \mathrm{E}(X_k - Y_k)^+}{2\mathrm{E}X_k}, && (7) \end{aligned}$$

where E is the expectation operator. The non-negative operator $(\cdot)^+ = \max\{\cdot, 0\}$ indicates that when $Y_k \geq X_k$, the AoS during the k-th and $(k+1)$-th visits is 0 since there is no update in this time period. Due to the memoryless property of geometric distribution, X_k and Y_k are independent. Then, the expectations in the numerator can be calculated as

$$\begin{aligned} &\mathrm{E}((X_k - Y_k)^+)^2 \\ =&\sum_{i=1}^{\infty}\sum_{j=1}^{i} (i-j)^2 (1-q_n)^{i-1} q_n (1-p_n)^{j-1} p_n && (8) \\ =&\sum_{j=1}^{\infty}\sum_{i=j}^{\infty} (i-j)^2 (1-q_n)^{i-1} q_n (1-p_n)^{j-1} p_n && (9) \\ =&\sum_{j=1}^{\infty} (1-q_n)^j (1-p_n)^{j-1} p_n \sum_{i=j}^{\infty} (i-j)^2 (1-q_n)^{i-j-1} q_n && (10) \\ =&\sum_{j=1}^{\infty} (1-q_n)^j (1-p_n)^{j-1} p_n \sum_{k=0}^{\infty} k^2 (1-q_n)^{k-1} q_n && (11) \\ =&\frac{(1-q_n)p_n}{1-(1-q_n)(1-p_n)} \cdot \frac{2-q_n}{q_n^2}, && (12) \end{aligned}$$

where (9) is obtained by changing the summation order, (11) is obtained by substituting the variable by $k = i - j$. Similarly, we have

$$\mathrm{E}(X_k - Y_k)^+ = \frac{(1-q_n)p_n}{1-(1-q_n)(1-p_n)} \cdot \frac{1}{q_n}. \tag{13}$$

As $\mathrm{E}X_k = 1/q_n$, we have

$$\bar{\Delta}_n = \frac{(1-q_n)p_n}{1-(1-q_n)(1-p_n)} \cdot \frac{1}{q_n} = \frac{1}{q_n} - \frac{1}{(1-p_n)q_n + p_n}. \tag{14}$$

The objective is to minimize the average AoS of all sensors by optimizing the sensors' visit probabilities $\{q_n\}_{n=1}^{N}$. The problem can be formulated as

$$\begin{aligned} \min_{\{q_n\}_{n=1}^{N}} \quad & \sum_{n=1}^{N} \frac{1}{N} \left(\frac{1}{q_n} - \frac{1}{(1-p_n)q_n + p_n}\right) && (15a) \\ \text{s.t.} \quad & \sum_{n=1}^{N} q_n \left(1 + \frac{E_n}{E_0}\right) \leq 1, && (15b) \\ & \sum_{n=1}^{N} q_n \leq 1, && (15c) \\ & q_n > 0, \quad \forall n \in \mathcal{N}, && (15d) \end{aligned}$$

where (15b) is derived from (4). In the following, we focus on solving the problem (15) and exploring its structural probabilities. We also consider the extension to the case that the UAV can visit multiple sensors in each time slot.

IV. Analysis and Algorithm

Firstly, the following lemma demonstrates the convex property of the problem (15).

Lemma 1. *The problem* (15) *is a convex optimization problem.*

Proof. Observe that

$$\begin{aligned}\frac{\partial^2 \bar{\Delta}_n}{\partial q_n^2} &= \frac{2}{q_n^3} - \frac{2(1-p_n)^2}{((1-p_n)q_n+p_n)^3} \\ &= \frac{2}{q_n^3} - \frac{2(1-p_n)^2}{(q_n+(1-q_n)p_n)^3} > 0. \end{aligned} \tag{16}$$

Therefore, $\bar{\Delta}_n$ is convex function of q_n [22, Sect. 3.1.4]. As the objective function (15a) is a weighted sum of $\bar{\Delta}_n$'s, it is also convex. In addition, all the constraints are linear. Therefore, the problem (15) is a convex optimization problem. □

There are a bunch of classical algorithms for convex optimization such as interior-point methods [22, Chap. 11]. Besides the algorithms, we are interested in the structural relation between the system parameters p_n's and the optimization variables q_n's. In the rest of this section, we try to explore the structures via Lagrangian duality analysis.

A. Lagrangian Duality Analysis

With multipliers $\lambda \geq 0$ and $\mu \geq 0$, we can introduce the Lagrangian function as

$$\begin{aligned}\mathcal{L} = &\sum_{n=1}^{N} \frac{1}{N}\left(\frac{1}{q_n} - \frac{1}{(1-p_n)q_n+p_n}\right) + \\ &\lambda\left(\sum_{n=1}^{N} q_n\left(1+\frac{E_n}{E_0}\right)-1\right) + \mu\left(\sum_{n=1}^{N} q_n - 1\right). \end{aligned} \tag{17}$$

The additional complementary slackness conditions are

$$\lambda\left(\sum_{n=1}^{N} q_n\left(1+\frac{E_n}{E_0}\right)-1\right) = 0, \tag{18}$$

$$\mu\left(\sum_{n=1}^{N} q_n - 1\right) = 0. \tag{19}$$

By applying the KKT optimality condition to the Lagrangian function (17), i.e., $\partial\mathcal{L}/\partial q_n = 0$, we obtain

$$\frac{1}{q_n^2} - \frac{1-p_n}{((1-p_n)q_n+p_n)^2} = \lambda N\left(1+\frac{E_n}{E_0}\right) + \mu N. \tag{20}$$

By multiplying the denominators on both sides of (20), a quartic equation can be obtained. Then, if the Lagrangian multipliers λ and μ are known, the optimal q_n can be either solved numerically or given in closed-form[1]. However, the closed-form expression of a root of the quartic equation is extremely complex. It is difficult to show how the UAV's optimal visit probabilities change with the sensors' update frequencies. To study the structural relationship, we introduce approximation to obtain a simple and intuitive solution.

[1] https://planetmath.org/QuarticFormula

B. Cubic Root Approximation

As (15a) is decreasing versus q_n, either (15b) or (15c) must be satisfied with equality, while the other is strictly unequal in general. Therefore, either λ or μ is equal to zero. If $\lambda = 0$ and $\mu > 0$, (15c) is satisfied with equality, meaning that no specific slots are allocated to purely recharge. In practice, this may happen when the recharge power is sufficiently high such that required energy can be recharged in the remaining time of each slot after flight. If $\lambda > 0$ and $\mu = 0$ on the other hand, a ratio of $(1-\sum_{n=1}^{N} q_n)$ slots are specified for power recharging. We have the following result.

Proposition 1. *When p_n is small, the solution of* (20) *can be approximated as*

$$q_n \approx \begin{cases} \sqrt[3]{\frac{2p_n(1-p_n)}{\mu N}}, & \text{if } \lambda = 0, \\ \sqrt[3]{\frac{2p_n(1-p_n)}{\lambda N}\left(1+\frac{E_n}{E_0}\right)^{-1}}, & \text{if } \mu = 0, \end{cases} \tag{21}$$

Proof. Denote $c_n = \lambda N\left(1+\frac{E_n}{E_0}\right) + \mu N$. Based on (20), define

$$f_n(x) = \frac{1}{((1-x)q_n+x)^2} = \frac{1}{((1-q_n)x+q_n)^2}. \tag{22}$$

We apply the Taylor's expansion to $f_n(x)$ for $x \to 0$ as

$$\begin{aligned} f_n(x) =& f_n(0) + f_n'(0)x + o(x) \\ \approx& \frac{1}{q_n^2} - \frac{2(1-q_n)}{q_n^3}x. \end{aligned} \tag{23}$$

Substituting $f_n(p_n)$ into (20), we have

$$c_n \approx \frac{1}{q_n^2} - (1-p_n)\left(\frac{1}{q_n^2} - \frac{2(1-q_n)}{q_n^3}p_n\right), \tag{24}$$

which can be transformed as a cubic equation

$$q_n^3 + \frac{p_n(1-2p_n)}{c_n}q_n - \frac{2p_n(1-p_n)}{c_n} = 0. \tag{25}$$

The real root to (25) is given by the well-known Cardano's Formula [23] as

$$q_n = \sqrt[3]{-\frac{b}{2}+\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{a}{3}\right)^3}} + \sqrt[3]{-\frac{b}{2}-\sqrt{\left(\frac{b}{2}\right)^2+\left(\frac{a}{3}\right)^3}}, \tag{26}$$

where $a = \frac{p_n(1-2p_n)}{c_n}, b = -\frac{2p_n(1-p_n)}{c_n}$. Since p_n is small, we can ignore the term $\left(\frac{a}{3}\right)^3$ compared with $\left(\frac{b}{2}\right)^2$. Joint with the value of λ and μ, we obtain (21). □

Based on (21), if $\lambda = 0$, which implies that the recharge power is sufficiently large as discussed before, the visit frequency q_n is proportional to $\sqrt[3]{p_n(1-p_n)}$ and is irrelevant to the energy conditions. Otherwise, the visit frequency q_n

is proportional to $\sqrt[3]{p_n(1-p_n)}$ and inversely proportional to $\sqrt[3]{1+\frac{E_n}{E_0}}$.

According Proposition 1, an approximate visit frequency allocation policy can be proposed. In particular, we first assume that (15c) is satisfied with equality. In this case, $\lambda = 0$ and the visit frequency can be approximated as

$$q_{\lambda,n} = \frac{\sqrt[3]{p_n(1-p_n)}}{\sum_{k=1}^{N} \sqrt[3]{p_n(1-p_n)}}. \tag{27}$$

If (27) satisfies (15b), the approximation is valid. Otherwise, (15b) must be satisfied with equality. Thus, we set

$$\tilde{q}_n = \sqrt[3]{p_n(1-p_n)\big(1+\frac{E_n}{E_0}\big)^{-1}}, \tag{28}$$

and by normalizing $\tilde{q}_n$'s, we have

$$q_{\mu,n} = \frac{\tilde{q}_n}{\sum_{k=1}^{N} \tilde{q}_n\big(1+\frac{E_n}{E_0}\big)}. \tag{29}$$

Due to approximation error, (27) and (29) may be infeasible. It is necessary to check the feasibility, i.e., whether (27) satisfies (15b), and whether (29) satisfies (15c). It is possible that both are infeasible, and the approximation should be modified by reducing q_n's proportionally. In this case, it is not obvious which approximation performs better. A simple way is to calculate the average AoS for both cases and compare to find the better one. The approximation algorithm is summarized in Algorithm 1.

Algorithm 1 Cubic Root Approximation Algorithm

Input: $N, E_0, p_n, E_n, n = 1, 2, \cdots, N$
Output: $q_n, n = 1, 2, \cdots, N$
Calculate $q_{\lambda,n}$'s according to (27).
if $\sum_{n=1}^{N} q_{\lambda,n}\left(1+\frac{E_n}{E_0}\right) > 1$ **then**
  $q_{\lambda,n} \leftarrow q_{\lambda,n}\left(\sum_{n=1}^{N} q_{\lambda,n}\left(1+\frac{E_n}{E_0}\right)\right)^{-1}$
end if
Calculate $q_{\mu,n}$'s according to (29).
if $\sum_{n=1}^{N} q_{\mu,n} > 1$ **then**
  $q_{\mu,n} \leftarrow q_{\mu,n}\left(\sum_{n=1}^{N} q_{\mu,n}\right)^{-1}$
end if
if $\sum_{n=1}^{N} \bar{\Delta}_n(q_{\lambda,n}) < \sum_{n=1}^{N} \bar{\Delta}_n(q_{\mu,n})$ **then**
  $q_n \leftarrow q_{\lambda,n}, \forall n = 1, 2, \cdots, N$
else
  $q_n \leftarrow q_{\mu,n}, \forall n = 1, 2, \cdots, N$
end if
return $q_n, n = 1, 2, \cdots, N$

V. Extension: Visit Multiple Sensors Per-Slot

In this section, we extend the model to the case that the UAV can visit multiple sensors in a single slot. Denote $\mathcal{S}_m \subset \mathcal{N}, m \in \mathcal{M} = \{1, 2, \cdots, M\}$ as a candidate set of sensors that can be jointly visited in one slot. It can be pre-determined by the locations of all sensors and the UAV's flight speed.

In each time slot, the UAV selects the set $\mathcal{S}_m$ with probability q_m, where $\sum_{m=1}^{M} q_m \leq 1$. The problem (15) can be viewed as a special case that the candidate sets are $\mathcal{S}_n = \{n\}, \forall n \in \mathcal{N}$. By introducing auxiliary variables, i.e., the effective visit probability α_n to sensor n, the AoS minimization problem can be formulated as

$$\min_{\substack{\{\alpha_n\}_{n=1}^{N},\\ \{q_m\}_{m=1}^{M}}} \sum_{n=1}^{N} \frac{1}{N}\left(\frac{1}{\alpha_n} - \frac{1}{(1-p_n)\alpha_n + p_n}\right) \tag{30a}$$

$$\text{s.t.} \quad \alpha_n = \sum_{m:n\in\mathcal{S}_m} q_m, \quad \forall n \in \mathcal{N}, \tag{30b}$$

$$\sum_{m=1}^{M} q_m\left(1+\frac{E_{\mathcal{S}_m}}{E_0}\right) \leq 1, \tag{30c}$$

$$\sum_{m=1}^{M} q_m \leq 1, \tag{30d}$$

$$q_m \geq 0, \quad \forall m \in \mathcal{M}, \tag{30e}$$

where α_n in (30b) is calculated based on the fact that the sensor n is visited if any set that contains n is visited, and $E_{\mathcal{S}_m}$ in (30c) is the required energy for visiting the sensors in set $\mathcal{S}_m$, which can be calculated as

$$E_{\mathcal{S}_m} = \min_{v_{\mathcal{S}_m}} P(v_{\mathcal{S}_m})\frac{2d_{\mathcal{S}_m}}{v_{\mathcal{S}_m}} - P_e\cdot\left(T - \frac{2d_{\mathcal{S}_m}}{v_{\mathcal{S}_m}}\right), \tag{31}$$

where $d_{\mathcal{S}_m}$ is the shortest path for visiting all sensors in $\mathcal{S}_m$. Again, we assume the flight speed $v_{\mathcal{S}_m}$ in a single slot is constant. The minimization in (31) is taken under the same constraints (2b) and (2c). Comparing (30e) with (15d), q_m in (30e) may be equal to zero, which means that the set $\mathcal{S}_m$ is never selected.

It is remarkable that the difference between problem (30) and (15) lies in the augmented optimization variables and the changed constraints. Nevertheless, the problem (30) is still convex as the objective function is convex, and all the constraints are linear. Thus, we can also apply the Lagrangian duality analysis similar to the previous section. A key observation is given as follows.

Lemma 2. *Suppose the recharge power is sufficiently large such that* (30d) *is satisfied with equality. For any $i, j \in \mathcal{M}$, if $\mathcal{S}_i \subsetneq \mathcal{S}_j$, we have $q_i = 0$ in the optimal solution of* (30)*, i.e., the subset $\mathcal{S}_i$ will never be selected.*

Proof. We introduce the Lagrangian function

$$\begin{aligned}\tilde{\mathcal{L}} =& \sum_{n=1}^{N} \frac{1}{N}\left(\frac{1}{\alpha_n} - \frac{1}{(1-p_n)\alpha_n + p_n}\right) \\ &+ \lambda\left(\sum_{m=1}^{M} q_m\left(1+\frac{E_{\mathcal{S}_m}}{E_0}\right) - 1\right) + \mu\left(\sum_{m=1}^{M} q_m - 1\right) \\ &+ \sum_{n=1}^{N} \nu_n\left(\alpha_n - \sum_{m:n\in\mathcal{S}_m} q_m\right) - \sum_{m=1}^{M} \zeta_m q_m,\end{aligned} \tag{32}$$

where $\lambda \geq 0, \mu \geq 0, \zeta_m \geq 0$, and ν_n are Lagrangian multipliers. The additional complementary slackness conditions are

$$\lambda \left(\sum_{m=1}^{M} q_m \left(1 + \frac{E_{\mathcal{S}_m}}{E_0}\right) - 1 \right) = 0, \tag{33}$$

$$\mu \left(\sum_{m=1}^{M} q_m - 1 \right) = 0, \tag{34}$$

$$\zeta_m q_m = 0, \quad \forall m \in \mathcal{M}. \tag{35}$$

By letting $\partial \mathcal{L}/\partial \alpha_n = 0$ and $\partial \mathcal{L}/\partial q_m = 0$, we obtain

$$\left(-\frac{1}{\alpha_n^2} + \frac{1-p_n}{((1-p_n)\alpha_n + p_n)^2} \right) + \nu_n = 0, \ \forall n \in \mathcal{N}, \tag{36}$$

$$\lambda \left(1 + \frac{E_{\mathcal{S}_m}}{E_0}\right) + \mu - \sum_{n \in \mathcal{S}_m} \nu_n - \zeta_m = 0, \ \forall m \in \mathcal{M}. \tag{37}$$

Similar to the single sensor case, either λ or μ is equal to zero depending on whether in-flight recharging is sufficient.

We prove the lemma by contradiction. If (30d) is satisfied with equality, we have $\mu > 0$ and $\lambda = 0$. For any $i, j \in \mathcal{M}$ such that $\mathcal{S}_i \subsetneq \mathcal{S}_j$, suppose $q_i > 0$. Based on (35), we have $\zeta_i = 0$. Then, according to (37), we have

$$\sum_{n \in \mathcal{S}_i} \nu_n = \mu. \tag{38}$$

For the subset $\mathcal{S}_j$, since $\zeta_j \geq 0$, we have

$$\sum_{n \in \mathcal{S}_j} \nu_n = \mu - \zeta_j \leq \mu. \tag{39}$$

Combining (38) and (39), we have

$$\sum_{n \in \mathcal{S}_j \setminus \mathcal{S}_i} \nu_n \leq 0. \tag{40}$$

However, it can be easily seen from (36) that $\nu_n > 0$, which contradicts (40) for $\mathcal{S}_i \subsetneq \mathcal{S}_j$. Therefore, we have $q_i = 0$. $\square$

The intuition of Lemma 2 is that if the energy recharged in-flight is sufficient, it is always beneficial to visit as many sensors as possible in each slot. On the other hand, if the in-flight recharge is insufficient, there is a trade-off between status update and energy recharging. Intuitively, visiting a larger set usually consumes more energy and remains less time for recharge because of longer flight distance. Thus, the above lemma may not hold. However, we can still design the visit frequency allocation policy considering only the largest visitable sets, which is shown to perform close to the optimal in the next section.

Based on Lemma 2, we only consider the largest visitable sets that are not subsets of any others. With these candidate sets, the problem (30) can also be solved by interior-point method. Besides, a heuristic policy can be proposed based on the cubic root relation obtained in the previous section. In particular, we approximate set-level update probability as

$$\beta_m = \frac{1}{|\mathcal{S}_m|} \sum_{n \in \mathcal{S}_m} \frac{p_n}{\sum_{j=1}^{M} I_{n \in \mathcal{S}_j}}, \tag{41}$$

where $|\mathcal{S}_m|$ is the size of set $\mathcal{S}_m$, and I_x is an indicator function. $I_x = 1$ if x is true, and $I_x = 0$ otherwise. Thus, each sensor's update probability is evenly allocated to the sets which contain the sensor, and each set averages the allocated update probabilities. Then, we apply the proposed approximate algorithm (Algorithm 1) in a set level, i.e., replace the sensor index n by the set index m, and replace p_n's by β_m's.

TABLE I
SYSTEM MODEL PARAMETERS

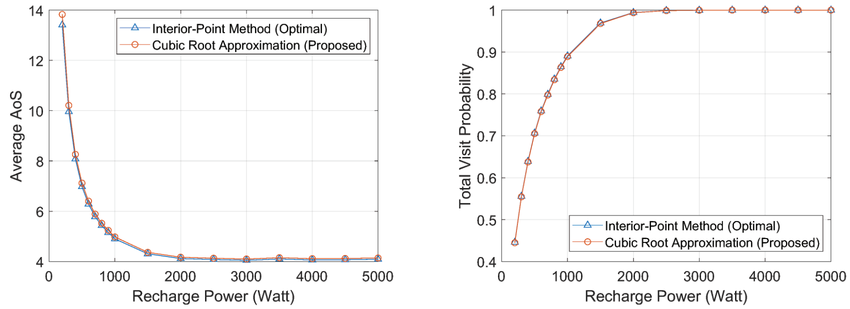
Notation	**value**	**Notation**	**value**
U_{tip}	120m/s	d_0	0.6
v_0	4.03m/s	ρ	1.225kg/m^3
s	0.05	A	0.503m^2
W	20kg	Ω	300rad/s
R	0.4m	δ	0.012
$v_{\min}$	0	$v_{\max}$	30m/s
T	120s	$d_{\max}$	1800m

VI. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed approximation method via numerical simulations. The system model parameters are listed in Table I. In the simulations, the distance between each sensor to the control center is set uniformly distributed between 1200m and 1800m, i.e., $d_n \approx U(1200\text{m}, 1800\text{m})$. The sensor update probability follows the uniform distribution as $p_n \sim U(0, 2\bar{p})$, where $\bar{p}$ is the average update probability.

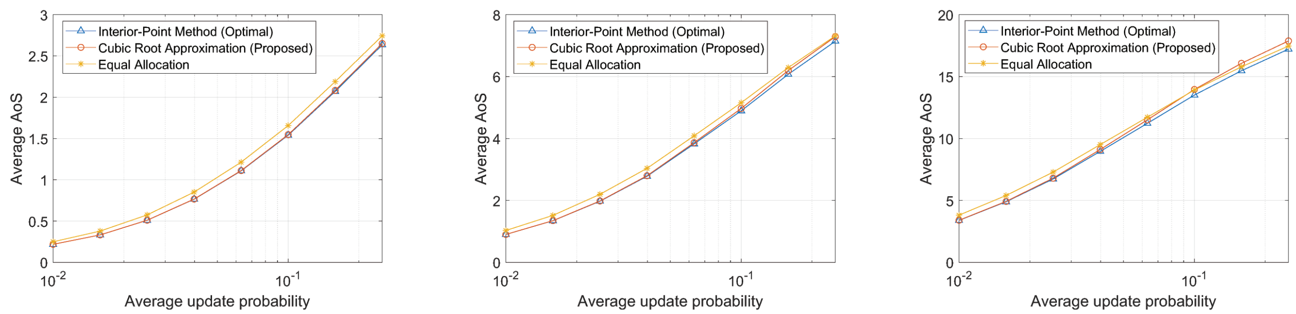
Firstly, we consider that the UAV visits a single sensor in one slot. Fig. 3(a) illustrates the average AoS performance with the change of average recharge power. It is shown that our proposed cubic root approximation well meets the optimal interior-point method. Furthermore, Fig. 3(b) shows the average total visit probability $\sum_{n=1}^{N} q_n$ performance. It can be seen that more than half of the slots are dedicated to recharge when the recharge power is as low as 200W. When the recharge power exceeds 2500W, the in-flight recharge power is sufficient and all the slots can be allocated for data collection.

Fig. 4 shows the performance of average AoS with different average update probabilities and sensor numbers. For comparison, we also depict the curves for *Equal Allocation* policy, where each sensor is visited with the same probability. It can be seen that the approximation performs close to the optimal when the average update probability is small, which validates our analysis. The difference is that with the increase of sensor number in the system, the range of $\bar{p}$ for cubic root approximation attaining the same precision narrows. When $N = 5$, the approximation is accurate for all $\bar{p} < 0.25$ and always better than the equal allocation. When $N = 10$, the performance gap between the proposed algorithm and the optimal one becomes apparent when $\bar{p} > 0.1$. In addition, the equal allocation performs almost the same as the proposed



(a) Average AoS (b) Average Total Visit Probability

Fig. 3. Performance versus average recharge power. The UAV can visit a single sensor in each slot. $N = 10, \bar{p} = 0.1$.



(a) $N = 5$ (b) $N = 10$ (c) $N = 20$

Fig. 4. Average AoS versus average update probability with different amounts of sensors. The UAV can visit a single sensor in each slot. $P_e = 1000$W.

approximation. While for $N = 20$, the equal allocation performs even better than the proposed approximation when $\bar{p} > 0.1$. Therefore, the proposed approximation algorithm works well for low update probability regime or small sensor number regime.

Then, we study the multi-visit case. In Fig. 5, the term *non-subset* refers to considering only the largest visitable sets that are not subsets of any others, and the term *all-sets* refers to considering all the candidate sets. It can be seen from Fig. 5(a) that executing interior-point method with the largest visitable sets performs almost the same as the global optimal one, although the number of visited sets (Fig. 5(c)) varies. In addition, the proposed set-level approximate algorithm considering only the largest visitable sets performs close to the optimal, while the one considering all candidate sets performs poor. In Fig. 5(b), it can be seen that the total visit probability of all-sets algorithm is higher than the others. The reason is that visiting small sets consumes less energy than visiting the largest visitable sets. Based on Fig. 5(c), it can be inferred that visiting subsets is extremely inefficient compared with visiting the largest sets.

We also evaluate the performance of average AoS with different average update probabilities and sensor numbers. As shown in Fig. 6, we also compare with equal allocation policy considering only the largest visitable sets. Different from visiting single sensor case, our proposed approximation algorithm always performs better than the equal allocation even with large number of sensors $N = 40$. The reason is that with more sensors, the largest visitable sets becomes larger as well. Thus, each sensor is equivalently visited with higher frequency. Hence, our proposed algorithm still performs well.

VII. Conclusion

This work studies the average AoS minimization problem with UAV collecting data from remote sensors under random visit policy. It is found that with low update probability, the UAV's visit probability is approximately proportional to a cubic root function. Based on this, a simple allocation policy is proposed, which is shown to perform well with low update probability and small number of sensors. The extension to visiting multi-sensor per-slot also works well, and is sufficient to only consider the largest visitable sensor sets. We also find that the proposed algorithm always performs better than the equal allocation policy, which illustrates the benefit of visiting multiple sensors in each slot. Future work can consider online learning problem and extension to multiple UAVs.

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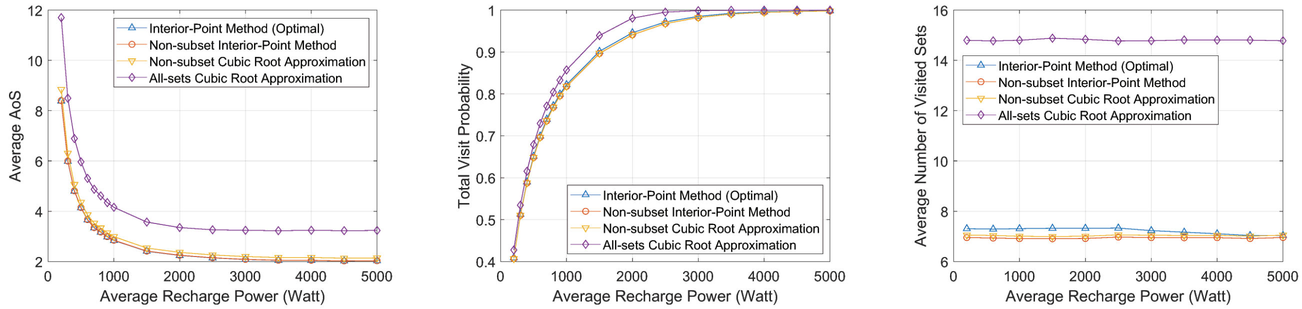
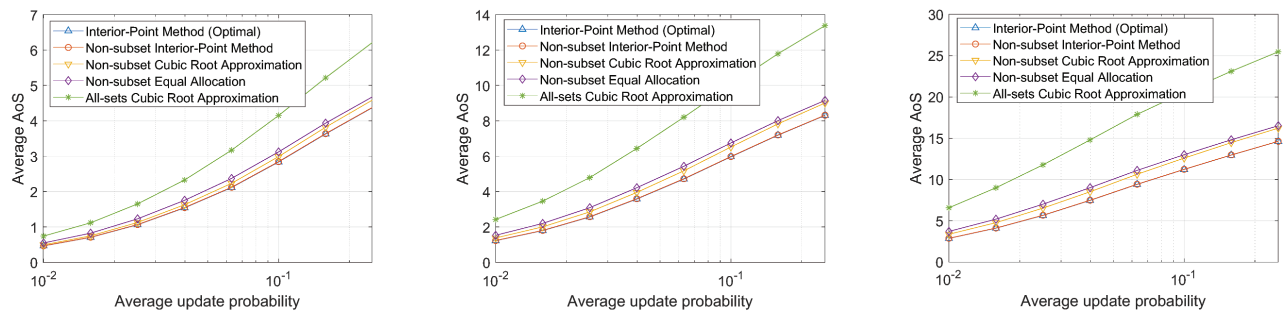


(a) Average AoS (b) Average Total Visit Probability (c) Average Number of Visited Sets

Fig. 5. Performance versus average recharge power. The UAV can visit multiple sensors in each slot. $N = 10, \bar{p} = 0.1$.



(a) $N = 10$ (b) $N = 20$ (c) $N = 40$

Fig. 6. Average AoS versus average update probability with different amounts of sensors. The UAV can visit multiple sensors in each slot. $P_e = 1000$W.

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