Version Innovation Age and Age of Incorrect Version for Monitoring Markovian Sources

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***Abstract*—In this paper, we study the problem of real-time remote tracking of a two-state Markov process. We introduce a modified randomized stationary sampling and transmission policy where the decision to perform sampling occurs probabilistically depending on the current state of the source and whether the system was in a sync state during the previous time slot or not. We then propose two new performance metrics, coined the *Version Innovation Age (VIA)* and the *Age of Incorrect Version (AoIV)*, and analyze their performance under the modified randomized stationary and other state-of-the-art sampling and transmission policies. Specifically, we derive closed-form expressions for the distribution and the average of VIA and AoIV under these policies. Furthermore, we formulate and solve two constrained optimization problems. The first optimization problem aims to minimize the average VIA subject to constraints on the time-averaged sampling cost and time-averaged reconstruction error. In the second problem, the objective is to minimize the average AoIV while considering a constraint on the time-averaged sampling cost. Finally, we compare the performance of various sampling and transmission policies and identify the conditions under which each policy outperforms the others in optimizing the proposed metrics.**

I. Introduction

Timely delivery of relevant status update packets from an information source has become increasingly crucial in various real-time communication systems, in which digital components remotely monitor and control physical entities [1], [2]. These systems require reliable and timely exchange of relevant information, coupled with efficient distributed processing, to facilitate optimal decision-making in applications such as industrial automation, collaborative robotics, and autonomous transportation systems. These challenging requirements gave rise to *goal-oriented semantics-empowered communications*, a novel paradigm that considers the usefulness, the timeliness [3]–[7] and the innate and contextual importance of information to generate, transmit, and utilize data in time-sensitive and data-intensive communication systems [8]. Recently, a new metric called Version Age of Information (VAoI) has been introduced in [9], where each update at the source is considered as a new version, thus quantifying how many versions out-of-date the information on the monitor is, compared to the version at the source. Several studies have considered the VAoI as a key performance metric of timeliness in information in networks [9]–[21]. The scaling of the average VAoI in gossip networks of different sizes and topologies is investigated in [9]–[18]. A learning-based approach to minimize the overall average VAoI of the worst-performing node in sparse gossip networks is employed in [19]. The authors in [20] studied the problem of minimizing the average VAoI using a Markov Decision Process (MDP) in a scenario where an energy harvesting (EH) sensor updates the gossip network via an aggregator equipped with caching capabilities. The work [21] considered the problem of minimizing VAoI over a fading broadcast channel, employing a non-orthogonal multiple access (NOMA) scheme.

Prior works on VAoI rely on the occurrence of *content change* at the source, while the destination nodes only demand the latest version of the information from the source, regardless of its *content*. In other words, VAoI exclusively focuses on changes occurring in the source's content, disregarding the significance and the utility of that information. Several semantics-aware metrics [22]–[29] have made a step in that direction, without though investigating the evolution of versions. In this work, we propose two new semantics-aware metrics, coined *Version Innovation Age* (VIA) and *Age of Incorrect Version* (AoIV), which take into account both the content and the version evolution of the information source[1].

In this paper, we examine a time-slotted communication system where a sampler makes decisions to perform sampling of a two-state Markov process, acting as the information source. After deciding to sample, the transmitter sends the sample in packet form to a remote receiver over an unreliable wireless channel. Then, at the receiver, real-time reconstruction of the information source is conducted based on the samples that have been successfully received. Furthermore, a specific action is executed at the receiver based on the estimated state of the information source. We introduce a new sampling and transmission policy, termed *modified randomized stationary* policy. To evaluate the system performance, we derive general expressions for the distribution and average of the VIA and AoIV under the modified randomized stationary policy and other state-of-the-art sampling and transmission

[1]VIA and AoIV are important in applications such as Internet of Things (IoT) software updates, distributed systems, and real-time monitoring because they offer a more accurate assessment of information freshness and accuracy, which affects system reliability and decision-making quality.

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policies introduced in [23], [25]. In addition, we formulate and solve two constrained optimization problems. The first optimization problem aims to minimize the average VIA with constraints on the time-averaged sampling cost, and time-averaged reconstruction error metric as proposed in [25]. In the second problem, we minimize the average AoIV under time-averaged sampling cost as a constraint[2].

II. System Model

We consider a time-slotted communication system in which a sampler conducts sampling of an information source, denoted as $X(t)$, at time slot t, as shown in Fig. 1. The transmitter then forwards the sampled information in packets over a wireless communication channel to the receiver. The information source is modeled as a two-state discrete-time Markov chain (DTMC) $\{X(t), t \in \mathbb{N}\}$. Therein, the state transition probability $\Pr\big[X(t+1)=j\big|X(t)=i\big]$ represents the probability of transitioning from state i to j and can be defined as $\Pr\big[X(t+1)=j\big|X(t)=i\big]=\mathbf{1}(i=0,j=0)(1-p)+\mathbf{1}(i=0,j=1)p+\mathbf{1}(i=1,j=0)q+\mathbf{1}(i=1,j=1)(1-q)$, where $\mathbf{1}(\cdot)$ is the indicator function. We denote the action of sampling at time slot t by $\alpha^{\mathrm{s}}(t)=\{0,1\}$, where $\alpha^{\mathrm{s}}(t)=1$ if the source is sampled and $\alpha^{\mathrm{s}}(t)=0$ otherwise[3]. At time slot t, the receiver constructs an estimate of the process $X(t)$, denoted by $\hat{X}(t)$, based on the successfully received samples. For the wireless channel between the source and the receiver, we assume that the channel state $h(t)$ equals 1 if the information source is sampled and successfully decoded by the receiver and 0 otherwise. We define the success probability as $p_s=\Pr[h(t)=1]$. At time slot t, when the source is sampled, we assume that with probability p_s, the system is in a sync state, i.e., $X(t)=\hat{X}(t)$. Otherwise, if the system is in an erroneous state, the state of the reconstructed source remains unchanged, i.e., $\hat{X}(t)=\hat{X}(t-1)$. Acknowledgment (ACK)/negative-ACK(NACK) packets are used to inform the transmitter about the success or failure of transmissions. ACK/NACK packets are assumed to be delivered to the transmitter instantly and without errors[4]. Therefore, the transmitter has accurate information about the reconstructed source state at time slot t, i.e., $\hat{X}(t)$. In addition, we assume that the corresponding sample is discarded in the event of a transmission failure (packet drop channel). Here, we propose a new sampling and transmission policy, named the *modified randomized stationary (MRS)* policy. Under this policy, at time slot t, the source is not sampled if the state of the source matches the state of the reconstructed source at the previous time slot, i.e., $X(t)=\hat{X}(t-1)$. This condition indicates that the system is in a sync state and thus sampling is unnecessary. Otherwise, the generation of a new sample at time slot t is triggered probabilistically. Here, we assume that when the system was in a sync state at time slot $t-1$ but the state of the source at time slot t differs from that at time slot $t-1$, the sampler performs sampling with probability $q_{\alpha_1^{\mathrm{s}}}$, where $q_{\alpha_1^{\mathrm{s}}}=\Pr[\alpha^{\mathrm{s}}(t)=1 \mid X(t)\neq X(t-1), X(t-1)=\hat{X}(t-1)]$. Furthermore, when the state of the source at time slot $t-1$ and t differs from the state of the reconstructed source at time slot $t-1$, the sampler performs sampling with probability $q_{\alpha_2^{\mathrm{s}}}$, where $q_{\alpha_2^{\mathrm{s}}}=\Pr[\alpha^{\mathrm{s}}(t)=1 \mid X(t)\neq\hat{X}(t-1), X(t-1)\neq\hat{X}(t-1)]$. For comparison purposes, we adopt three other relevant sampling policies, namely the *randomized stationary (RS)*, *change-aware*, and *semantics-aware* policies proposed in [23], [25].

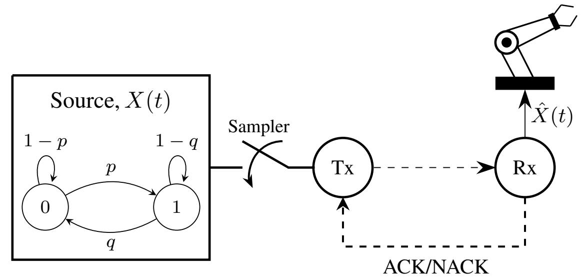


Fig. 1. Real-time monitoring of a Markovian source over a wireless channel.

III. Performance Metrics and Analysis

In this section, we study the effect of the *semantics of information* at the receiver. For that, we propose and analyze two new performance metrics, termed VIA and AoIV, which jointly quantify *both the timing and importance aspects of information*.

A. Version Innovation Age (VIA)

Before introducing the first new metric, we review the VAoI proposed in [9]. We assume that each update at the information source represents a version. At time slot t, $V_{\mathrm{S}}(t)$ represents the version at the source, while $V_{\mathrm{R}}(t)$ represents the version at the receiver. The VAoI at the receiver is then defined as $\mathrm{VAoI}(t)=V_{\mathrm{S}}(t)-V_{\mathrm{R}}(t)$. This metric relies on changes in the information source, but the content of the information source is not considered important. Therefore, we introduce a new metric named VIA, which measures the number of outdated versions at the receiver compared to the source when the source is in a specific state. In other words, the VIA differs when the source is in state i compared to when it is in state j. Since the VAoI solely considers changes in the source state and does not depend on the specific state of the source, this metric cannot be defined for a specific state. We define the evolution of VIA as follows

$$\mathrm{VIA}(t)=\begin{cases}\mathrm{VIA}(t-1), & X(t)=X(t-1)\text{ and }\{\alpha^{\mathrm{s}}(t)=0,\\ & \text{or }(\alpha^{\mathrm{s}}(t)=1, h(t)=0)\},\\ \mathrm{VIA}(t-1)+1, & X(t)\neq X(t-1)\text{ and }\{\alpha^{\mathrm{s}}(t)=0,\\ & \text{or }(\alpha^{\mathrm{s}}(t)=1, h(t)=0)\},\\ 0, & \alpha^{\mathrm{s}}(t)=1, h(t)=1.\end{cases} \tag{1}$$

Using (1), for a two-state DTMC information source depicted in Fig. 1, and applying the total probability theorem,

[2]An extended version of this work can be found in [30].

[3]We assume that when sampling occurs at time slot t, the transmitter sends the sample immediately at that time slot.

[4]An ACK/NACK feedback channel is required for the modified randomized stationary and semantics-aware policies.

we can express the probability that the VIA at time slot t equals $i \geqslant 0$ as follows

$$\Pr\big[\mathrm{VIA}(t)=i\big]=\Pr\big[X(t)=0,\mathrm{VIA}(t)=i\big] +\Pr\big[X(t)=1,\mathrm{VIA}(t)=i\big]=\pi_{0,i}+\pi_{1,i}. \quad (2)$$

Note that $\pi_{0,i}$ and $\pi_{1,i}$ in (2) are the probabilities obtained from the stationary distribution of the two-dimensional DTMC describing the joint status of the original source regarding the current state of the VIA, i.e., $\big(X(t),\mathrm{VIA}(t)\big)$.

Lemma 1. *For a two-state DTMC information source, the stationary distribution $\pi_{j,i}$ for the RS policy is given by*[5]

$$\begin{aligned}\pi_{0,i}&=\frac{p^k q^w p_{\alpha^s}p_s\big(1-p_{\alpha^s}p_s\big)^i}{(p+q)\big(p+(1-p)p_{\alpha^s}p_s\big)^w\big(q+(1-q)p_{\alpha^s}p_s\big)^k}, i=0,1,\ldots.\\ \pi_{1,i}&=\frac{p^w q^k p_{\alpha^s}p_s\big(1-p_{\alpha^s}p_s\big)^i}{(p+q)\big(p+(1-p)p_{\alpha^s}p_s\big)^k\big(q+(1-q)p_{\alpha^s}p_s\big)^w}, i=0,1,\ldots.\end{aligned} \quad (3)$$

where k and w are given by

$$k=\begin{cases}\frac{i}{2}, & \bmod\{i,2\}=0,\\ \frac{i+1}{2}, & \bmod\{i,2\}\neq 0.\end{cases}\qquad w=\begin{cases}\frac{i+2}{2}, & \bmod\{i,2\}=0,\\ \frac{i+1}{2}, & \bmod\{i,2\}\neq 0.\end{cases} \quad (4)$$

Proof: See Appendix A. ■

Using (3) and (4), we calculate the average VIA, $\overline{\mathrm{VIA}}$, as[6]

$$\begin{aligned}\overline{\mathrm{VIA}}&=\sum_{i=0}^{\infty} i\Pr\big[\mathrm{VIA}(t)=i\big]=\sum_{i=1}^{\infty} i\big(\pi_{0,i}+\pi_{1,i}\big)\\ &=\frac{2pq\big(1-p_{\alpha^s}p_s\big)}{(p+q)p_{\alpha^s}p_s}.\end{aligned} \quad (5)$$

Remark 1. *Using* (5) *and* (28)*, we can prove that for $0<p_s\leqslant 1$, the RS policy has a lower average VIA compared to the change-aware policy if $\frac{2pq}{p+q+\left(2pq-p-q\right)p_s}\leqslant p_{\alpha^s}\leqslant 1$. Otherwise, the change-aware policy outperforms the RS policy in terms of average VIA.*

Remark 2. *Using (39) in [25], we can express $\overline{VIA}$ given in* (5) *and* (28) *as a function of the time-averaged reconstruction error, which is the probability that the system is in an erroneous state. For the RS policy,* (5) *is*

$$\overline{VIA}(P_E)=\frac{\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]P_E}{p_{\alpha^s}p_s} \quad (6)$$

where P_E represents the time-averaged reconstruction error and in (6) *is given by*

$$P_E=\frac{2pq(1-p_{\alpha^s}p_s)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]}. \quad (7)$$

[5]Since, at each time slot, the state of the VIA does not depend on the state of the reconstructed source, we only investigate the performance of this metric for RS and change-aware policies.

[6]The convergence condition for (5) is $\frac{\sqrt{pq}\left(1-p_{\alpha^s}p_s\right)}{\sqrt{\left(p+(1-p)p_{\alpha^s}p_s\right)\left(q+(1-q)p_{\alpha^s}p_s\right)}}<1$.

Furthermore, for the change-aware policy, the expression given in (28) *can be written as*

$$\overline{VIA}(P_E)=\left(\frac{2}{p_s}-1\right)P_E \quad (8)$$

where P_E in (8) *is calculated as*

$$P_E=\frac{1-p_s}{2-p_s}. \quad (9)$$

B. Age of Incorrect Version (AoIV)

A major issue with VIA is that when the state of the source changes and transmission fails, the VIA increases by one. However, the system may be in a sync state, i.e., $X(t)=\hat{X}(t)$. In other words, the VIA can still increase even if the system has perfect knowledge of the source's state. For that, we introduce another metric, termed AoIV, defined as the number of outdated versions at the receiver compared to the source when the system is in an erroneous state, i.e., $X(t)\neq\hat{X}(t)$. The evolution of the AoIV is

$$\mathrm{AoIV}(t)=\begin{cases}\mathrm{AoIV}(t-1), & X(t)=X(t-1),\\ & X(t)\neq\hat{X}(t)\\ \mathrm{AoIV}(t-1)+1, & X(t)\neq X(t-1),\\ & X(t)\neq\hat{X}(t)\\ 0, & X(t)=\hat{X}(t).\end{cases} \quad (10)$$

Using (10), for a two-state DTMC information source as in Fig. 1, and applying the total probability theorem, $\Pr\big[\mathrm{AoIV}(t)\neq 0\big]$ is calculated as[7]

$$\begin{aligned}\Pr\big[\mathrm{AoIV}(t)\neq 0\big]&=\Pr\big[X(t)=0,\hat{X}(t)=1,\mathrm{AoIV}(t)=1\big]\\ &\quad+\Pr\big[X(t)=1,\hat{X}(t)=0,\mathrm{AoIV}(t)=1\big]\\ &=\pi_{0,1,1}+\pi_{1,0,1}\end{aligned} \quad (11)$$

where $\pi_{0,1,1}$ and $\pi_{1,0,1}$ are the probabilities obtained from the stationary distribution of the three-dimensional DTMC describing the joint status of the original and reconstructed source regarding the current state of the AoIV, i.e., $\big(X(t),\hat{X}(t),\mathrm{AoIV}(t)\big)$.

Lemma 2. *For a two-state DTMC information source, the stationary distribution $\pi_{i,j,k},\forall i,j,k\in\{0,1\}$ for the RS policy is given by*

$$\begin{aligned}\pi_{0,0,0}&=\frac{q\big[q+(1-q)p_{\alpha^s}p_s\big]}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\ \pi_{0,1,1}&=\frac{pq\big(1-p_{\alpha^s}p_s\big)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\ \pi_{1,1,0}&=\frac{p\big[p+(1-p)p_{\alpha^s}p_s\big]}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\ \pi_{1,0,1}&=\frac{pq\big(1-p_{\alpha^s}p_s\big)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\ \pi_{0,0,1}&=\pi_{0,1,0}=\pi_{1,0,0}=\pi_{1,1,1}=0.\end{aligned} \quad (12)$$

[7]For a two-state DTMC information source, the maximum value of AoIV is 1.

Furthermore, for the MRS policy, we can write

$$\pi_{0,0,0} = \frac{q\big[q_{\alpha_2^s} + p\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]\big[q + (1-q)q_{\alpha_2^s}p_s\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},$$
$$\pi_{0,1,1} = \frac{pq\big(1 - q_{\alpha_1^s}p_s\big)\big[q_{\alpha_2^s} + q\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},$$
$$\pi_{1,1,0} = \frac{p\big[q_{\alpha_2^s} + q\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]\big[p + (1-p)q_{\alpha_2^s}p_s\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},$$
$$\pi_{1,0,1} = \frac{pq\big(1 - q_{\alpha_1^s}p_s\big)\big[q_{\alpha_2^s} + p\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},$$
$$\pi_{0,0,1} = \pi_{0,1,0} = \pi_{1,0,0} = \pi_{1,1,1} = 0 \tag{13}$$

where $F(\cdot,\cdot)$ *in* (13) *is given by*

$$F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big) = (1-p)(1-q)p_s q_{\alpha_2^s}^2 + (p+q-2pq)q_{\alpha_2^s} + pq(2 - p_s q_{\alpha_1^s})q_{\alpha_1^s}. \tag{14}$$

Proof: See Appendix B. ■

Now, using (12) we can calculate (11) for the RS policy as follows

$$\Pr\big[\mathrm{AoIV}(t) = 1\big] = \pi_{0,1,1} + \pi_{1,0,1} = \frac{2pq\big(1 - p_{\alpha^s}p_s\big)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]}. \tag{15}$$

Using (15), the average AoIV, $\overline{\mathrm{AoIV}}$, for the RS policy can be obtained as

$$\overline{\mathrm{AoIV}} = \sum_{i=1}^{\infty} i\Pr\big[\mathrm{AoIV}(t) = i\big] = \frac{2pq\big(1 - p_{\alpha^s}p_s\big)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]}. \tag{16}$$

Furthermore, using (13), (11) can be written as

$$\Pr\big[\mathrm{AoIV}(t) = 1\big] = \pi_{0,1,1} + \pi_{1,0,1} = \frac{pq\big(1 - q_{\alpha_1^s}p_s\big)\big[(p+q)q_{\alpha_1^s} + (2-p-q)q_{\alpha_2^s}\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)}. \tag{17}$$

Now, using (17), the average AoIV, $\overline{\mathrm{AoIV}}$, for the MRS policy can be calculated as

$$\overline{\mathrm{AoIV}} = \sum_{i=1}^{\infty} i\Pr\big[\mathrm{AoIV}(t) = i\big] = \frac{pq\big(1 - q_{\alpha_1^s}p_s\big)\big[(p+q)q_{\alpha_1^s} + (2-p-q)q_{\alpha_2^s}\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)} \tag{18}$$

where $F(\cdot,\cdot)$ in (17) and (18) is obtained in (14).

Remark 3. *In what follows, RSC and MRSC policies represent the randomized stationary and modified randomized stationary policies in constrained optimization problems, respectively.*

IV. Constrained Optimization Problems

In this section, we formulate and solve two constrained optimization problems. In the first one, presented in Section IV-A, we aim to find the optimal RSC sampling policy that minimizes the average VIA while considering constraints on both the time-averaged sampling cost and the time-averaged reconstruction error, as given in (7). In the second optimization problem, presented in Sections IV-B, the objective is to find the optimal RSC and MRSC sampling policies that minimize the average AoIV while constraining the time-averaged sampling cost to be less than a certain threshold.

A. Minimizing the average VIA

The objective of this optimization problem is to find the optimal value of the probability of sampling in the RS sampling policy, p_{α^s}, to minimize the average VIA. We assume that each attempted sampling incurs a cost denoted by δ, and the time-averaged sampling cost is constrained not to surpass a specified threshold $\delta_{\max}$. Furthermore, we consider a constraint that the time-averaged reconstruction error cannot exceed a certain threshold $E_{\max}$. We formulate the following optimization problem

$$\underset{p_{\alpha^s}}{\text{minimize}} \quad \overline{\mathrm{VIA}} \tag{19a}$$
$$\text{subject to} \quad \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^s(t) = 1\} \leqslant \delta_{\max}, \tag{19b}$$
$$P_{\mathrm{E}} \leqslant E_{\max} \tag{19c}$$

where the constraint in (19b) is the time-averaged sampling cost, which can be simplified as

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^s(t) = 1\} = \delta p_{\alpha^s}. \tag{20}$$

Now, using (5), (7) and (20) the optimization problem can be formulated as

$$\underset{p_{\alpha^s}}{\text{minimize}} \quad \frac{2pq\big(1 - p_{\alpha^s}p_s\big)}{(p+q)p_{\alpha^s}p_s} \tag{21a}$$
$$\text{subject to} \quad \frac{2pq - E_{\max}(p+q)^2}{2pqp_{\mathrm{s}} + E_{\max}(p+q)(1-p-q)p_{\mathrm{s}}} \leqslant p_{\alpha^s} \leqslant \eta \tag{21b}$$

where $\eta = \delta_{\max}/\delta \leqslant 1$ and $E_{\max} \leqslant 1$.

To solve this optimization problem, we first note that the objective function in (21a) is decreasing with p_{α^s}. In other words, the objective function has its minimum value when p_{α^s} has its maximum. Using the constraint given in (21b), the maximum value of sampling probability is η. However, η is the optimal value of sampling probability when $\frac{2pq-E_{\max}(p+q)^2}{2pqp_{\mathrm{s}}+E_{\max}(p+q)(1-p-q)p_{\mathrm{s}}} \leqslant \eta \leqslant 1$; otherwise, we cannot find a sampling probability that satisfies the constraint of the optimization problem, and thus, an optimal solution does not exist.

B. *Minimizing the average AoIV*

The goal of this optimization problem is to determine the optimal values for the sampling probabilities, p_{α^s}, $q_{\alpha_1^s}$, and $q_{\alpha_2^s}$ for both the RSC and MRSC sampling policies, respectively, which minimize the average AoIV while considering the time-averaged sampling cost constraint. Here, δ represents the cost of each attempted sampling, and $\delta_{\max}$ is the total average cost. Using (16), this optimization problem for the RSC policy is formulated as follows

$$\underset{p_{\alpha^s}}{\text{minimize}} \quad \frac{2pq\left(1-p_{\alpha^s}p_s\right)}{(p+q)\left[p+q+(1-p-q)p_{\alpha^s}p_s\right]} \tag{22a}$$

$$\text{subject to} \quad \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^s(t)=1\}\leqslant\delta_{\max}. \tag{22b}$$

To solve this optimization problem, we note that the objective function given in (22a) decreases as p_{α^s} increases. In other words, to minimize the objective function, we must find the maximum value of p_{α^s} that satisfies the constraint in (22b). Using (20), for the RSC policy, the maximum value of the sampling probability that minimizes the average AoIV is given by $p^*_{\alpha^s}=\eta$, where $0\leqslant\eta=\delta_{\max}/\delta\leqslant 1$. Now, using (18) for the MRSC policy, we formulate the optimization problem as follows

$$\underset{q_{\alpha_1^s},q_{\alpha_2^s}}{\text{minimize}} \quad \frac{pq\left(1-q_{\alpha_1^s}p_s\right)\left[(p+q)q_{\alpha_1^s}+(2-p-q)q_{\alpha_2^s}\right]}{(p+q)F\left(q_{\alpha_1^s},q_{\alpha_2^s}\right)} \tag{23a}$$

$$\text{subject to} \quad \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^s(t)=1\}\leqslant\delta_{\max} \tag{23b}$$

where $F(\cdot,\cdot)$ is given in (14). Since this optimization problem is not convex with respect to $q_{\alpha_1^s}$ and $q_{\alpha_2^s}$, and the optimization parameters are interdependent, a closed-form solution is hard or impossible to be found for global optimization. Therefore, we solve (23) numerically. Furthermore, as a special case, by using the following Lemma, we can obtain the optimal value of the probability of sampling for the MRSC policy when $q_{\alpha_1^s}=q_{\alpha_2^s}$.

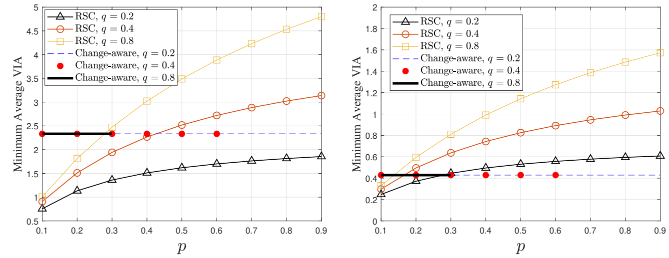
Lemma 3. *In the MRSC sampling policy, when* $q_{\alpha_1^s}=q_{\alpha_2^s}=q_{\alpha^s}$ *the optimal value of the sampling probability, i.e.,* $q^*_{\alpha^s}$ *is given by*

$$q^*_{\alpha^s} = \begin{cases} 1, & \eta(p+q)(1-p-q)p_s \geqslant 2pq, \\ \min\left\{1, \frac{\eta(p+q)^2}{2pq-\eta(p+q)(1-p-q)p_s}\right\}, & \eta(p+q)(1-p-q)p_s < 2pq. \end{cases}$$

Proof: See Appendix C. ■

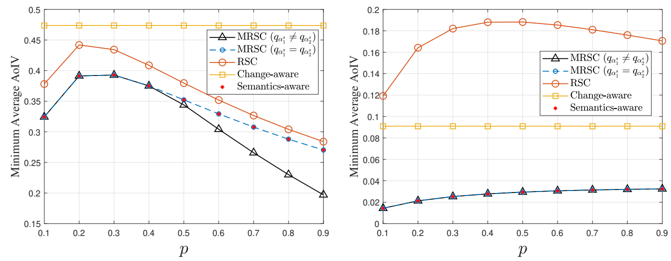
V. Numerical Results

In this section, we numerically validate our analytical results and assess the performance of the sampling policies in terms of average VIA, and average AoIV under various system parameters. Simulation results are obtained by averaging over 10^7 time slots, and the initial values for the state of the source and the reconstructed source are set to $X(1)=0$ and $\hat{X}(1)=0$, respectively. Fig. 2 illustrate the minimum average VIA under time-averaged sampling cost and reconstruction error constraints for $\eta=0.5$ and $E_{\max}=0.5$, considering various values of p, q, and p_s. As seen in this figure, with both low and high success probabilities, the optimal RSC outperforms the change-aware policy in scenarios where the source changes slowly and rapidly. In contrast, the change-aware policy exhibits superior performance for a moderately changing source. This is because when the source changes slowly, the change-aware policy takes fewer sampling and transmission actions, resulting in a higher average VIA. In contrast, when the source changes rapidly, the change-aware policy generates more samples, violating the imposed constraint, and in that case, the optimal RSC performs better. Moreover, according to Remark 1, the RS policy achieves a lower average VIA compared to the change-aware policy when $\frac{2pq}{p+q+\left(2pq-p-q\right)p_s}\leqslant p_{\alpha^s}<1$. However, based on the constraint given in (21b), the maximum sampling probability is limited to η. Consequently, for values of p and q where $\eta<\frac{2pq}{p+q+\left(2pq-p-q\right)p_s}$, the change-aware policy outperforms the optimal RSC policy in terms of average VIA. Figs. 3 and 4 illustrate the minimum average AoIV under a time-averaged sampling cost constraint as a function of p for



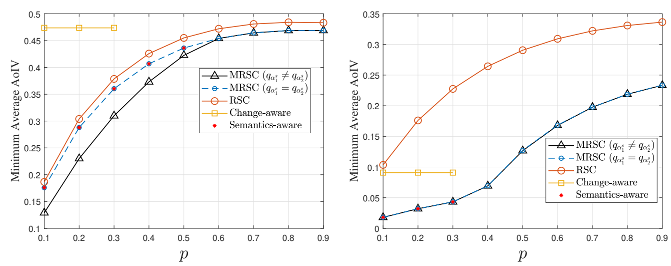
(a) $p_s = 0.3$ (b) $p_s = 0.7$

Fig. 2. Minimum Average VIA in a constrained optimization problem as a function of p and q.



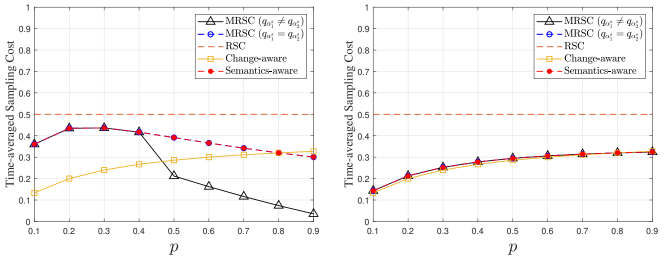
(a) $p_s = 0.1$ (b) $p_s = 0.9$

Fig. 3. Minimum Average AoIV as a function of p for $q = 0.2$ and $\eta = 0.5$.



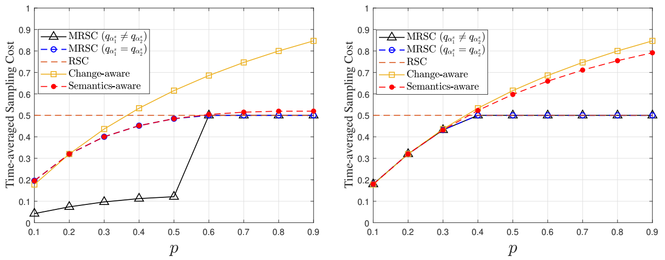
(a) $p_s = 0.1$ (b) $p_s = 0.9$

Fig. 4. Minimum Average AoIV as a function of p for $q = 0.8$ and $\eta = 0.5$.



(a) $p_s = 0.1$ (b) $p_s = 0.9$

Fig. 5. Time-averaged sampling cost as a function of p for $q = 0.2$ and $\eta = 0.5$.



(a) $p_s = 0.1$ (b) $p_s = 0.9$

Fig. 6. Time-averaged sampling cost as a function of p for $q = 0.8$ and $\eta = 0.5$.

$\eta = 0.5$, and selected values of p_s and q. We observe that the optimal MRSC, regardless of whether $q_{\alpha_1^s}$ is equal to $q_{\alpha_2^s}$ or not, outperforms all other policies across scenarios of slow, moderate, and rapid source state changes. The reason behind this is that in the MRSC policy, at time slot t, we perform sampling *probabilistically*, only when $X(t) \neq \hat{X}(t-1)$. Therefore, this policy requires fewer sampling actions than the change-aware and semantics-aware policies, which in turn can violate the imposed constraint for rapidly changing sources. Furthermore, in the optimal MRSC policy, we can perform sampling with a higher probability compared to the optimal RSC policy while incurring either fewer or, in the worst case, the same time-averaged sampling cost for a rapidly changing source, as shown in Figs. 5 and 6. Therefore, the optimal MRSC has a lower average AoIV than the optimal RSC policy. Moreover, for the MRSC, varying the sampling probabilities and considering a higher value for $q_{\alpha_2^s}$ and a lower value for $q_{\alpha_1^s}$ results in lower average AoIV compared to when the same sampling probabilities are considered, particularly for a lower success probability and when the source remains in one state with high probability. This is because, when the total value of time-averaged sampling cost is limited, refraining from sampling when the system is in an erroneous state is more detrimental than when the system is in sync and no sampling is performed. Therefore, it is reasonable to consider $q_{\alpha_2^s} \geqslant q_{\alpha_1^s}$.

VI. Conclusion

We studied a time-slotted communication system where a sampler performs sampling, and the transmitter forwards the samples over an unreliable wireless channel to a receiver that monitors the evolution of a two-state DTMC. We introduced a modified randomized stationary sampling and transmission policy. We then proposed two new metrics, namely VIA and AoIV, and we analyzed the average system performance regarding these metrics for the proposed modified randomized stationary, as well as for randomized stationary, change-aware, and semantics-aware policies. Furthermore, two cost-constrained optimization problems were formulated to find the optimal randomized stationary and modified randomized stationary policies. Our results showed that the optimal randomized stationary policy outperforms the change-aware policy for slowly and rapidly evolving sources in terms of the average VIA. However, the change-aware policy could perform better for moderately changing sources under certain conditions. Furthermore, regarding the average AoIV, the optimal modified randomized stationary policy outperforms all other policies across varying states of the source and the channel. This superior performance holds irrespective of whether the source exhibits slow, moderate, or rapid changes and regardless of the channel quality.

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Appendix

A. Proof of Lemma 1

To obtain $\pi_{j,i}$, we represent the two-dimensional DTMC describing the joint status of the original source regarding the current state of the VIA, i.e., $\big(X(t), \mathrm{VIA}(t)\big)$, where the transition probabilities $P_{i,j/m,n} = \Pr\big[X(t+1)=m, \mathrm{VIA}(t+1) = n \big| X(t) = i, \mathrm{VIA}(t) = j\big]$, $\forall i, m \in \{0, 1\}$ and $\forall j, n \in \{0, 1, \cdots\}$ are given by

$$
\begin{aligned}
&P_{0,0/0,0} = 1 - p, \qquad P_{0,j/0,0} = (1-p)p_{\alpha^s}p_s,\\
&P_{0,j/1,0} = pp_{\alpha^s}p_s, \qquad P_{0,j/0,j+1} = 0,\\
&P_{0,j/1,j+1} = p(1-p_{\alpha^s}p_s), \; P_{0,j/0,j} = (1-p)(1-p_{\alpha^s}p_s),\\
&P_{1,0/1,0} = 1 - q, \qquad P_{1,j/1,0} = (1-q)p_{\alpha^s}p_s,\\
&P_{1,j/1,j+1} = 0, \qquad P_{1,j/0,0} = qp_{\alpha^s}p_s,\\
&P_{1,j/0,j+1} = q(1-p_{\alpha^s}p_s), P_{1,j/1,j} = (1-q)(1-p_{\alpha^s}p_s).
\end{aligned} \tag{24}
$$

Now, using (24), one can derive the state stationary distribution $\pi_{j,i}$ $\forall j \in \{0, 1\}$ and $i \geqslant 0$ as follows

$$
\begin{aligned}
\pi_{0,i} &= \frac{p^k q^w p_{\alpha^s}p_s\big(1-p_{\alpha^s}p_s\big)^i}{(p+q)\big(p+(1-p)p_{\alpha^s}p_s\big)^w\big(q+(1-q)p_{\alpha^s}p_s\big)^k}, i = 0, 1, \ldots.\\
\pi_{1,i} &= \frac{p^w q^k p_{\alpha^s}p_s\big(1-p_{\alpha^s}p_s\big)^i}{(p+q)\big(p+(1-p)p_{\alpha^s}p_s\big)^k\big(q+(1-q)p_{\alpha^s}p_s\big)^w}, i = 0, 1, \ldots.
\end{aligned} \tag{25}
$$

where k and w are given by

$$
k = \begin{cases} \frac{i}{2}, & \mod\{i,2\} = 0,\\ \frac{i+1}{2}, & \mod\{i,2\} \neq 0. \end{cases} \quad w = \begin{cases} \frac{i+2}{2}, & \mod\{i,2\} = 0,\\ \frac{i+1}{2}, & \mod\{i,2\} \neq 0. \end{cases} \tag{26}
$$

Similarly, for the change-aware policy, $\pi_{0,i}$ and $\pi_{1,i}$ in (25) can be written as

$$
\pi_{0,i} = \frac{qp_s(1-p_s)^i}{p+q}, \pi_{1,i} = \frac{pp_s(1-p_s)^i}{p+q}, \quad i = 0, 1, \cdots. \tag{27}
$$

Using (27), we can obtain the average VIA, $\overline{\mathrm{VIA}}$, for the change-aware policy as

$$
\overline{\mathrm{VIA}} = \sum_{i=0}^{\infty} i\Pr[\mathrm{VIA}(t) = i] = \sum_{i=0}^{\infty} i\big(\pi_{0,i} + \pi_{1,i}\big) = \frac{1-p_s}{p_s}. \tag{28}
$$

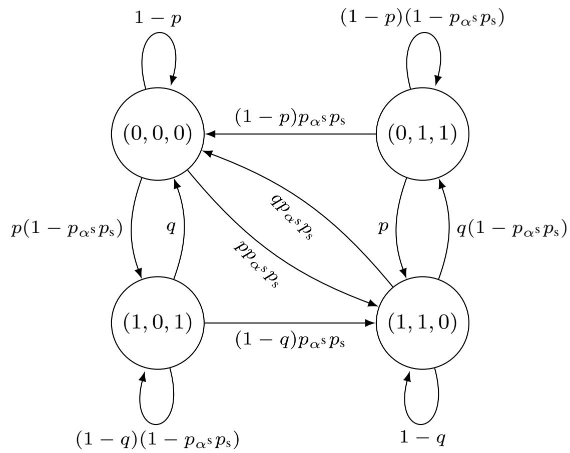

Fig. 7. Three-dimensional DTMC describing the joint status of the original and reconstructed source regarding the current state of the AoIV, i.e., $\big(X(t), \hat{X}(t), \mathrm{AoIV}(t)\big)$.

B. Proof of Lemma 2

To derive $\pi_{i,j,k}, \forall i, j, k \in \{0, 1\}$, we represent the three-dimensional DTMC describing the joint status of the original and reconstructed source regarding the current state of the AoIV, i.e., $\big(X(t), \hat{X}(t), \mathrm{AoIV}(t)\big)$ as Fig. 7. Now, using Fig. 7, we obtain the state stationary $\pi_{i,j,k}$ for the RS policy as

$$
\begin{aligned}
\pi_{0,0,0} &= \Pr\big[X(t) = 0, \hat{X}(t) = 0, \mathrm{AoIV}(t) = 0\big]\\
&= \frac{q\big[q + (1-q)p_{\alpha^s}p_s\big]}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\
\pi_{0,1,1} &= \Pr\big[X(t) = 0, \hat{X}(t) = 1, \mathrm{AoIV}(t) = 1\big]\\
&= \frac{pq\big(1-p_{\alpha^s}p_s\big)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\
\pi_{1,1,0} &= \Pr\big[X(t) = 1, \hat{X}(t) = 1, \mathrm{AoIV}(t) = 0\big]\\
&= \frac{p\big[p + (1-p)p_{\alpha^s}p_s\big]}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\
\pi_{1,0,1} &= \Pr\big[X(t) = 1, \hat{X}(t) = 0, \mathrm{AoIV}(t) = 1\big]\\
&= \frac{pq\big(1-p_{\alpha^s}p_s\big)}{(p+q)\big[p+q+(1-p-q)p_{\alpha^s}p_s\big]},\\
\pi_{0,0,1} &= \pi_{0,1,0} = \pi_{1,0,0} = \pi_{1,1,1} = 0.
\end{aligned} \tag{29}
$$

Similarly, for the MRS policy, we can write (29) as follows

$$
\begin{aligned}
\pi_{0,0,0} &= \frac{q\big[q_{\alpha_2^s} + p\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]\big[q + (1-q)q_{\alpha_2^s}p_s\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},\\
\pi_{0,1,1} &= \frac{pq\big(1-q_{\alpha_1^s}p_s\big)\big[q_{\alpha_2^s} + q\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},\\
\pi_{1,1,0} &= \frac{p\big[q_{\alpha_2^s} + q\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]\big[p + (1-p)q_{\alpha_2^s}p_s\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},\\
\pi_{1,0,1} &= \frac{pq\big(1-q_{\alpha_1^s}p_s\big)\big[q_{\alpha_2^s} + p\big(q_{\alpha_1^s} - q_{\alpha_2^s}\big)\big]}{(p+q)F\big(q_{\alpha_1^s}, q_{\alpha_2^s}\big)},\\
\pi_{0,0,1} &= \pi_{0,1,0} = \pi_{1,0,0} = \pi_{1,1,1} = 0
\end{aligned} \tag{30}
$$

where $F(\cdot,\cdot)$ in (30) is given by

$$F\big(q_{\alpha_1^{\rm s}}, q_{\alpha_2^{\rm s}}\big) = (1-p)(1-q)p_s q_{\alpha_2^{\rm s}}^2 + (p+q-2pq)q_{\alpha_2^{\rm s}} + pq(2-p_s q_{\alpha_1^{\rm s}})q_{\alpha_1^{\rm s}}. \quad (31)$$

Furthermore, for the change-aware policy, one can write (29) as follows

$$\pi_{0,0,0} = \frac{q}{(p+q)(2-p_{\rm s})}, \pi_{0,1,1} = \frac{q(1-p_{\rm s})}{(p+q)(2-p_{\rm s})}, \quad (32a)$$

$$\pi_{1,1,0} = \frac{p}{(p+q)(2-p_{\rm s})}, \pi_{1,0,1} = \frac{p(1-p_{\rm s})}{(p+q)(2-p_{\rm s})}, \quad (32b)$$

$$\pi_{0,0,1} = \pi_{0,1,0} = \pi_{1,0,0} = \pi_{1,1,1} = 0. \quad (32c)$$

Now, using (32), the average AoIV for the change-aware policy is given by

$$\overline{\text{AoIV}} = \sum_{i=1}^{\infty} i\Pr[\text{AoIV}(t)=i] = \pi_{0,1,1} + \pi_{1,0,1} = \frac{1-p_{\rm s}}{2-p_{\rm s}}. \quad (33)$$

Moreover, for the semantics-aware policy, we can obtain (29) as follows

$$\pi_{0,0,0} = \frac{q\big[q+(1-q)p_s\big]}{(p+q)\big[p+q+(1-p-q)p_s\big]}, \quad (34a)$$

$$\pi_{0,1,1} = \frac{pq\big[1-p_s\big]}{(p+q)\big[p+q+(1-p-q)p_s\big]}, \quad (34b)$$

$$\pi_{1,1,0} = \frac{p\big[p+(1-p)p_s\big]}{(p+q)\big[p+q+(1-p-q)p_s\big]}, \quad (34c)$$

$$\pi_{1,0,1} = \frac{pq\big[1-p_s\big]}{(p+q)\big[p+q+(1-p-q)p_s\big]}, \quad (34d)$$

$$\pi_{0,0,1} = \pi_{0,1,0} = \pi_{1,0,0} = \pi_{1,1,1} = 0. \quad (34e)$$

Using (34), the average AoIV for the semantics-aware policy can be written as

$$\overline{\text{AoIV}} = \sum_{i=1}^{\infty} i\Pr[\text{AoIV}(t)=i] = \pi_{0,1,1} + \pi_{1,0,1} = \frac{2pq\big(1-p_s\big)}{(p+q)\big[p+q+(1-p-q)p_s\big]}. \quad (35)$$

C. Proof of Lemma 3

For the MRSC policy, the constraint in (23b) can be written as

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^{\rm s}(t)=1\}\\
&=\delta\Pr\Big[\alpha^{\rm s}(t)=1\Big|X(t)\neq\hat{X}(t-1), X(t-1)=\hat{X}(t-1)\Big]\\
&\times\Pr\Big[X(t)\neq\hat{X}(t-1), X(t-1)=\hat{X}(t-1)\Big]\\
&+\delta\Pr\Big[\alpha^{\rm s}(t)=1\Big|X(t)\neq\hat{X}(t-1), X(t-1)\neq\hat{X}(t-1)\Big]\\
&\times\Pr\Big[X(t)\neq\hat{X}(t-1), X(t-1)\neq\hat{X}(t-1)\Big]\\
&=\delta q_{\alpha_1^{\rm s}}\Pr\Big[X(t)\neq\hat{X}(t-1), X(t-1)=\hat{X}(t-1)\Big]\\
&+\delta q_{\alpha_2^{\rm s}}\Pr\Big[X(t)\neq\hat{X}(t-1), X(t-1)\neq\hat{X}(t-1)\Big]. \quad (36)
\end{aligned}$$

Using the total probability theorem we have

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^{\rm s}(t)=1\}\\
&=\delta q_{\alpha_1^{\rm s}}\Pr\Big[X(t)\neq\hat{X}(t-1)\Big|X(t-1)=\hat{X}(t-1)\Big]\\
&\times\Pr\Big[X(t-1)=\hat{X}(t-1)\Big]\\
&+\delta q_{\alpha_2^{\rm s}}\Pr\Big[X(t)\neq\hat{X}(t-1)\Big|X(t-1)\neq\hat{X}(t-1)\Big]\\
&\times\Pr\Big[X(t-1)\neq\hat{X}(t-1)\Big]\\
&=\delta q_{\alpha_1^{\rm s}}\Big(p\pi_{0,0}^{\rm A}+q\pi_{1,1}^{\rm A}\Big)+\delta q_{\alpha_2^{\rm s}}\Big((1-p)\pi_{0,1}^{\rm A}+(1-q)\pi_{1,0}^{\rm A}\Big) \quad (37)
\end{aligned}$$

where $\pi_{i,j}^{\rm A}, \forall i,j\in\{0,1\}$ in (37) are the probabilities derived from the stationary distribution of the two-dimensional DTMC describing the joint status of the originl source regarding the current state at the reconstructed source, i.e., $\big(X(t),\hat{X}(t)\big)$. Using a procedure similar to the one proposed in Appendix B, we can obtain $\pi_{i,j}^{\rm A}$ as follows

$$\begin{aligned}
\pi_{0,0}^{\rm A} &= \frac{q\big[q_{\alpha_2^{\rm s}}+p\big(q_{\alpha_1^{\rm s}}-q_{\alpha_2^{\rm s}}\big)\big]\big[q+(1-q)q_{\alpha_2^{\rm s}}p_s\big]}{(p+q)F\big(q_{\alpha_1^{\rm s}},q_{\alpha_2^{\rm s}}\big)},\\
\pi_{0,1}^{\rm A} &= \frac{pq\big(1-q_{\alpha_1^{\rm s}}p_s\big)\big[q_{\alpha_2^{\rm s}}+q\big(q_{\alpha_1^{\rm s}}-q_{\alpha_2^{\rm s}}\big)\big]}{(p+q)F\big(q_{\alpha_1^{\rm s}},q_{\alpha_2^{\rm s}}\big)},\\
\pi_{1,0}^{\rm A} &= \frac{pq\big(1-q_{\alpha_1^{\rm s}}p_s\big)\big[q_{\alpha_2^{\rm s}}+p\big(q_{\alpha_1^{\rm s}}-q_{\alpha_2^{\rm s}}\big)\big]}{(p+q)F\big(q_{\alpha_1^{\rm s}},q_{\alpha_2^{\rm s}}\big)}\\
\pi_{1,1}^{\rm A} &= \frac{p\big[q_{\alpha_2^{\rm s}}+q\big(q_{\alpha_1^{\rm s}}-q_{\alpha_2^{\rm s}}\big)\big]\big[p+(1-p)q_{\alpha_2^{\rm s}}p_s\big]}{(p+q)F\big(q_{\alpha_1^{\rm s}},q_{\alpha_2^{\rm s}}\big)} \quad (38)
\end{aligned}$$

where $F(\cdot,\cdot)$ given in (38) is obtained in (31). Now, using (38), (37) is given by

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^{\rm s}(t)=1\} = \frac{2pq\delta\Big[q_{\alpha_2^{\rm s}}+p\big(q_{\alpha_1^{\rm s}}-q_{\alpha_2^{\rm s}}\big)\Big]\Big[q_{\alpha_2^{\rm s}}+q\big(q_{\alpha_1^{\rm s}}-q_{\alpha_2^{\rm s}}\big)\Big]}{(p+q)F\big(q_{\alpha_1^{\rm s}},q_{\alpha_2^{\rm s}}\big)} \quad (39)$$

where $F(\cdot,\cdot)$ given in (39) is obtained in (31). Now, assuming $q_{\alpha_1^{\rm s}} = q_{\alpha_2^{\rm s}} = q_{\alpha^{\rm s}}$, we can simplify (39) as follows

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\delta\mathbf{1}\{\alpha^{\rm s}(t)=1\} = \frac{2pq\delta q_{\alpha^{\rm s}}}{(p+q)\big[p+q+(1-p-q)q_{\alpha^{\rm s}}p_s\big]}. \quad (40)$$

Using (40), we can write the constraint of the optimization problem given in (23b) as follows

$$\big[2pq-\eta(p+q)(1-p-q)p_s\big]q_{\alpha^{\rm s}} \leqslant \eta(p+q)^2 \quad (41)$$

where $\eta = \delta_{\max}/\delta$. Now, using (41), and considering that $0\leqslant q_{\alpha^{\rm s}}\leqslant 1$, the optimal value of the sampling probability for the MRSC policy is given by

$$q_{\alpha^{\rm s}}^{*} = \begin{cases} 1, & \eta(p+q)(1-p-q)p_s \geqslant 2pq,\\ \min\Big\{1, \frac{\eta(p+q)^2}{2pq-\eta(p+q)(1-p-q)p_s}\Big\}, & \eta(p+q)(1-p-q)p_s < 2pq. \end{cases}$$

Furthermore, using (90) and (91) in [25], the constraint given in (23b) for the change-aware and semantics-aware policies is simplified as $\frac{2pq\delta}{p+q}$ and $\frac{2pq\delta}{(p+q)\big[p+q+(1-p-q)p_s\big]}$, respectively.