# Version Age of Information Minimization over Fading Broadcast Channels

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***Abstract*—We consider a base station (BS) receiving version update packets from multiple streams and broadcasting them to users using non-orthogonal multiple access (NOMA) over a fading broadcast channel. Sequentially indexed packets arrive randomly in each stream, rendering previous ones obsolete. To measure information freshness, we define the version age of information (VAoI) at a user as the difference in the version index of the latest available packet at the BS and that at the user. Our objective is to minimize a weighted sum of average VAoI across users, subject to an average power constraint at the BS, by optimally scheduling and transmitting update packets from various streams with sufficient power to ensure successful delivery. We consider channel-only stationary randomized policies (CO-SRP), which make transmission decisions solely based on channel power gains. We solve the resulting convex problem and demonstrate that the VAoI achieved with the proposed CO-SRP is within twice the optimal achievable VAoI. Additionally, we obtained a Constrained Markov Decision Process (CMDP)-based solution and analyzed its structural properties. Numerical simulations demonstrate close performance between the CO-SRP and CMDP-based solutions. Time-division multiple-access (TDMA) scheme, allowing transmission to at most one user at a time, matches NOMA's performance under strict power constraints but is outperformed as constraints are relaxed.**

## I. Introduction

In the emerging communication networks, fresh information delivery is of great importance. As such, researchers have made tremendous efforts over the past decade to quantify, analyze, and optimize the freshness of information over networks. Metrics such as the age of information (AoI) and its variants have been considered and optimized in various communication scenarios [1]–[3].

Recently, a metric called Version Age of Information (VAoI), defined as the difference in the version indices of the latest information packets available at the transmitter and receiver, has been considered [4]–[6]. Unlike the AoI, which starts to increase after receiving a packet irrespective of whether or not there is a new packet available at the source, the VAoI increases only when a packet arrives at the transmitter. We consider the optimization of the VAoI metric in a downlink broadcast channel (BC), in which a single base station (BS) transmits information to multiple receivers simultaneously, using non-orthogonal multiple access (NOMA).

In the literature, optimization of AoI and its variants over BCs have been considered under various assumptions [7]–[15]. The works [8]–[12] and [13]–[15] consider optimization of AoI over BCs under NOMA and time-division multiple access (TDMA) schemes, respectively. Communications over noiseless BC [16]–[19] and erasure BC [20] have been considered. The works [11], [13], [14], [20]–[22] consider simple on-off fading channel where a transmitted packet is considered to be successful with some probability and fails otherwise, and [8]–[10], [18] consider a more general fading channel where the channel power gain realizations can take more than two states and the probability of success of a transmission depends on the transmission power. Different levels of knowledge about channel power gains at the BS have been assumed. For instance, [8], [9] consider a BC in which the BS has statistical knowledge of channel state information only, without perfect channel state information at the transmitter (CSIT), and [10], [12], [18], [19] consider AoI minimization with perfect CSIT at the BS. In the above works, AoI minimization is studied using different tools. For instance, [8], [9], [15], [16], [21] adopt the Markov Decision Process (MDP) framework, [12] adopts a Lyapunov optimization based drift-plus-penalty framework. Index-based [13]–[17], [21] and stationary randomized policies [11], [13], [14], [22] have also been considered. The works, such as [23] and [24], analyze the average AoI.

In some of the above works, including [8], [9], both user scheduling and transmit power control are considered, as in the current work. However, our work differs from these, as we consider perfect CSIT, as in [10], [12], [18], [19] and adopt the NOMA scheme. In applying NOMA with perfect CSIT, we employ superposition coding at the transmitter and successive interference cancellation at the receiver, with the decoding order determined by the channel power gains of all users. In this case, the transmit powers that are required to reliably deliver a required number of bits of packets depend on the channel power gain realizations and decoding order. As the channel realizations can change over slots, the decoding order also needs to be changed in every slot. Thus, the power required for the successful delivery of update packets varies in every slot, in turn impacting the streams that should be scheduled for transmission in a slot. In [10], non-stationary channels have been considered, and in [12], only two users are considered, in which the channel power gain of a user is assumed to be always greater than the other, due to which the set of users and their decoding order need not be changed in every slot. The works [18], [19], which consider perfect CSIT, obtain optimal precoding schemes for AoI minimization. Moreover, almost

ISBN 978-3-903176-65-2 © 2024 IFIP

all the above works, including [8], [9], [12], [18], [19], [23], [24] consider the AoI as the metric, unlike in the current work where VAoI is considered as the metric.

In summary, none of the earlier works consider the minimization of VAoI over a BC with perfect CSIT at the BS using NOMA, which we consider in the current work. The results from earlier works, even those that consider NOMA with perfect CSIT, cannot be generalized or specialized to obtain results for the considered problem. Our main contributions are as follows:

- To minimize a long-term expected average weighted sum of VAoI across users subject to an average transmit power constraint, we consider a class of channel-only stationary randomized policies (CO-SRP), in which the transmission decisions are made based only on the channel power gain realizations. We construct the induced Markov chain and derive its stationary distribution to obtain the optimal CO-SRP. Using this, we express the average VAoI and average power under the CO-SRP and reformulate the original problem as a convex optimization problem and solve it using standard techniques. We show that the VAoI achieved by the CO-SRP is within twice the optimal VAoI.
- By considering the VAoI and channel power gains of all the users as a state, we obtain a constrained MDP (CMDP) based solution by Lagrangian relaxation. We also obtain the structural properties of the solution to the Lagrangian relaxed problem.
- Through numerical simulations, we show that our proposed CO-SRP solution performs competitively with the CMDP-based approach while being significantly simpler. Furthermore, a TDMA scheme allows transmission to only one user at a time and achieves comparable performance to NOMA when the bound on the average power constraint is low. However, as the bound is increased, NOMA outperforms TDMA.

## II. System Model

We consider a BS that receives status update packets from $N$ streams to be communicated to $N$ destination users. The BS consists of a transmitter and $N$ *single-packet* queues, each for storing the latest packet of a user, where a queue can store at most one packet. Time is slotted with slot index $t \in \{1, 2, ...\}$. Next, we describe the packet arrival model, channel model, and the NOMA strategy considered, followed by a description of the performance metric adopted and the problem formulation.

### A. Packet Arrival Model

At the start of every slot $t$, a new packet from stream $i \in \mathcal{N} \triangleq \{1, 2, ..., N\}$ arrives with a certain probability, the packet needs to be delivered to User $i \in \mathcal{N}$. Let $A_i(t) \in \{0, 1\}$ be the indicator function that is equal to 1 when a packet from the $i^{\text{th}}$ stream arrives in slot $t$, and $A_i(t) = 0$ otherwise. The packet arrival follows a Bernoulli process that is independent and identically distributed (i.i.d.) across time, with $\mathbb{P}(A_i(t) = 1) = \lambda_i, \ \forall i \in \mathcal{N}$ and $t \in \{1, 2, \ldots\}$. The latest packet from the $i^{\text{th}}$ stream is stored in Queue $i$. We assume that each packet arriving at the $i^{\text{th}}$ stream contains $R_i^0$ bits, that is, the packet length is $R_i^0$.

### B. Channel Model and NOMA Strategy

We consider a block fading model where the channel power gain remains constant over a block (coherence duration) and varies independently across blocks [25, Section 5.4.5]. A version update packet is coded over a single block, where the channel remains constant, that is, the codeword length is equal to a slot/coherence duration, implying slow fading.

Let $H_i(t) \in \mathcal{H}$ be the channel power gain between the BS and User $i$ in slot $t$ for all $i \in \mathcal{N}$. We consider that $H_i(1), H_i(2) \ldots$, are i.i.d. for each User $i$ and that $\mathcal{H}$ is a finite set with positive elements. Let $u_i(t) \in \{0, 1\}$ be the indicator function, which equals 1 when the BS transmits a packet from the $i^{\text{th}}$ stream during slot $t$, and $u_i(t) = 0$ otherwise. For transmission, the BS adopts superposition coding, which allocates power $P_i(t)$ to the message corresponding to User $i$ in time slot $t$, encodes and superimposes the codewords, and transmits them over the fading BC. Each user employs SIC to decode its message from the received symbols. Under this strategy, we describe the relationship between transmit powers and achievable rates next.

Let $O_1(t)$ be the index of the user with the highest channel power gain, $O_2(t)$ be the index of the user with the next highest channel power gain, and so on, in slot $t$. That is, $h_{O_1}(t) \geq h_{O_2}(t) \geq \ldots h_{O_N}(t)$ is satisfied.[1] As mentioned, the users adopt SIC for decoding, where the user with index $O_N(t)$ decodes its message by considering codewords from users $O_{N-1}(t), \ldots, O_1(t)$ as noise, the User $O_{N-1}(t)$ first decodes the message of User $O_N(t)$ (which is possible because the channel for User $O_{N-1}(t)$ is better than that for User $O_N(t)$), subtracts the encoded version of the message from the received symbols and then decodes its message by considering the codewords from users $O_{N-2}(t), \ldots, O_1(t)$ as noise. The users $O_{N-2}(t), O_{N-3}(t), \ldots, O_1(t)$, decode the messages in the similar manner [26], [27].

Under the superposition coding at the BS and SIC at the users, the signal-to-interference-plus-noise ratio, considering unit noise variance, experienced at User $i$ is given by

$$S_i(t) = \frac{P_i(t) h_i(t)}{1 + h_i(t) \sum_{k < O_i^{-1}(t)} P_{O_k}(t)},$$

where $h_i(t)$ is the realization of the random channel power gain $H_i(t)$, and $O_i^{-1}(t)$ is the position of User $i$ when the users are arranged according to decreasing values of their channel power gains. In this case, we consider that the maximum achievable rate for User $i$ is given by $f(S_i(t))$, where $f(\cdot)$ is a concave, non-decreasing function with $f(0) = 0$ [27].

In the above, when transmitting a packet for User $i$, it is optimal to transmit a minimum of $R_i^0$ bits of the packet; otherwise, the update will not be received successfully. Hence,

[1] Multiple orderings are possible if channel power gain realizations of more than one user are same. In such cases, we consider the ordering that places the user with a lower index before the user with a higher index having the same channel power gain.

the power consumed for transmitting less than $R_i^0$ bits does not result in any reward, or in other words, the reward earned will be the same as when no bits are transmitted. Hence, with the knowledge of the channel power gains of all users, the BS allocates powers such that it delivers all the bits of a packet. Thus, $P_{O_1}, P_{O_2}, \ldots$, are chosen such that

$$f(h_{O_1}P_{O_1}u_{O_1}) = R^0_{O_1}u_{O_1},$$
$$f\left(\frac{h_{O_2}P_{O_2}u_{O_2}}{1+h_{O_2}P_{O_1}u_{O_1}}\right) = R^0_{O_2}u_{O_2},$$
$$\vdots$$
$$f\left(\frac{h_{O_i}P_{O_i}u_{O_i}}{1+h_{O_i}\sum_{k=1}^{i-1}P_{O_k}u_{O_k}}\right) = R^0_{O_i}u_{O_i},$$

and so on until $i = N$. From the above, we obtain

$$P_{O_i}u_{O_i} = \frac{f^{-1}\left(R^0_{O_i}u_{O_i}\right)}{h_{O_i}} + f^{-1}\left(R^0_{O_i}u_{O_i}\right)\sum_{k=1}^{i-1}P_{O_k}u_{O_k},$$

for all $i = 1, 2, \ldots, N$. (1)

### C. Performance Metric and Problem Formulation

Let $z_i(t)$ denote the version of the packet in the $i^{\text{th}}$ queue at the beginning of slot $t$. Recall that $A_i(t)$ is the indicator variable that indicates the arrival of a new packet from the $i^{\text{th}}$ stream in slot $t$. Then, we have

$$z_i(t) = \sum_{\tau=1}^{t} A_i(\tau).$$

$z_i(t)$ value changes only when a new packet arrives in the queue. Let $y_i(t)$ be the version of the most recently received packet at Destination $i$ at time $t$. Then, the instantaneous VAoI is defined as $\Delta_i(t) = z_i(t) - y_i(t)$. We measure instantaneous VAoI at the end of a slot.

In this work, we are interested in obtaining transmission scheduling policy, $\pi$, which gives the rule for obtaining the sequence of decisions $\{u_i(t)\}_{i=1}^N$ at the BS, for minimization of the long-term expected average VAoI subject an average power constraint. For given weights $w_1, \ldots, w_N$, this can be accomplished by solving the following optimization problem

$$V_{\text{opt}} = \min_{\pi} \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} w_i \mathbb{E}[\Delta_i(t)], \tag{2a}$$

$$\text{subject to} \quad \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} \mathbb{E}[P_i(t)u_i(t)] \le \bar{P}, \tag{2b}$$

Note that for any choice of $\{u_i(t)\}_{i=1}^N$, we can obtain the transmit powers required for ensuring all the bits of a packet are delivered from (1) for any $t$.

In this work, we also consider a TDMA scheme, where, at most, one user can transmit in a given time slot. Specifically, we derive the TDMA scheme-based solution by solving (2) with the additional constraint:

$$\sum_{i=1}^{N} u_i(t) \le 1, \forall t \in \{1, 2, \ldots, T\}. \tag{3}$$

Next, we obtain solutions to (2).

## III. Optimization Analysis

In this section, we first obtain a policy among the class of channel-only stationary randomized policies (CO-SRP) to solve (2). We also solve (2) using the CMDP framework.

### A. Channel-Only Stationary Randomized Policy (CO-SRP)

Let $2^{\mathcal{N}}$ be the power set of $\mathcal{N}$ and $\mathbf{h} \triangleq (h_1, h_2, \ldots, h_N)$. Our policy is:

*CO-SRP: In a slot, when the channel power gain is $\mathbf{h}$, the BS transmits to users in $\mathcal{W} \in 2^{\mathcal{N}}$ with probability $\mu_{\mathbf{h}}^{\mathcal{W}}$.*

The policy does not require the BS to have knowledge of the instantaneous VAoIs and queue backlogs (queue status) of the users for making transmission decisions, so its execution becomes simple. Under the policy, the conditional probability of successful delivery of a packet to User $i$, conditioned on $\mathbf{h}$, is $\sum_{\mathcal{W}\in 2^{\mathcal{N}}:\mathcal{W}\cap\{i\}\neq\emptyset}\mu_{\mathbf{h}}^{\mathcal{W}}$, where $\emptyset$ is the null set. Hence, the probability of successfully delivering packets to User $i$, $p_i^{\boldsymbol{\mu}} = \sum_{\mathcal{W}\in 2^{\mathcal{N}}:\mathcal{W}\cap\{i\}\neq\emptyset}\mathbb{E}_{\mathbf{H}}[\mu_{\mathbf{h}}^{\mathcal{W}}] = \sum_{\mathbf{h}\in\mathcal{H}^N}\sum_{\mathcal{W}\in 2^{\mathcal{N}}:\mathcal{W}\cap\{i\}\neq\emptyset}\mu_{\mathbf{h}}^{\mathcal{W}}\,\mathbb{P}(\mathbf{h})$, where $\mathbb{P}(\mathbf{h})$ is the probability of channel being $\mathbf{h}$ and $\boldsymbol{\mu} = \{\mu_{\mathbf{h}}^{\mathcal{W}} |\ \forall\mathcal{W}\in 2^{\mathcal{N}}, \forall\mathbf{h}\in\mathcal{H}^N\}$. When the information is transmitted to users, the transmit power allocated for codeword corresponding to User $i \in \mathcal{W}$ when the channel is $\mathbf{h}$, which we denote $P_{i,\mathbf{h}}^{\mathcal{W}}$ can be computed via (1) and the expected power consumed for User $i$ will be equal to $P_i^{\boldsymbol{\mu}} = \sum_{\mathcal{W}\in 2^{\mathcal{N}}:\mathcal{W}\cap\{i\}\neq\emptyset}\mathbb{E}_{\mathbf{H}}[\mu_{\mathbf{h}}^{\mathcal{W}}P_{i,\mathbf{h}}^{\mathcal{W}}]$.

Under this policy, the VAoI evolves as a Markov chain shown in Fig. 1. Below, we derive the long-term expected average VAoI (objective function of (2)) under the policy by obtaining the stationary distribution of the Markov chain, reformulate (2) under the CO-SRP, which turns out to be a convex optimization problem, and obtain a bound on its performance.

**Theorem 1.** *The long-term expected average VAoI under the CO-SRP are given by*

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[\Delta_i(t)] = \frac{\lambda_i(1-p_i^{\boldsymbol{\mu}})}{p_i^{\boldsymbol{\mu}}}.$$

*Proof.* Define $\pi_{n,i} \triangleq \mathbb{P}(\Delta_i = n)$ for all $i \in \mathcal{N}$, $n \in \{0, 1, \ldots\}$. Then, we have

$$\pi_{0,i} = \pi_{0,i}\ (1-\lambda_i(1-p_i^{\boldsymbol{\mu}})) + p_i^{\boldsymbol{\mu}}\sum_{m=1}^{\infty}\pi_{m,i}$$
$$\pi_{n,i} = \pi_{n-1,i}\ \lambda_i(1-p_i^{\boldsymbol{\mu}}) + \pi_{n,i}\ (1-\lambda_i)(1-p_i^{\boldsymbol{\mu}}),$$

for $n \in \{1, 2, \ldots\}$. Since $\sum_{n=0}^{\infty}\pi_{n,i} = 1$, we get,

$$\pi_{n,i} = \frac{p_i^{\boldsymbol{\mu}}}{\lambda_i(1-p_i^{\boldsymbol{\mu}})+p_i^{\boldsymbol{\mu}}}\left(\frac{\lambda_i(1-p_i^{\boldsymbol{\mu}})}{\lambda_i(1-p_i^{\boldsymbol{\mu}})+p_i^{\boldsymbol{\mu}}}\right)^n,$$

for $n \in \{0, 1, \ldots\}$. Thus, the expected VAoI is

$$\sum_{n=0}^{\infty} n\mathbb{P}(\Delta_i = n) = \sum_{n=0}^{\infty} n\pi_{n,i} = \frac{\lambda_i(1-p_i^{\boldsymbol{\mu}})}{p_i^{\boldsymbol{\mu}}}. \tag{4}$$

$\square$

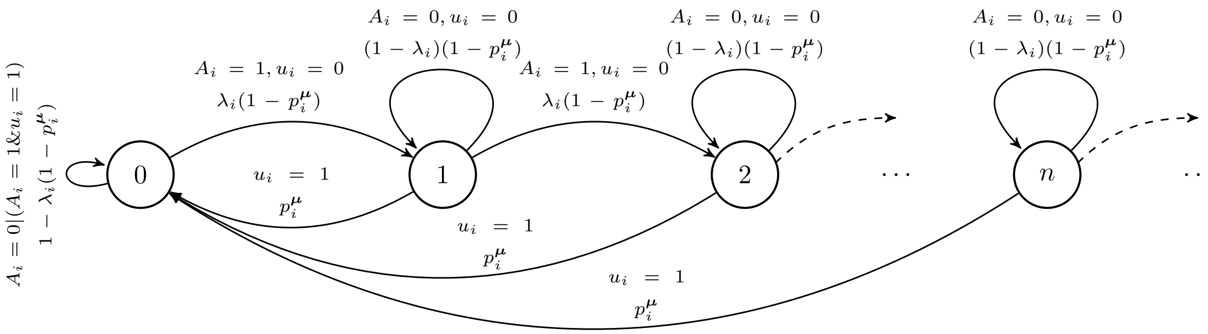


Fig. 1: VAoI evolution for the CO-SRP. The expressions closer to the arrows represent the state transition probability; the other expressions above are the events that cause the transitions.

Similarly, using $P_i^{\boldsymbol{\mu}}$, the expected average transmit power consumed by BS for under the CO-SRP is expressed as

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\mathbb{E}[P_i(t)] = \sum_{i=1}^{N} P_i^{\boldsymbol{\mu}}. \tag{5}$$

Using Theorem 1, and (5), the optimization problem in (2) can be reformulated as

$$V_{\text{SRP}} = \min_{0\le\mu_{\mathbf{h}}^{\mathcal{W}}\le 1} \sum_{i=1}^{N} w_i\lambda_i\left(\frac{1}{p_i^{\boldsymbol{\mu}}}-1\right), \tag{6a}$$

$$\text{subject to } \sum_{i=1}^{N} P_i^{\boldsymbol{\mu}} \le \bar{P}, \tag{6b}$$

$$\sum_{\mathcal{W}\in 2^{\mathcal{N}}} \mu_{\mathbf{h}}^{\mathcal{W}} = 1,\ \forall \mathbf{h}\in\mathcal{H}^N, \tag{6c}$$

where, both $p_i^{\boldsymbol{\mu}}$ and $P_i^{\boldsymbol{\mu}}$ are affine function of $\mu_{\mathbf{h}}^{\mathcal{W}}$ (for given $\mathcal{W}$ and $\mathbf{h}$, $P_{i,\mathbf{h}}^{\mathcal{W}}$ values which can be computed from (1), are constants) and take only positive values. Since the composition of a convex non-increasing function with a concave function gives a convex function [28], $1/p_i^{\boldsymbol{\mu}}$ is convex. Hence, (6) is a convex optimization problem that can be solved via standard numerical techniques.

As mentioned, we are also interested in obtaining the CO-SRP under the TDMA scheme. Under the TDMA scheme, the CO-SRP can be expressed as follows: In a slot, when the channel power gain is $\mathbf{h}$, the BS transmits to User $i\in\mathcal{N}$ with probability $\mu_{\mathbf{h}}^{i}$. In this case, the equivalent of (6) under the TDMA scheme can be obtained by constraining $\mathcal{W}$ to take the values in $\{\emptyset,\{1\},\{2\},\dots,\{N\}\}$, instead of all the values in the power set $2^{\mathcal{N}}$.

*1) Performance Bound:* We show in the following that the performance of the CO-SRP remains within a bounded multiplicative gap from that of the optimal achievable average VAoI as the solution to (2).

**Theorem 2.** *The VAoI of the CO-SRP,* $V_{\text{SRP}}$ *in* (6) *is less than or equal to twice the optimal VAoI,* $V_{\text{opt}}$ *in* (2)*, i.e.,* $V_{\text{SRP}} \le 2V_{\text{opt}}$.

*Proof.* The proof consists of two parts. First, we obtain a lowerbound for the objective function of (2) using an optimization problem. Second, we compare this lowerbound with the objective function of the CO-SRP to prove the theorem. See Appendix A in [29] for details. $\square$

### *B. Constrained MDP (CMDP) Based Solution*

In this section, we cast the problem (2) under the CMDP framework. We then relax it using the Lagrangian approach and obtain the optimal solution for the relaxed problem. The parameters and decision variables of a CMDP associated with (2) are as follows: Let $\mathbf{s}\triangleq(\boldsymbol{\Delta},\mathbf{h})$ be the state of the system, where $\boldsymbol{\Delta}\triangleq(\Delta_1,\dots,\Delta_N)$ and $\mathbf{h}\triangleq(h_1,\dots,h_N)$ are the set of VAoIs and set of channel power gain realizations of all users in time slot $t$, respectively. The set of actions for all users at a time step is denoted as $\mathbf{u}\triangleq(u_1,\dots,u_N)$, where $u_i=0$ indicates that the packet is not transmitted from Queue $i$ and $u_i=1$ indicates that the packet is successfully transmitted from Queue $i$, which requires sufficient power to be allocated to deliver $R_i^0$ number of bits. We can compute the probability transition matrix based on the packet arrival and channel power gain distributions for each $\mathbf{s}$ and $\mathbf{u}$. We define a stationary policy, $\pi$, which is a mapping from state space to action space. For a given stationary policy $\pi$, the long-term expected average weighted sum of VAoI and the expected power consumption cost will be:

$$V_\pi = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} w_i\mathbb{E}_\pi[\Delta_i(t)], \quad \text{and}$$

$$P_\pi = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N}\mathbb{E}_\pi[P_i(t)u_i(t)],$$

respectively. Now, (2) can be rewritten as the CMDP problem:

$$V_{\text{opt}} = \min_{\pi}\ V_\pi, \tag{7a}$$

$$\text{subject to } P_\pi \le \bar{P}. \tag{7b}$$

To obtain the solution for (7), we transform it into an unconstrained MDP problem using the Lagrangian relaxation method, which is discussed below.

*1) Lagrange Relaxation of the CMDP:* We now relax (7) to an unconstrained MDP problem using Lagrangian relaxation. The average Lagrangian cost function is defined as

$$\begin{aligned}\mathcal{L}_\pi(\theta) &= V_\pi + \theta P_\pi \\ &= \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\sum_{i=1}^{N} \mathbb{E}_\pi[w_i\Delta_i(t) + \theta u_i(t)P_i(t)],\end{aligned}$$

where $\theta$ is the Lagrange multiplier and the random instantaneous cost is $C(\mathbf{s},\mathbf{u},\theta) = \sum_{i=1}^{N}(w_i\Delta_i + \theta u_i P_i)$. Now, (7) can be transformed as an unconstrained problem,

$$\mathcal{L}_\pi^{\mathrm{opt}}(\theta) = \min_{\pi\in\Pi} \mathcal{L}_\pi(\theta), \tag{8}$$

where $\mathcal{L}_\pi^{\mathrm{opt}}(\theta)$ is the minimum average Lagrangian cost obtained by the optimal policy $\pi_\theta^{\mathrm{opt}}$ for a given $\theta$. We first obtain the optimal policy $\pi_\theta^{\mathrm{opt}}$ of the unconstrained MDP problem by solving the Bellman equation.

In the optimal case, the solution to (8) will result in a bounded VAoI and bounded instantaneous cost for each User $i \in \mathcal{N}$. This is because if the VAoI of User $i$ becomes greater than $\theta P_i^{\max}$, where $P_i^{\max}$ is the worst case maximum power required to deliver $R_i^0$ bits of User $i$, we can always transmit the packet (since the VAoI is non-zero, there will be a packet for transmission) and reduce the overall cost. Hence, in our case, the MDP can be considered to have finite state and action spaces with a bounded instantaneous cost. To obtain our results, we consider the following discounted total cost criterion

$$V_\gamma^\theta(\mathbf{s}) = \min_\pi \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\gamma^{t-1}\sum_{i=1}^{N}\mathbb{E}_\pi[w_i\Delta_i(t) + \theta u_i(t)P_i(t)],$$

where $0<\gamma<1$ is the discount factor. It can be shown that $V_\gamma^\theta(\mathbf{s})$ satisfies

$$V_\gamma^\theta(\mathbf{s}) = \min_{\mathbf{u}} Q_\gamma(\mathbf{s},\mathbf{u},\theta), \tag{10}$$

where $Q_\gamma(\mathbf{s},\mathbf{u},\theta) = \mathbb{E}[C(\mathbf{s},\mathbf{u},\theta) + \gamma V_\gamma^\theta(\mathbf{s}')|\mathbf{s},\mathbf{u}]$, where $\mathbf{s}'$ is the next state reached from state $\mathbf{s}$, when action $\mathbf{u}$ is taken, as in Proposition 5 of [16]. Further, we define,

$$V_{\gamma,m+1}^\theta(\mathbf{s}) = \min_{\mathbf{u}} \mathbb{E}[C(\mathbf{s},\mathbf{u},\theta) + \gamma V_{\gamma,m}^\theta(\mathbf{s}')|\mathbf{s},\mathbf{u}], \tag{11}$$

for all $m \in \{0,1,2,\ldots\}$ with $V_{\gamma,0}^\theta(\mathbf{s}) = 0$ for all $\mathbf{s}$. It can be shown that $\lim_{m\to\infty} V_{\gamma,m}^\theta(\mathbf{s}) = V_\gamma^\theta(\mathbf{s})$ for each $\mathbf{s}$ and $\gamma \in (0,1)$ and that $\lim_{\gamma\to 1} V_\gamma^\theta(s)$ will give the optimal objective value to (8).

Let $\boldsymbol{x}_{-i} \triangleq (x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_N)$. Below, we show the optimal policy obtained by the MDP is of threshold-type.

**Theorem 3.** *For any $i \in \{1,2,\ldots,N\}$, suppose $\boldsymbol{\Delta}_{-i}$, $\mathbf{h}$ and $\mathbf{u}_{-i}$ are fixed. If it is optimal to transmit $i^{\text{th}}$ stream when its VAoI is $\Delta_i$, then it must be optimal to transmit $i^{\text{th}}$ stream when its VAoI is $\Delta_i+1$. Similarly, for any $i \in \{1,2,\ldots,N\}$, suppose $\boldsymbol{\Delta}$, $\mathbf{h}_{-i}$ and $\mathbf{u}_{-i}$ are fixed, if it is optimal to transmit $i^{\text{th}}$ stream when the channel for User $i$ is $h_i$, then it must be optimal to transmit $i^{\text{th}}$ stream, when the channel is greater than $h_i$.*

*Proof.* See Appendix C in [29]. □

Due to the above theorem, the complexity of solving and executing the considered problem using the MDP-based approach reduces.

*2) Bisection Search for $\theta$:* Given the optimal policy $\pi_\theta^{\mathrm{opt}}$ for (8) for a given $\theta$, the objective function of the CMDP problem (7) increases with $\theta$, and $P_{\pi_\theta^{\mathrm{opt}}}$ in (7b) decreases with $\theta$. To determine a suitable value of $\theta$ that makes the result of the unconstrained problem to satisfy the constraint (7b) with equality, we employ the bisection algorithm.

## IV. Numerical Results

In this section, we present numerical results for the two-user case. As mentioned, while our theoretical results apply to a more general concave non-decreasing function with $f(0)=0$, we use $f(x) = \log(1+x)$ for concreteness in our simulations, as in [27].

In Fig. 2, we show the variation of the long-term expected average VAoI as a function of the average transmit power, $\bar{P}$, for NOMA and TDMA schemes under CO-SRP and CMDP-based policy. Firstly, we observe that as $\bar{P}$ increases, the average VAoI decreases. This is because more and more of the arriving packets get delivered as soon as they arrive. Interestingly, at lower powers, NOMA and TDMA perform the same. NOMA starts performing better than TDMA when power increases. This is because, even in NOMA, in the optimal case, packets are not transmitted to both users simultaneously at lower powers, thereby making it similar to TDMA. However, at higher powers, it is optimal for NOMA to transmit to both users simultaneously. Secondly, we observe that the average VAoI tends to zero in the NOMA case as $\bar{P}$ increases, but for the TDMA scheme, it reaches a saturation point at a value of $\lambda$. This behavior can be explained as follows. The expression for the long-term expected average weighted sum of VAoI under CO-SRP is $\sum_{i=1}^{N} w_i(\lambda_i(p_i^{-1}-1))$, where $p_i \in [0,1]$ is the probability of successful delivery of a packet to User $i$. For the two-user case, we can rewrite expression as $0.5\lambda(p_1^{-1}+p_2^{-1}-2)$ when $w_1=w_2=0.5$ and $\lambda_1=\lambda_2=\lambda$. With the TDMA scheme, even with infinite available power, the BS can transmit a packet to only one user at any time $t$; this implies $p_2 = (1-p_1)$. When $p_1 \to 1$ and $p_2 \to 0$, the average VAoI for the first user goes to zero, while for the second user, it tends towards $\infty$. So, the optimal value for the probability of successful delivery with infinite $\bar{P}$, by symmetry, is $p_1=p_2=0.5$. From this, we conclude that the minimum achievable average VAoI in the TDMA scheme for the two-user case, at high $\bar{P}$, is $\lambda$. However, in the NOMA scheme, there is no restriction on selecting only one user to transmit at any given time. With higher available power, the BS can simultaneously transmit to multiple users. So, the optimal value for the probability of successful delivery of a packet for User $i$ in the NOMA scheme is $p_i=1$. Using this condition, we conclude that the minimum achievable average VAoI in the NOMA scheme is zero. In the CMDP-based solution, the

TABLE I: Variation of difference of long-term expected average weighted sum of VAoI under CO-SRP with NOMA and TDMA schemes with respect to $H_{\max}$ (maximum value of channel power gain), $\mathbb{P}(H = 0.1)$ and $R^0$ considering $R_1^0 = R_2^0 = R^0$, $H_1 = H_2 = H$, $\mathcal{H} = \{H_{\max}, 0.1\}$, $\lambda_1 = \lambda_2 = 0.9$, and $w_1 = w_2 = 0.5$. We denote $\bar{\Delta}_{\text{TDMA}}$, and $\bar{\Delta}_{\text{NOMA}}$ as the expected average VAoI under TDMA and NOMA schemes respectively.

| $H_{\max}$ | $\mathbb{P}(H = 0.1)$ | $R^0$ | $\bar{P}$ | $\bar{\Delta}_{\text{TDMA}}$ | $\bar{\Delta}_{\text{NOMA}}$ | $\bar{\Delta}_{\text{TDMA}} - \bar{\Delta}_{\text{NOMA}}$ |
|---|---|---|---|---|---|---|
| 1 | 0.9 | 2 | 45 | 1.1563 | 1.1502 | 0.0061 |
| 1 | 0.5 | 2 | 45 | 0.9 | 0.4150 | 0.485 |
| 2 | 0.5 | 2 | 45 | 0.9 | 0.3032 | 0.5968 |
| 2 | 0.5 | 3 | 45 | 0.9981 | 0.9980 | 0 |
| 2 | 0.5 | 3 | 100 | 0.9 | 0.5569 | 0.3431 |

trends observed in the CO-SRP continue to hold. Accordingly, in the rest of the section, we obtain results only for CO-SRP.

We study the variation of the probability of simultaneous transmission to both users, ($\mathbb{E}_{\mathbf{H}}\left[\mu_{\mathbf{h}}^{\mathcal{W}=\{1,2\}}\right]$), in the NOMA scheme as a function of average transmit power, in Fig. 3. From the figure, we observe that at lower $\bar{P}$, the probability of simultaneous transmission to both users is zero, and hence NOMA behaves exactly like TDMA. However, this probability increases as $\bar{P}$ rises. We also examined this behavior under various scenarios involving the probability of encountering a bad channel (channel with lower channel power gain, $H = 0.1$). When $\mathbb{P}(H = 0.1)$ is higher, the power required for transmitting to both users is also high, so NOMA transmits to a single user at any given time with less available power rather than attempting simultaneous transmission to both users. In Fig. 4, we show the variation of the long-term expected average VAoI with respect to the probability of packet arrival, $\lambda$, for both NOMA and TDMA schemes under the CO-SRP. NOMA achieves a lower average VAoI compared to the TDMA scheme. In Fig. 5, we show the largest achievable long-term expected average VAoI region for both TDMA and NOMA schemes. We varied the weights associated with users $(w_1, w_2)$ between 0 and 1, subject to the constraint $w_1 + w_2 = 1$. The figure shows that the achievable region of the NOMA scheme includes that of the TDMA scheme. In TDMA, if the average VAoI of any of one user is low, it results in a significantly higher average VAoI for the other user because TDMA allows the BS to transmit to only one user at a given time.

We also study the variation of the long-term expected average VAoI for TDMA and NOMA schemes with respect to different parameters. The results are summarized in Table I. At higher values of $\bar{P}$, we observe a decrease in both the average VAoI under the TDMA scheme and the NOMA scheme as the probability of bad channel occurrence, $\mathbb{P}(H = 0.1)$, decreases. The average VAoI under NOMA decreases with increasing $H_{\max}$ (the maximum value of channel power gain) and decreasing $R^0$. However, these changes in $H_{\max}$ and $R^0$ do not significantly impact the average VAoI under TDMA at higher values of $\bar{P}$ and $\mathbb{P}(H = 0.1)$.

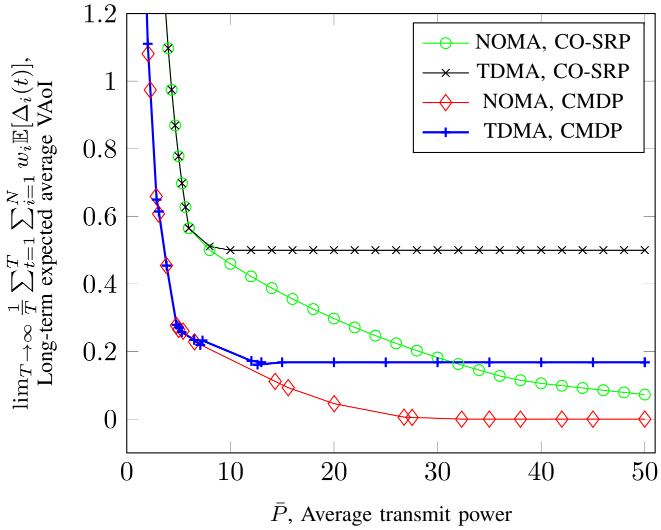


Fig. 2: Variation of the long-term expected average weighted sum of VAoI, with the average transmit power, $\bar{P}$, under the CO-SRP and the CMDP-based policy with NOMA and TDMA schemes. Parameters adopted are $\lambda_1 = \lambda_2 = 0.5$, $R_1^0 = R_2^0 = 2$, $\mathcal{H} = \{1, 0.1\}$, $\mathbb{P}(H_1 = 0.1) = \mathbb{P}(H_2 = 0.1) = 0.2$, and $w_1 = w_2 = 0.5$.

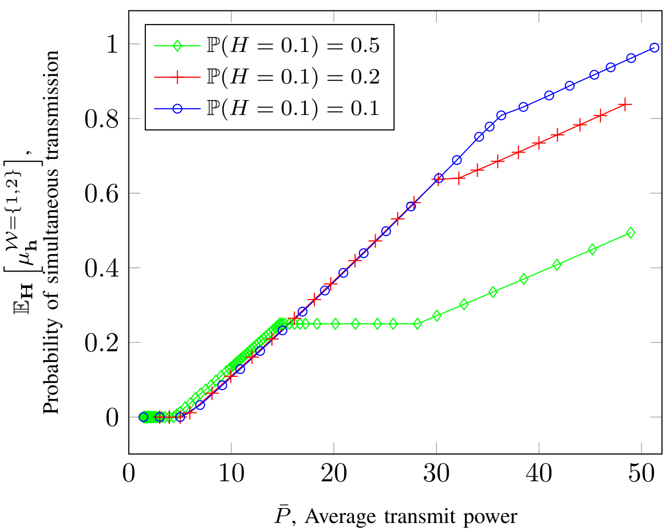


Fig. 3: Probability of simultaneous transmissions to both users versus the average transmit power, $\bar{P}$, under the CO-SRP with NOMA scheme for $R_1^0 = R_2^0 = 2$, $\mathcal{H} = \{1, 0.1\}$, $H_1 = H_2 = H$, $\lambda_1 = \lambda_2 = 0.8$ and $w_1 = w_2 = 0.5$.

## V. Conclusions

We considered the problem of scheduling the transmission of version update packets from exogenous streams to different users via a fading broadcast channel using the NOMA scheme to minimize the long-term expected average weighted sum of VAoI across the users subject an average power constraint at the BS. To address this problem, we derived a simpler channel-only stationary randomized policy by solving a convex optimization problem and obtained its performance bounds. We also obtained a CMDP-based solution and characterized its structural properties. Through numerical simulations, we conclude that our proposed CO-SRP solution achieves competitive performance with the CMDP-based approach while

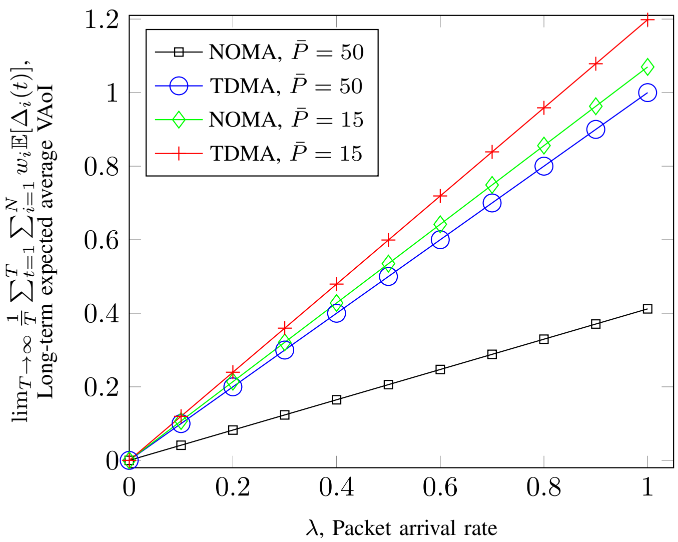


Fig. 4: Variation of the long-term expected average weighted sum of VAoI with respect to probability of packet arrival, $\lambda_1 = \lambda_2 = \lambda$, under the CO-SRP with NOMA and TDMA schemes. Parameters are set to $R_1^0 = R_2^0 = 2$, $\mathcal{H} = \{1, 0.1\}$, $\mathbb{P}(H_1 = 0.1) = \mathbb{P}(H_2 = 0.1) = 0.2$, $w_1 = w_2 = 0.5$.

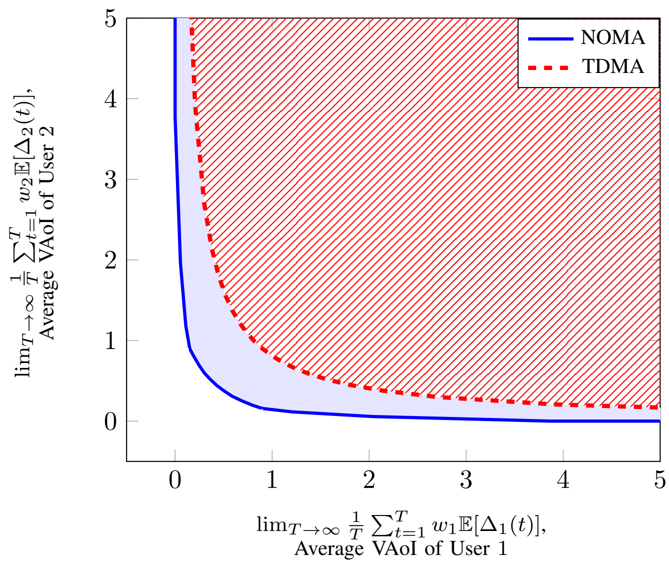


Fig. 5: The largest achievable long-term expected average VAoI region by NOMA and TDMA schemes under CO-SRP by varying the weights associated with users $(w_1, w_2)$. Parameters adopted are $\bar{P} = 40$, $\lambda_1 = \lambda_2 = 0.9$, $R_1^0 = R_2^0 = 2$ and $\mathcal{H} = \{1, 0.1\}$, and $\mathbb{P}(H_1 = 0.1) = \mathbb{P}(H_2 = 0.1) = 0.5$.

being significantly simpler, and the VAoI achieved by CO-SRP is within twice the optimal VAoI. Additionally, based on our comparison of NOMA to a TDMA scheme, we conclude that NOMA outperforms TDMA when bound on the power consumption is high. However, NOMA and TDMA exhibit similar performance when this bound low.

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