Spectral Co-Clustering Based Wireless Network Decomposition for Resource Scheduling

Yiyu Liu, Yilin Xiao, Ming Tang, Lin Gao, Jianwei Huang

***Abstract*—Large-scale wireless networks pose significant challenges in resource scheduling, where the solution space grows exponentially with network size. While network decomposition offers a promising solution by breaking networks into manageable subnetworks, existing approaches, including spectral clustering, fail to effectively capture the complex service relationships between base stations (BSs) and users, particularly in networks with massive user populations. This paper presents BSCCD (Bidirectional Spectral Co-Clustering Based Decomposition), a new decomposition scheme that addresses these challenges through two key innovations: (i) a two-round spectral co-clustering framework that captures bidirectional BS-user relationships, and (ii) a user node merging strategy that handles massive user populations. Extensive experiments on real-world datasets from multiple Chinese cities demonstrate that BSCCD reduces computation latency by up to 61.91% compared to global optimization, while achieving more than 10% improvement in solution quality over traditional clustering approaches. The advantage is particularly pronounced in medium-scale networks, where BSCCD outperforms traditional methods by 28.76%. Our results demonstrate BSCCD's practical viability for resource scheduling in contemporary wireless networks, especially in scenarios with complex BS-user interactions and large user populations.**

***Index Terms*—Network decomposition, spectral co-clustering, network resource scheduling problem.**

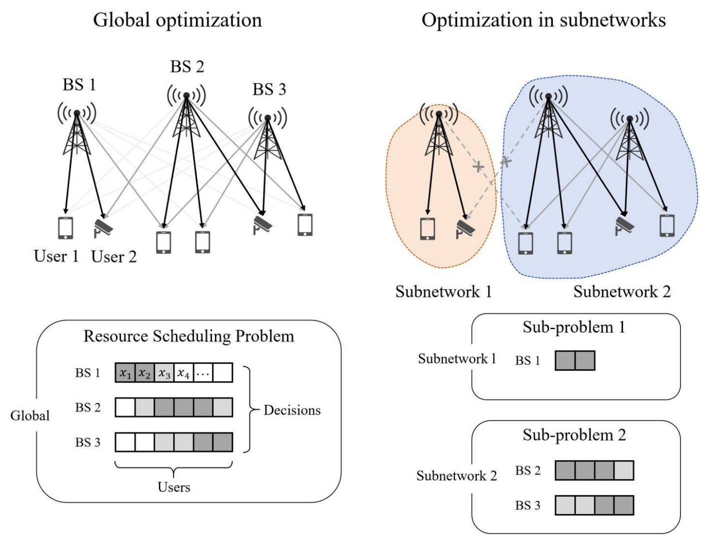

Fig. 1. An illustration of network decomposition, where a network is decomposed into multiple subnetworks. Due to the reduced network size, scheduling within the subnetworks separately enhances efficiency compared to scheduling over the entire network, while incurring solution quality degradation.

I. Introduction

A. Background and Motivation

The explosive growth of 5G networks has fundamentally transformed the scale and complexity of wireless communications. With billions of connected devices worldwide, contemporary wireless networks face unprecedented challenges in resource management and scheduling. Network optimization problems, which are essential for ensuring high-quality communication services, become computationally intractable as the solution space grows exponentially with network size. This computational barrier has sparked intense interest in network decomposition approaches (e.g., [1], [2]) (see Fig. 1), which break large networks into manageable subnetworks that can operate independently and in parallel, thereby significantly reducing computational complexity while striving to maintain solution quality.

Network decomposition fundamentally represents a clustering problem, where both base stations (BSs) and users must be intelligently grouped into subnetworks based on their characteristics and communication relationships. Current approaches span multiple paradigms: similarity-based methods [3]–[5] that group nodes based on predefined metrics, greedy algorithms [6], [7] that iteratively optimize local objectives, and game-theoretical approaches [8] that model decomposition as strategic interactions. Among these, *spectral clustering* [9], [10] has emerged as a particularly promising direction due to its ability to capture non-linear relationships and handle irregular cluster shapes [11]. By modeling network topology as a graph and minimizing inter-cluster connections, spectral

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clustering offers a mathematically principled approach to network decomposition.

However, existing spectral clustering approaches face two critical limitations in wireless network decomposition, leading to two fundamental research questions.

- First, their BS-centric or user-centric nature, while providing stability in clustering, fails to adequately capture the complex service relationships between BSs and users. This oversight often leads to suboptimal decomposition, particularly manifesting as the cluster-edge problem [12], where users at cluster boundaries experience severe interference from neighboring clusters. This raises our first key question: *How can we effectively incorporate both BS and user cluster characteristics in the spectral clustering process while maintaining computational tractability?*
- Second, when attempting to address this through bipartite graph modeling [13], these approaches struggle with the practical reality that user populations typically vastly outnumber BSs. This population imbalance can overwhelm BS characteristics, resulting in poorly structured subnetworks that compromise both computational efficiency and solution quality. This leads to our second key research question: *How can we handle the inherent scale disparity between BS and user populations while preserving the essential network structure information?*

Addressing these two questions is crucial for developing an effective decomposition scheme for contemporary wireless networks.

B. Solution and Approach

To address these fundamental research questions, we propose BSCCD (Bidirectional Spectral Co-Clustering Based Decomposition), a new decomposition scheme specifically designed for large-scale wireless network resource scheduling. In response to the first research question of incorporating both BS and user characteristics, BSCCD introduces a two-round spectral co-clustering approach that captures bidirectional relationships between BSs and users. To tackle the second question regarding population disparity, we develop a user node merging strategy that effectively alleviates the unbalanced clustering while preserving the network's structural characteristics.

The key contributions of this work are:

- **Bidirectional Co-clustering Framework:** We introduce a fundamentally new perspective on wireless network decomposition by treating it as a bidirectional co-clustering problem rather than traditional one-sided clustering. This shift presents technical challenges due to the inherent coupling between BS and user cluster characteristics. We address these challenges through a two-round spectral co-clustering approach that alternates between BS-centric and user-centric methods, effectively capturing the BS-user service relationship and maintaining solution quality.
- **Population Disparity Handling:** The presence of massive user populations introduces unbalanced clustering results that lose BS cluster characteristics. We develop a user node merging strategy that alleviates the impact of the BS-user population disparity while preserving network structure information. Our key technical innovation lies in systematically leveraging the bidirectional property of spectral co-clustering to hierarchically identify and merge similar users.
- **Performance Insights:** Based on real-world datasets, our extensive experiments reveal practical and consistent advantages of BSCCD over existing approaches. Surprisingly, medium-sized networks (5000-10000 nodes) benefit more from our decomposition scheme than smaller or larger networks in terms of solution quality, achieving up to 89.63% solution quality maintenance of global optimization and surpassing traditional methods by 28.76%. This counter-intuitive result suggests an optimal network size range that benefits the most from decomposition performance, which has not been previously identified.

The rest of this paper is organized as follows: Section II presents the network decomposition problem. Section III details our BSCCD scheme, with particular focus on how it addresses the two key research questions through bidirectional spectral co-clustering and user node merging. Section IV presents the performance evaluation. Section V concludes this work.

II. Network Decomposition Problem

In this section, we first present the general class of network resource scheduling problems and then formulate the network decomposition problem, highlighting the key technical challenges that motivate our bidirectional co-clustering approach.

A. Network Resource Scheduling Problem

Network resource scheduling in wireless systems encompasses a broad class of optimization problems involving resource allocation (e.g., bandwidth, power, spectrum) between BSs and users [14]. These problems appear in various contexts, including traffic scheduling [15] and energy efficiency optimization [16]. While seemingly diverse, these problems share fundamental structural properties that enable a unified treatment through our proposed framework.

Consider a wireless network with N BSs and M users, represented by sets $\mathcal{B} = \{1, 2, \cdots, N\}$ and $\mathcal{U} = \{1, 2, \cdots, M\}$, respectively. The key components of the resource scheduling problem $\mathcal{P}$ are:

- Decision Variables: $\mathbf{X} \triangleq (x_{n,m} \mid n \in \mathcal{B}, m \in \mathcal{U})$, representing resource allocation decisions for each BS-user pair;
- Preference Matrix: $\mathbf{Q} \triangleq (q_{n,m} \mid n \in \mathcal{B}, m \in \mathcal{U})$, given according to problem-specific demands (e.g., transmission rate, geographic preference), where $q_{n,m} \geq 0$ quantifies the allocation preference between BS n and user m;
- Objective Function: $f(\mathbf{X} \odot \mathbf{Q})$, capturing system performance metrics, where $\odot$ represents element-wise multiplication;
- Constraint Sets: Resource limitations and user demands.

The general formulation is:

$$\max_{\mathbf{X}} \quad f(\mathbf{X} \odot \mathbf{Q}) \tag{1a}$$

$$\text{s.t.} \quad \sum_{m\in\mathcal{U}} g_n(x_{n,m}) \le 0, \quad n \in \mathcal{B}, \tag{1b}$$

$$\sum_{n\in\mathcal{B}} h_m(x_{n,m}) \le 0, \quad m \in \mathcal{U}, \tag{1c}$$

$$x_{n,m} \ge 0, \quad n \in \mathcal{B},\ m \in \mathcal{U}, \tag{1d}$$

where constraints (1b) and (1c) represent BS resource limitations and user service requirements, respectively.

The solution space of $\mathcal{P}$ grows exponentially with network size, which poses challenges in efficiency and optimality, underscoring the necessity of decomposition. The unified framework of $\mathcal{P}$ reveals two critical characteristics that influence the decomposition strategy:

- Coupling Through Preferences: The objective function couples decisions $\mathbf{X}$ with preferences $\mathbf{Q}$, making decomposition sensitive to preference structure;
- Bidirectional Constraints: As the decisions pertain to BS-user pairs, the constraints must be satisfied in accordance with both BS and user characteristics.

B. Network decomposition Problem

Network decomposition aims to divide the network into K subnetworks while preserving solution quality and minimizing computational overhead. Each subnetwork corresponds to a sub-problem of the same form as $\mathcal{P}$, and can therefore be solved using the same algorithm. Let $\mathcal{V} \triangleq \mathcal{B} \cup \mathcal{U}$ represent the complete network, and $\mathcal{V}_k \subseteq \mathcal{V}$ denote subnetwork $k \in \mathcal{K} = \{1, 2, \cdots, K\}$. The decomposition problem can be formulated as:

$$\min_{\mathcal{V}_k, k\in\mathcal{K}} \quad \sum_{k\in\mathcal{K}} T_{\mathcal{P}}(\mathcal{V}_k) \tag{2a}$$

$$\text{s.t.} \quad F_{\mathcal{P}}(\mathcal{V}) - F_{\mathcal{P}}^{\text{SUB}}(\mathcal{V}_k, k \in \mathcal{K}) \le \epsilon, \tag{2b}$$

$$\cup_{k\in\mathcal{K}} \mathcal{V}_k = \mathcal{V}, \tag{2c}$$

where $T_{\mathcal{P}}(\mathcal{V}_k)$ represents the computational latency for solving the sub-problem in subnetwork $\mathcal{V}_k$; $F_{\mathcal{P}}(\mathcal{V})$ and $F_{\mathcal{P}}^{\text{SUB}}(\mathcal{V}_k, k \in \mathcal{K})$ denote the solution quality (the objective value f in (1a)) of the original problem and after decomposition, respectively; ϵ represents the quality degradation tolerance.

This formulation presents two fundamental technical challenges:

- Quality-Complexity Trade-off: Smaller subnetworks reduce computational complexity but may degrade solution quality due to fragmentary global information;
- Preference Structure Preservation: Decomposition distorts the known preference structure, degrading solution quality, while problem-specific objective functions make it difficult to achieve a unified modeling of the solution quality so as to minimize the impact on the preference structure.

These challenges motivate our bidirectional co-clustering approach, which explicitly addresses: (i) the preservation of preference structure in decomposition; (ii) the necessity for simultaneous consideration of both BS and user cluster characteristics; (iii) the handling of the BS-user population disparity.

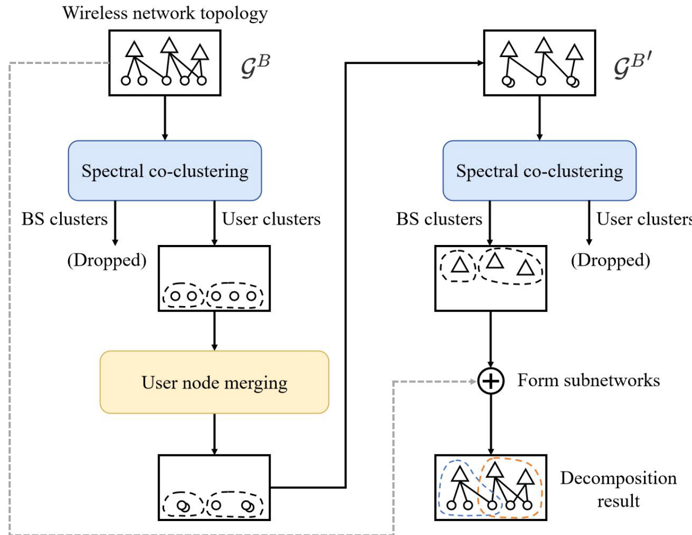

Fig. 2. An illustration of BSCCD scheme.

III. Bidirectional Spectral Co-Clustering Based Decomposition Scheme

This section presents the Bidirectional Spectral Co-Clustering Based Decomposition (BSCCD), designed to tackle network decomposition challenges for large-scale wireless network resource scheduling. Section III-A provides an overview of the scheme, followed by the bipartite graph representation of the network in Section III-B. The scheme employs two rounds of spectral co-clustering (Sections III-C and III-E) to extract bidirectional cluster characteristics and incorporates a user node merging strategy (Section III-D) to address the BS-user population disparity, ensuring balanced clustering and enhanced resource scheduling efficiency. Section III-F analyzes the computational complexity of BSCCD.

A. Scheme Overview

The BSCCD scheme models the wireless network topology as a bipartite graph and comprises two main modules: the spectral co-clustering module and the user node merging module, as shown in Fig. 2. The spectral co-clustering module clusters nodes in the bipartite graph, producing bidirectional cluster characteristics, including both BS clusters and user clusters. The user node merging module refines the bipartite graph by merging similar users within each user cluster obtained in the first round of spectral co-clustering. The decomposition process proceeds as follows:

1) The spectral co-clustering module processes the original network topology and performs the first round of clustering, generating user clusters.
2) The user node merging module adjusts the bipartite graph based on these user clusters.

3) The spectral co-clustering module conducts a second round of clustering on the modified network topology, producing BS clusters.
4) Subnetworks are formed by incorporating connected users into each BS cluster.

The two rounds of spectral co-clustering capture bidirectional BS-user relationships by alternately leveraging BS and user clusters. In contrast, adopting BS clusters alone may lead to unbalanced results due to the influence of BS-user population disparity. In such cases, BS cluster characteristics are overwhelmed by massive user populations.

B. Network Topology Representation

To decompose the wireless network for the general network resource scheduling problem $\mathcal{P}$, we model the network topology as a bipartite graph between two types of nodes: BSs and users. Drawing on insights from [13], we aim to reflect the solution quality of sub-problems in each subnetwork by considering subnetwork interdependence. We measure the interdependence by quantifying the interconnections among network nodes from the perspective of the graph structure, where lower interdependence indicates higher solution quality. To ensure the graph representation aligns with the preference matrix $\mathbf{Q}$, we focus on real-world interactions between BSs and users, establishing connections exclusively between BS-user pairs.

Let $\mathcal{G}^B = (\mathcal{B},\mathcal{U},\mathcal{E})$ represent the BS-user bipartite graph, where $\mathcal{B}$ denotes the set of BSs, $\mathcal{U}$ the set of users, and $\mathcal{E}$ the set of edges. Each edge in $\mathcal{G}^B$ reflects the association between a BS and a user. Specifically, we define the edge weight between BS $n \in \mathcal{B}$ and user $m \in \mathcal{U}$ using two key indicators: reference signal received power (RSRP) and downlink throughput. RSRP captures the geographical proximity between the BS and the user, while downlink throughput indicates the frequency and quality of the user's connection to the BS. Together, these indicators quantify the closeness of a BS-user pair from both physical and service perspectives, effectively modeling the interdependence of subnetworks. Meanwhile, these indicators can be easily collected from measurement report (MR) data in practical scenarios.

Let $\text{RSRP}_{n,m}$ (in dBm) denote the RSRP on user m with respect to BS n. Let $\text{DLT}_{n,m}$ (in bits) denote the downlink throughput between them. Then, the weight of edge $(n,m) \in \mathcal{E}$ in defined as follows:

$$z_{n,m} = \alpha \cdot 10^{\frac{\text{RSRP}_{n,m}}{50}} + (1-\alpha) \cdot \frac{\log(1+\text{DLT}_{n,m})}{\log(1+\text{DLT}_{\max})}, \quad (3)$$

where $\text{DLT}_{\max} = \max_{n\in\mathcal{B},m\in\mathcal{U}} \text{DLT}_{n,m}$, and α is a weighting factor. We have empirically validated the performance of such a weight representation for network resource scheduling problems in Section IV.

When performing clustering analysis, adjacency matrices serve as a standard representation format for graphs, where each element represents the edge weight between a pair of nodes. In particular, for bipartite graphs, we can define the adjacency matrix through the biadjacency matrix, which only characterizes the connection between different types of nodes. Let us define the biadjacency matrix $\mathbf{Z} \triangleq [z_{n,m}]_{n\in\mathcal{B},m\in\mathcal{U}} \in \mathbb{R}^{N\times M}$ for the bipartite graph $\mathcal{G}^B$. Then, the corresponding adjacency matrix $\mathbf{W} \in \mathbb{R}^{(N+M)\times(N+M)}$ can be expressed as:

$$\mathbf{W} = \begin{bmatrix} \mathbf{0}_{N\times N} & \mathbf{Z} \\ \mathbf{Z}^T & \mathbf{0}_{M\times M} \end{bmatrix}, \quad (4)$$

where $\mathbf{0}_{N\times N}$ and $\mathbf{0}_{M\times M}$ represent zero matrices with dimensions $N\times N$ and $M \times M$ respectively.

C. First Round of Spectral Co-Clustering

Based on the BS-user bipartite graph, we perform spectral co-clustering to obtain bidirectional cluster characteristics for BSs and users.

Building upon spectral clustering principles, our objective is to minimize interconnections between subgraphs by partitioning graph $\mathcal{G}^B$, so that the preference structure within the network can be preserved. This can be achieved by minimizing the sum of the normalized-cut in graph $\mathcal{G}^B$ [17]. For a K-partition configuration, let $\mathcal{V}_k \subseteq \mathcal{B}\cup\mathcal{U}$ represent the node subset in subgraph k. With edge weight $w_{i,j}$ between nodes i and j in $\mathbf{W}$, we define:

- $\text{cut}(\mathcal{V}_k, \mathcal{V}_{k'}) \triangleq \sum_{i\in\mathcal{V}_k, j\in\mathcal{V}_{k'}} w_{i,j}$, quantifying inter-subgraph connections;
- $\text{vol}(\mathcal{V}_k) \triangleq \sum_{i\in\mathcal{V}_k, j\in\mathcal{B}\cup\mathcal{U}} w_{i,j}$, representing sum of degrees of the subgraph.

The spectral co-clustering problem is formulated as:

$$\min_{\mathcal{V}_1,\mathcal{V}_2,\cdots,\mathcal{V}_K} \sum_{k<l} \left(\frac{\text{cut}(\mathcal{V}_k,\mathcal{V}_l)}{\text{vol}(\mathcal{V}_k)} + \frac{\text{cut}(\mathcal{V}_k,\mathcal{V}_l)}{\text{vol}(\mathcal{V}_l)} \right). \quad (5)$$

This formulation naturally groups strongly connected BSs and users by minimizing cuts across weakly connected edges.

Following established methods in [18], we solve this problem in two steps:

1) Generate low-dimensional node representations by solving a generalized eigenvalue problem;
2) Cluster nodes using the low-dimensional representation.

In the first step, we generate the low-dimensional node representations by solving the generalized eigenvalue problem:

$$\mathbf{L}\mathbf{x} = \lambda\mathbf{D}\mathbf{x}, \quad (6)$$

where:

- $\mathbf{D} \in \mathbb{R}^{(N+M)\times(N+M)}$ is the degree matrix of graph $\mathcal{G}^B$, constructed as a diagonal matrix with elements $d_{i,i} = \sum_l w_{i,l}$, where $w_{i,l}$ represents the edge weight between nodes i and l;
- $\mathbf{L} \triangleq \mathbf{D} - \mathbf{W}$ is the Laplacian matrix, where $\mathbf{W}$ is the adjacency matrix defined in III-B;
- $\mathbf{x} \in \mathbb{R}^{N+M}$ represents an eigenvector corresponding to the eigenvalue λ.

We solve $L = \lceil \log_2 K \rceil$ eigenvectors $\mathbf{x}_2, \mathbf{x}_3, \cdots, \mathbf{x}_{L+1} \in \mathbb{R}^{N+M}$, corresponding to the L smallest eigenvalues $\lambda_2 \le \lambda_3 \le \cdots \le \lambda_{L+1}$ excluding $\lambda_1 = 0$. These eigenvectors are assembled into matrix $\mathbf{X} = \begin{bmatrix} \mathbf{x}_2 & \mathbf{x}_3 & \cdots & \mathbf{x}_{L+1} \end{bmatrix} \in \mathbb{R}^{(N+M)\times L}$, where row i represents the L-dimensional feature vector of node $i \in \mathcal{B} \cup \mathcal{U}$ in the low-dimensional space. The dimension L is chosen as $\lceil \log_2 K \rceil$ to ensure sufficient dimensionality reduction while preserving essential cluster structure for K partitions.

To solve the generalized eigenvalue problem in (6) and obtain $\mathbf{X}$, we employ singular value decomposition (SVD). The process involves computing the normalized biadjacency matrix $\mathbf{D}_{\mathcal{B}}^{-1/2}\mathbf{Z}\mathbf{D}_{\mathcal{U}}^{-1/2}$, where $\mathbf{D}_{\mathcal{B}} \in \mathbb{R}^{N\times N}$ is the degree matrix of BS nodes, comprising the first N rows and columns of $\mathbf{D}$, and $\mathbf{D}_{\mathcal{U}} \in \mathbb{R}^{M\times M}$ is the degree matrix of user nodes, comprising the last M rows and columns of $\mathbf{D}$. We decompose the normalized biadjacency matrix as:

$$\mathbf{D}_{\mathcal{B}}^{-1/2}\mathbf{Z}\mathbf{D}_{\mathcal{U}}^{-1/2} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{-1}, \tag{7}$$

obtaining left singular vectors $\mathbf{u}_2, \mathbf{u}_3, \cdots, \mathbf{u}_{L+1}$ and right singular vectors $\mathbf{v}_2, \mathbf{v}_3, \cdots, \mathbf{v}_{L+1}$ corresponding to the L largest singular values (excluding the first) of the normalized biadjacency matrix. This allows us to approximate $\mathbf{X}$ as:

$$\mathbf{X} = \begin{bmatrix} \mathbf{D}_{\mathcal{B}}^{-\frac{1}{2}}\mathbf{U}_L \\ \mathbf{D}_{\mathcal{U}}^{-\frac{1}{2}}\mathbf{V}_L \end{bmatrix}, \tag{8}$$

where:

$$\mathbf{U}_L \triangleq \begin{bmatrix} \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \mathbf{u}_{L+1} \end{bmatrix} \in \mathbb{R}^{N\times L}$$
$$\mathbf{V}_L \triangleq \begin{bmatrix} \mathbf{v}_2 & \mathbf{v}_3 & \cdots & \mathbf{v}_{L+1} \end{bmatrix} \in \mathbb{R}^{M\times L}$$

are constructed with L singular vectors respectively, similar to the structure of $\mathbf{X}$.

After determining $\mathbf{X}$, the second step involves using K-means to cluster the rows of $\mathbf{X}$ (which correspond to the low-dimensional representations of the nodes), thus solving problem (5). Within each subgraph, BS nodes and user nodes are grouped into the same BS cluster and user cluster, respectively.

The SVD approach ensures both types of nodes (BSs and users) contribute with equal standing to the dimensionality reduction process through their respective singular vectors ($\mathbf{U}_L$ and $\mathbf{V}_L$). The clustering of $\mathbf{X}$ yields bidirectional results, revealing both BS clusters and user clusters for the original network. This bidirectional property enables selective cluster utilization. Specifically, in the first round of spectral co-clustering, we obtain user clusters, which serve as input for subsequent user merging operations.

D. User Node Merging

The user node merging process modifies user clusters obtained from the first round of spectral co-clustering by merging similar users within each cluster, thereby adjusting the bipartite graph. This process comprises two key steps.

1) Similarity-Based Sub-Clustering: Within each user cluster from the previous spectral co-clustering, we perform an additional similarity-based sub-clustering to identify highly similar users. There are two reasons for this hierarchical clustering: (i) due to the bidirectional property of spectral co-clustering, users within the same cluster share concentrated connections to a specific BS group (Fig. 3); (ii) these connections differ primarily in association strengths, which can be identified using similarity metrics. We employ HDBSCAN [19] for this sub-clustering due to its robust parameter selection and adaptive cluster number determination capabilities. This sub-clustering method ensures the similarity of users in both cluster characteristics and connection patterns.

Algorithm 1 BSCCD procedure

Input the biadjacency matrix $\mathbf{Z}$.
Compute singular vectors $\mathbf{u}_2, \mathbf{u}_3, \cdots, \mathbf{u}_{L+1}$ and $\mathbf{v}_2, \mathbf{v}_3, \cdots, \mathbf{v}_{L+1}$ based on (7). ▷ First round co-clustering
Compute $\mathbf{X}$ based on (8).
Apply K-means on rows of $\mathbf{X}$, obtaining user clusters.
for each user cluster $\mathcal{C}$ **do** ▷ User node merging
    Obtain sub-clusters through HDBSCAN within $\mathcal{C}$.
    Compute $\mathbf{Z}'$ based on (9).
end for
Compute singular vectors $\mathbf{u}'_2, \mathbf{u}'_3, \cdots, \mathbf{u}'_{L+1}$ and $\mathbf{v}'_2, \mathbf{v}'_3, \cdots, \mathbf{v}'_{L+1}$ based on (7). ▷ Second round co-clustering
Compute $\mathbf{X}'$ based on (8).
Apply K-means on rows of $\mathbf{X}'$, obtaining BS clusters.
Form subnetworks based on BS clusters.

2) Virtual User Creation: Each identified sub-cluster is consolidated into a new virtual user through logarithmic summation. This process updates the network's user set (Fig. 2), resulting in an updated biadjacency matrix $\mathbf{Z}'$ and revised adjacency matrix $\mathbf{W}'$ (per equation (4)). For a sub-cluster containing s users $\{m_1, m_2, \cdots, m_s\}$, we create a new virtual user m_{new}. The association (i.e., the edge weight) between any BS n and m_{new} is defined as:

$$z'_{n,m_{\text{new}}} = \log(1+z_{n,m_1}+z_{n,m_2}+\cdots+z_{n,m_s}), \quad n \in \mathcal{B}. \tag{9}$$

The logarithmic summation approach aggregates user features within sub-clusters to form representative virtual users while characterizing concentrated associations between user groups and specific BS groups. Its logarithmic nature provides diminishing marginal effects, preventing unbalanced user distribution post-merging. This merging strategy reduces noise point impact while preserving user cluster characteristics, ultimately enhancing the BS clustering results in the second round of spectral co-clustering.

E. Second Round of Spectral Co-Clustering

Following user node merging, we proceed with a second round of spectral co-clustering on the modified network topology. This iteration is expected to yield improved BS clustering results compared to the first round for two primary reasons: the mitigation of the BS-user population disparity effects and the preservation of user cluster characteristics. The second round follows the identical spectral co-clustering procedure outlined in Section III-C, utilizing the updated adjacency matrix $\mathbf{W}'$. Based on the resulting BS clusters, we form the final network decomposition. Given the procedural similarity to the first round, we omit the details here.

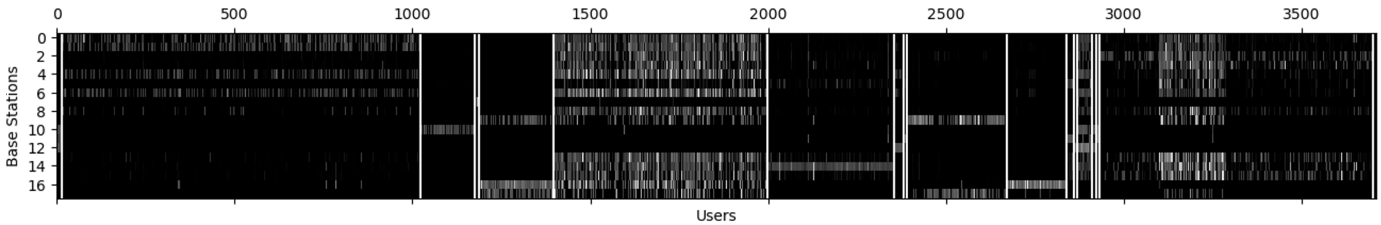

Fig. 3. User clusters obtained from the first round of spectral co-clustering. The rectangle depicts the BS-user biadjacency matrix, where brighter points reflect stronger associations between the corresponding BSs (rows) and users (columns). After the first round of spectral co-clustering, the columns are rearranged. Users in the same cluster are placed together, and different clusters are separated with white vertical lines.

F. Computation Complexity Analysis

The computation complexity of BSCCD is primarily determined by following steps:

- SVD in spectral co-clustering: computing the L singular vectors of an $N \times M$ matrix has complexity $\mathcal{O}(NML)$;
- K-means in spectral co-clustering: Clustering rows of an $N \times M$ matrix into K clusters in fixed small iterations requires $\mathcal{O}(NMK)$ computation time;
- Sub-clustering in user node merging: Clustering an $M_1 \times N$ matrix through HDBSCAN requires $\mathcal{O}(NM_1^2)$.

Suppose the number of clusters of the two rounds of spectral co-clustering is K_1 and K_2 (typically $K_1 > K_2$), respectively. Suppose the largest cluster in the first round of spectral co-clustering has M_1 users. Then, the computation complexity of BSCCD is $\mathcal{O}(NM\log K_1 + NMK_1 + NM_1^2 + NM\log K_2 + NMK_2) = \mathcal{O}(NMK_1 + NM_1^2)$. If the K_1 clusters in the first round each contain an equal number of users, the complexity becomes $\mathcal{O}(NMK_1 + \frac{NM^2}{K_1^2})$.

IV. Experiment

We assess our network decomposition scheme's effectiveness through two approaches: (i) analyzing theoretical performance metrics and (ii) conducting practical case studies of network resource scheduling problems. Our evaluation focuses on two key aspects: computational efficiency (measured by latency) and the solution quality (measured by objective values of the resource scheduling problem).

A. Experiment Settings

1) Dataset Description: Our evaluation utilizes real-world datasets collected by a world-leading telecom company across several major Chinese cities, comprising engineering parameter data and measurement report (MR) data. The engineering parameter data encompasses BS configurations, including geographic locations, frequencies, bandwidths, and cell parameters. From the MR data, which contains real-time performance records from terminal devices, we primarily utilize RSRP (in dBm) and downlink throughput (in bits) to model BS-user associations (as defined in (3)). We evaluate our BSCCD scheme across nine independent network systems, categorized into small (datasets 1-3), medium (datasets 4-6), and large networks (datasets 7-9), as shown in Table I.

TABLE I
DESCRIPTION OF EXPERIMENT DATASETS.

Dataset	Records	Base stations	Users
1	1697	13	802
2	14042	17	2660
3	18335	18	3710
4	29694	19	6092
5	38453	22	6998
6	30825	23	9621
7	100780	16	24931
8	68759	19	22011
9	116243	30	23858

TABLE II
WEIGHTED INTERFERENCE OF THE 6 CLUSTERING APPROACHES. LOWER VALUES INDICATE BETTER PERFORMANCE.

Dataset	BSCCD	SCCD	(Location) K-means	(Location) Spectral	(Association) K-means	(Association) Spectral
1	**0.04**	**0.04**	**0.04**	**0.04**	1.14	0.40
2	**0.77**	**0.77**	10.25	10.25	2.38	3.16
3	**2.25**	2.54	5.65	5.65	2.58	2.47
4	6.93	7.59	8.08	7.26	4.19	**3.17**
5	**2.81**	3.32	5.99	5.99	4.29	3.55
6	1.73	**1.55**	3.74	3.88	1.66	1.89
7	**1.28**	**1.28**	3.90	2.52	1.67	1.83
8	**0.35**	**0.35**	4.27	5.40	3.07	3.03
9	**4.04**	4.84	5.58	7.53	6.87	4.47

2) Comparative Analysis: We benchmark BSCCD against three BS clustering-based decomposition algorithms: (i) spectral co-clustering (SCCD), which differs from BSCCD by directly applying BS clustering without user node merging; (ii) K-means clustering [20]; and (iii) spectral clustering [21]. For K-means and spectral clustering, we implement two distinct BS feature characterization approaches: a geographic approach using BS location data, and an association approach using the BS-user biadjacency matrix, where a BS's feature vector comprises its associations with all users.

Table I presents the dataset, detailing the number of records, BSs, and users for each network system. Table II compares the weighted interference metrics across all six clustering approaches. Table IV and III summarize their computational efficiency and solution quality of the case study, respectively.

B. Evaluation on Weighted Interference

We assess the effectiveness of network decomposition by measuring the interdependence between subnetworks using the

TABLE III
OBJECTIVE VALUE AND DEGRADATION OF THE 6 CLUSTERING APPROACHES ON THE NUM PROBLEM.

Dataset	Total User Utility							Degradation					
	Global	BSCCD	SCCD	(Location)		(Association)		BSCCD	SCCD	(Location)		(Association)	
				K-means	Spectral	K-means	Spectral			K-means	Spectral	K-means	Spectral
1	493.77	**484.51**	**484.51**	**484.51**	**484.51**	465.68	478.51	**-0.0188**	**-0.0188**	**-0.0188**	**-0.0188**	-0.0569	-0.0309
		472.84	**472.84**	**472.84**	**472.84**	427.11	470.05	**-0.0424**	**-0.0424**	**-0.0424**	**-0.0424**	-0.1350	-0.0480
2	3031.06	**2706.69**	**2706.69**	2475.46	2475.46	1766.91	2026.45	**-0.1070**	**-0.1070**	-0.1833	-0.1833	-0.4171	-0.3314
		2476.52	**2476.52**	1858.86	1858.86	878.88	1201.53	**-0.1830**	**-0.1830**	-0.3867	-0.3867	-0.7100	-0.6036
3	3683.10	**3516.40**	3478.80	2665.73	2665.73	2278.54	2306.79	**-0.0453**	-0.0555	-0.2762	-0.2762	-0.3814	-0.3737
		3327.16	3202.00	1706.78	1706.78	1239.12	1295.70	**-0.0966**	-0.1306	-0.5366	-0.5366	-0.6636	-0.6482
4	4469.72	**3791.35**	3694.91	3331.75	2921.20	2275.47	2176.96	**-0.1518**	-0.1733	-0.2546	-0.3464	-0.4909	-0.5130
		3343.60	3125.37	2536.69	1661.45	1081.42	770.61	**-0.2519**	-0.3008	-0.4325	-0.6283	-0.7581	-0.8276
5	6092.89	**4891.74**	4808.33	3413.94	3413.94	2947.66	2706.94	**-0.1971**	-0.2108	-0.4397	-0.4397	-0.5162	-0.5557
		3529.84	3382.89	1891.66	1891.66	1311.64	1024.89	**-0.4207**	-0.4448	-0.6895	-0.6895	-0.7847	-0.8318
6	6889.90	**6175.15**	5971.11	4468.98	4739.95	4422.58	3988.37	**-0.1037**	-0.1334	-0.3514	-0.3120	-0.3581	-0.4211
		5602.82	4971.00	2754.17	2654.38	2488.43	1882.90	**-0.1868**	-0.2785	-0.6003	-0.6147	-0.6388	-0.7267
7	12757.97	**8643.48**	**8643.48**	7510.02	7043.80	7353.59	7234.67	**-0.3225**	**-0.3225**	-0.4113	-0.4479	-0.4236	-0.4329
		5005.89	**5005.89**	3584.94	2804.36	3770.89	3540.30	**-0.6076**	**-0.6076**	-0.7190	-0.7802	-0.7044	-0.7225
8	10432.81	**10121.81**	**10121.81**	8495.87	7667.50	5438.26	5327.58	**-0.0298**	**-0.0298**	-0.1857	-0.2651	-0.4787	-0.4893
		10075.00	**10075.00**	7229.78	5546.36	2490.74	2359.76	**-0.0343**	**-0.0343**	-0.3070	-0.4684	-0.7613	-0.7738
9	19433.33	**11332.25**	11265.45	9921.77	9443.20	10647.14	8670.77	**-0.4169**	-0.4203	-0.4894	-0.5141	-0.4521	-0.5538
		5808.99	5736.16	4659.22	3974.52	5520.58	3224.22	**-0.7011**	-0.7048	-0.7602	-0.7955	-0.7159	-0.8341

sum of interference-to-signal-power-ratio [13]. We refer to this metric as weighted interference, defined as:

$$\delta = \sum_{k=1}^{K} \frac{\mathrm{cut}(\mathcal{V}_k, \overline{\mathcal{V}_k})}{\mathrm{vol}(\mathcal{V}_k) - \mathrm{cut}(\mathcal{V}_k, \overline{\mathcal{V}_k})}, \tag{10}$$

where K represents the number of subnetworks, and $\overline{\mathcal{V}_k}$ denotes the complementary set of $\mathcal{V}_k$, comprising all nodes not in $\mathcal{V}_k$ from $\mathcal{B} \cup \mathcal{U}$.

This metric quantifies the interference levels among subnetworks and serves as a key indicator of subnetwork performance in wireless network systems, reflecting the decomposition quality. In the equation, the numerator represents the absolute interference between internal and external nodes, while the denominator reflects the subnetwork's internal cohesion. A lower weighted interference value indicates reduced interdependence between subnetworks, suggesting more effective network decomposition. The theoretical optimum of zero indicates completely independent subnetworks.

Observation 1 (Weighted Interference)**.** *In Table II, BSCCD generally outperforms existing benchmarks. Compared to SCCD, the weighted interference of BSCCD is 11.61% smaller at the maximum negative difference and 16.53% larger at the maximum positive difference. Compared to the best-performing traditional approaches, these values are 118.61% smaller and 88.45% larger, respectively. For small and large networks, BSCCD always achieves the best performance. For medium networks, BSCCD ranks at a high level.*

C. Evaluation on Case Study

We evaluate the performance of our decomposition scheme for solving network resource scheduling problem $\mathcal{P}$ through a case study, where we apply this scheme to solve a generally defined instance—the network utility maximization (NUM) problem [22]. Specifically, we compare the objective value and the computation latency between globally solving the problem over the entire network and solving the problem in subnetworks using our decomposition scheme.

The NUM problem aims at maximizing the overall system utility through reasonable network resource scheduling. According to [23], the NUM problem has a unique optimal solution under mild conditions, making it suitable for performance evaluation. Consider an arbitrary resource to be allocated. BS n with available resources r_n allocates $x_{n,m}$ resources to user m. Given the preference matrix $\mathbf{Q}$, the NUM problem maximizes the total utility of users:

$$\max_{\mathbf{X}} \quad u(\mathbf{X}) = \sum_{m\in\mathcal{U}} u_m\Big(\sum_{n\in\mathcal{B}} x_{n,m} q_{n,m}\Big) \tag{11a}$$

$$\text{s.t.} \quad \sum_{m\in\mathcal{U}} x_{n,m} \le r_n, \quad n \in \mathcal{B}, \tag{11b}$$

$$x_{n,m} \ge 0, \quad n \in \mathcal{B}, m \in \mathcal{U}. \tag{11c}$$

The utility of each user is an increasing function of $\sum_{n\in\mathcal{B}} x_{n,m} q_{n,m}$ with a diminishing growth rate, reflecting the diminishing marginal utility. This objective function conforms to the definition of the objective function of $\mathcal{P}$ in Section II-A.

We evaluate the computation latency and objective value degradation (i.e., the total user utility in (11a)) when solving the NUM problem using our network decomposition scheme. For subnetwork solutions, we calculate computation latency by summing the latency of each sub-problem. For users that appear in different subnetworks, we synchronize their utility

TABLE IV
COMPUTATION LATENCY OF THE NUM PROBLEM.

Dataset	Global (s)	BSCCD (s)	Subnetworks	Speed up
1	0.09	0.07	2	22.22%
2	0.67	0.35	4	47.76%
3	1.57	0.76	4	51.59%
4	1.83	1.46	5	20.22%
5	2.43	1.72	5	29.22%
6	4.90	2.61	5	46.73%
7	18.97	14.11	4	25.62%
8	19.14	11.35	5	40.70%
9	32.95	12.55	7	61.91%

values through average values and worst values across these subnetworks, as shown in Table III. "Global" in Tables IV and III refers to solving the NUM problem over the entire network without decomposition.

Observation 2 (Objective Value Degradation)**.** *In Table III, BSCCD always leads to minimum objective value degradation. Compared to the best-performing traditional approaches, BSCCD achieves an average improvement of 12.68% for average utility synchronization and 21.01% for worst utility synchronization. For medium-sized networks, BSCCD outperforms SCCD by 2.16% and 5.49%, and surpasses the best-performing traditional approaches by 18.46% and 28.76%, in terms of average and worst utility synchronization, respectively.*

Observation 3 (Speed Up)**.** *As shown in Table IV, solving the NUM problem after decomposition through BSCCD reduces the computation latency by up to 61.91%. Such a reduction is more significant under larger network systems when the number of subnetworks is large.*

These observations demonstrate the effectiveness of our BSCCD scheme in reducing computation latency while maintaining solution quality. The evaluation of solution quality aligns with that of weighted interference. Furthermore, it demonstrated the superiority of the BSCCD scheme, whose advantage in computation latency is most evident on large networks, while its advantage in solution quality is more pronounced on medium networks. This pattern arises because small networks are manageable even for basic approaches, where different approaches can give the same decomposition result; while large networks are more complex and require decomposition into more subnetworks, improving efficiency at the cost of some solution quality.

V. Conclusion

In this work, we introduced the Bidirectional Spectral Co-Clustering Based Decomposition (BSCCD) scheme, which systematically decomposes large-scale wireless networks into subnetworks for parallel resource scheduling. Our scheme advances beyond traditional decomposition methods through two key innovations: (i) the incorporation of BS-user service relationship analysis, and (ii) a user node merging strategy that efficiently handles real-world scenarios with high user density. Through comprehensive experiments on real-world datasets, we demonstrated that our scheme achieves up to 61.91% speedup while providing over 10% improvement on average over existing methods. These results validate BSCCD's effectiveness in addressing the fundamental trade-off between computational efficiency and solution quality in resource scheduling for contemporary wireless networks. Building upon these outcomes, future research directions include investigating dynamic adjustment mechanisms and enhancing robustness, with particular emphasis on generalizing the framework to diverse network optimization problems and resolving synchronization challenges in overlapping subnetworks.

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