# Frequency Assignment for Guaranteed QoS in Two-Ray Models with Limited Location Information

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***Abstract*—We consider a two-ray channel model in which the distance between transmitter and receiver is only known up to an interval. Due to the unknown distance, destructive interference might occur which significantly reduces the receive power. To mitigate this problem, multiple frequencies can be used in parallel. In this work, we consider a worst-case design approach which allows maximizing the guaranteed quality of service (QoS) despite the uncertainty about the channel. First, we derive the worst-case receive power within the uncertainty region. Next, we compare different approaches to assign frequencies to the user such that the worst-case is maximized. We propose a greedy algorithm, which significantly outperforms standard baseline schemes while also being resource efficient. With this, the communication system can be designed such that a certain performance can always be guaranteed, and ultra-reliability is practically achieved.**

## I. Introduction

Many modern applications of wireless communications require high reliability [1], [2]. This includes networks with autonomous vehicles, e.g., self-driving cars and unmanned aerial vehicles (UAVs). However, in many scenarios only limited information about the state of the system is available, e.g., the transmitter has no or only partial channel state information (CSI), such as rough location information. In such cases, it can be difficult to ensure a high reliability.

It has been observed and experimentally verified that negative dependency between channel gains can significantly improve reliability when only limited CSI is available [3]–[7]. The basic idea is to establish diversity and ensure that always at least one communication link is available. In this work, we will apply this idea to assign frequencies in a multi-carrier system such that the worst-case performance is improved and, thus, a certain quality of service (QoS) can always be guaranteed.

Throughout this work, we assume a two-ray model, in which only a line-of-sight (LoS) connection and one significant multipath component exist. This second component could be caused by a single reflection on the ground, e.g., in flat outdoor terrain [8], on large concrete areas, e.g., airports [9], and for a UAV flying above water [10]–[12]. At higher frequencies, the multi-path channels become increasingly sparse [13] and it has been observed that this model is appropriate for high frequency bands like millimeter wave (mmWave) [14]–[18].

The recent trend joint communications and sensing (JCAS) for 6G wireless networks allows a system to obtain location information for terminals [19]. In the case of the two-ray model, this localization information is an estimation of the distance between transmitter and receiver and can be exploited as partial CSI. However, the estimation is not perfect and is modeled by an uncertainty interval. Besides using JCAS, the distance interval could also be obtained from geographic information, e.g., from GPS information or if it is known that the receiver is within a certain area.

When varying the distance between transmitter and receiver, the relative phase of the two received signal components varies and they may interfere constructively or destructively. A destructive interference causes a drop of receive power, which in turn could cause an outage of the communication link. In order to mitigate drops of the signal power on one frequency, multiple frequencies are used in parallel. In [20], it is analyzed and experimentally verified that frequency diversity improves the performance of distance measurements in outdoor ground reflection scenarios. Instead of using multiple frequencies in parallel, it was shown experimentally in [9] that using multiple antennas and carefully choosing the spacing between them can also improve the received power.

In order to achieve a reliable connection between transmitter and receiver despite the uncertainty about the distance, we consider a worst-case design. With this, the communication system can be designed such that a certain performance can always be guaranteed, even in the worst case. Similar scenarios have been considered in [21] and [22]. However, in these previous works, only two parallel frequencies were considered and their spacing was optimized, whereas we allow an arbitrary number of parallel frequencies in this work. In addition to this, we consider different solution algorithms to assign frequencies to a user. Furthermore, we show that the presented results hold both for maximum ratio combining (MRC) and selection combining (SC). The results presented in this work can enable ultra-reliability without the need for perfect CSI at the transmitter.

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Our main contributions and the outline of the paper are summarized as follows.

- We analyze the received power and derive a worst-case bound for the considered two-ray scenario with location uncertainty, when multiple frequencies are employed in parallel and MRC is used. (Section III)
- We show that the derived results for MRC also provide a valid worst-case design for the case that SC is used at the receiver. (Section IV)
- We compare multiple algorithms for assigning subcarrier frequencies such that the worst-case receive power is maximized. We propose a greedy algorithm, which outperforms all standard baseline algorithms in our numerical evaluations. (Section V)

*Notation:* In order to simplify the notation, we will omit variables on which functions depend when their value is clear from the context, e.g., we will write $f(x)$ instead of $f(x, y)$ when the value of $y$ is fixed. Since the angular frequency $\omega = 2\pi f$ is a simple scaling of the frequency $f$, we will treat them somewhat interchangeably. Especially for calculations, it is more convenient to use $\omega$, while $f$ is relevant for actual system design. We will therefore use the frequency $f$ for the numerical examples while expressing all formulas in terms of the angular frequency $\omega$.

## II. System Model and Problem Formulation

In this work, we consider the classical two-ray ground reflection model [23, Chap. 4.6]. The propagation environment in this scenario is approximated as a plane reflecting ground surface. A single-antenna transmitter is placed at height $h_{\mathrm{Tx}}$ above the ground. The single-antenna receiver is located at height $h_{\mathrm{Rx}}$ at a distance $d$. This geometrical model is depicted in Figure 1. The setup shows that the signal propagation to the receiver occurs through two separate paths. There is a LoS propagation with path length $\ell$. Additionally, the signal reflects off the ground, creating a second path. The total length of the second path is $\tilde{\ell}$. Finally, these two components superimpose at the receiver. The path lengths can be calculated based on basic trigonometric considerations as

$$\ell^2 = (h_{\mathrm{Tx}} - h_{\mathrm{Rx}})^2 + d^2 \quad \text{and} \quad \tilde{\ell}^2 = (h_{\mathrm{Tx}} + h_{\mathrm{Rx}})^2 + d^2. \tag{1}$$

When transmitting on a single frequency $\omega_i = 2\pi f_i$, the received power $P_{\mathrm{R},i}$ at distance $d$ is given as [3, Eq. (2)], [23]

$$P_{\mathrm{R},i}(d, \omega_i; h_{\mathrm{Tx}}, h_{\mathrm{Rx}}, P_{\mathrm{T},i}) = P_{\mathrm{T},i} \left(\frac{c}{2\omega_i}\right)^2 \left(\frac{1}{\ell^2} + \frac{1}{\tilde{\ell}^2} - \frac{2}{\ell\tilde{\ell}} \cos\left[\frac{\omega_i}{c}(\tilde{\ell} - \ell)\right]\right), \tag{2}$$

with the transmit power $P_{\mathrm{T},i}$ on angular frequency $\omega_i$, and the speed of light $c$. Throughout this work, we assume a perfectly reflecting ground, since it has been shown in [22, Prop. 1] that this creates the worst destructive interference for this model and is therefore an appropriate assumption for the worst-case design that we are considering in this work.

It is well-known that the interference between the two components at the receiver can be either constructive or destructive, depending on the distance $d$. This results in local minima of the receive power at certain distances, which in turn can lead to outages in the transmission. Multiple frequencies can be used in parallel to mitigate the receive power drops. The exact problem formulation is given in the following.

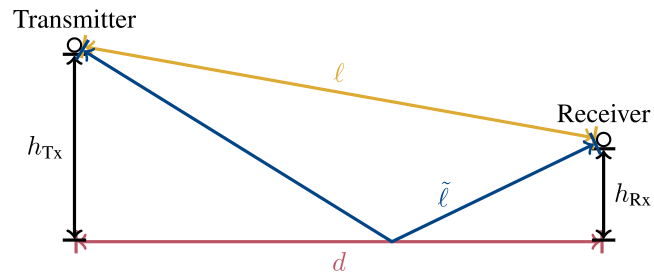


Figure 1. Geometrical model of the considered two-ray ground reflection scenario. The transmitter is placed at height $h_{\mathrm{Tx}}$ above the ground. The receiver is located at height $h_{\mathrm{Rx}}$ at a (ground) distance $d$ away from the transmitter. The LoS path and reflection path have lengths $\ell$ and $\tilde{\ell}$, respectively.

### A. Problem Formulation

Throughout the following, we will consider a two-ray ground reflection scenario as described above. The distance between transmitter and receiver $d$ varies and is known only up to an interval, i.e., $d \in [d_{\min}, d_{\max}]$. In order to ensure a high reception quality at any distance within the interval, the transmitter employs multiple frequencies $f_0, f_1, \dots, f_{N-1}$ as subcarriers such that we can compensate for possible destructive interference of the two rays. This leads to the following challenge.

**Problem 1.** Find a frequency assignment that maximizes the *worst-case* receive power in the distance interval $[d_{\min}, d_{\max}]$ when the receiver is employing MRC. This optimization problem can be formulated as

$$\max_{\omega_0, \dots, \omega_{N-1}} \min_{d \in [d_{\min}, d_{\max}]} P_{\mathrm{S}}(d, \omega_0, \dots, \omega_{N-1}), \tag{3}$$

where $P_{\mathrm{S}}(d, \omega_0, \dots, \omega_{N-1}) = \sum_{i=0}^{N-1} P_{\mathrm{R},i}(d, \omega_i)$ is the received sum power based on (2).

Similarly, the receiver can use SC, in which only the strongest component is used, i.e., the total receive power is then given as [24]

$$P_{\mathrm{SC}}(d, \omega_0, \dots, \omega_{N-1}) = \max_{i=0,1,\dots,N-1} P_{\mathrm{R},i}(d, \omega_i). \tag{4}$$

This leads to the following second problem that will be addressed in this work.

**Problem 2.** Find a worst-case frequency assignment for the described communication system if the receiver uses SC.

We address the above problems with the following three steps. 1) In Section III, we derive the minimum receive power in a given distance interval for fixed frequencies and MRC at the receiver. 2) We generalize the results for MRC to SC in Section IV. 3) Next, we present methods in Section V to assign frequencies to maximize the minimum receive power.

## III. Worst-Case Receive Power

We start with addressing Problem 1 and the assumption that the receiver uses MRC, i.e., we consider the receive power to be the sum power $P_{\mathrm{S}}$ throughout this section. As a first step to solve problem (3), we calculate a worst-case power for the given distance interval $[d_{\min}, d_{\max}]$. In order to allow for tractable calculations, we replace the actual receive power $P_{\mathrm{S}}$ by a tight lower bound $\underline{P_{\mathrm{S}}}$. This will still yield a valid worst-case bound, since

$$\min_{d\in[d_{\min},d_{\max}]} P_{\mathrm{S}}(d) \geq \min_{d\in[d_{\min},d_{\max}]} \underline{P_{\mathrm{S}}}(d)\,. \tag{5}$$

In the following, we derive a tight lower bound $\underline{P_{\mathrm{S}}}$ and show how the worst-case receive power in (5) can be efficiently computed.

In this work, we assume equal power allocation over all frequencies, i.e., we have $P_{\mathrm{T},i} = P_{\mathrm{T}}/N$. With this, we get the total receive power $P_{\mathrm{S}}$ as

$$P_{\mathrm{S}} = \frac{P_{\mathrm{T}}}{N}\left(\frac{c}{2}\right)^2 \sum_{i=0}^{N-1} \frac{1}{\omega_i^2}\left(\frac{1}{\ell^2} + \frac{1}{\tilde{\ell}^2} - \frac{2}{\ell\tilde{\ell}}\cos\frac{\omega_i}{c}(\tilde{\ell}-\ell)\right). \tag{6}$$

A tight lower bound on this achieved received sum power is presented in the following lemma.

**Lemma 1** (Lower Bound on the Total Receive Power)**.** *For the described two-ray ground reflection model using frequencies $\omega_0, \omega_1, \ldots, \omega_{N-1}$ in parallel, the received (sum) power $P_{\mathrm{S}}$ from* (6) *is lower bounded by* (7) *at the bottom of this page.*

*Proof Sketch.* The lower bound $\underline{P_{\mathrm{S}}}$ is calculated as the (lower) envelope of $P_{\mathrm{S}}$, which is given as the absolute value of its analytic function [25]. This can be calculated using the Hilbert transform of the oscillating part in (6).

Throughout the following, we assume a uniform spacing between all possible subcarrier frequencies $f_i$ with frequency spacing $\Delta f > 0$, i.e.,

$$f_i = f_0 + i\Delta f, \quad i = 0, 1, \ldots, N-1\,. \tag{8}$$

First, this assumption reflects most current standards, in which the subcarriers are equally spaced. Furthermore, when considering a small spacing $\Delta f$ and a large number of available frequencies $N$, this model can be used to approximate the case of continuous frequencies.

Under this assumption, we can calculate the worst-case receive power within distance interval $[d_{\min}, d_{\max}]$ as follows. The minimum sum power will occur either on the boundaries ($d_{\min}$ or $d_{\max}$), or at a local minimum of $\underline{P_{\mathrm{S}}}$ within $[d_{\min}, d_{\max}]$, where the components interfere destructively. Destructive interference occurs when the oscillating term under the square root in (7) is maximal. With the uniform frequency spacing from (8), we can reformulate this oscillating part as

$$\begin{aligned}
&\sum_{i=0}^{N-1}\frac{1}{\omega_i^4} + 2\sum_{i=0}^{N-1}\sum_{j<i}\frac{\cos\left((\omega_i-\omega_j)\frac{\tilde{\ell}-\ell}{c}\right)}{\omega_i^2\omega_j^2} && (9)\\
&\qquad = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1}\frac{\cos\left((i-j)\Delta\omega\frac{\tilde{\ell}-\ell}{c}\right)}{\omega_i^2\omega_j^2} && (10)\\
&\qquad = \sum_{n=0}^{N-1} a_n \cos(nt)\,, && (11)
\end{aligned}$$

where we use the shorthand $t = \Delta\omega(\tilde{\ell}-\ell)/c$ and introduce the variable $n = |i-j|$ to describe the difference between the individual frequencies. The coefficients $a_n$ are therefore given as

$$a_n = \sum_{|i-j|=n} \frac{1}{\omega_i^2\omega_j^2}, \quad i,j = 0,\ldots,N-1\,. \tag{12}$$

This leads to the following proposition.

**Proposition 1.** *The oscillating part* (9) *in the lower bound $\underline{P_{\mathrm{S}}}$ has the form of a Fourier series as shown in* (11) *and* (12).

This fact can be used to calculate the distances at which destructive interference occurs.

**Lemma 2.** *The distances $d_k$ at which local maxima and minima of the oscillating part* (9) *of the lower bound $\underline{P_{\mathrm{S}}}$ occur, are calculated as*

$$d_k = \frac{\sqrt{\left(4h_{\mathrm{Tx}}^2 - \Delta\ell_k^2\right)\left(4h_{\mathrm{Rx}}^2 - \Delta\ell_k^2\right)}}{2\Delta\ell_k} \tag{13}$$

*with $\Delta\ell_k = ct_k/\Delta\omega$ and $t_k$ being the solutions of the equation*

$$\sum_{n=1}^{N-1} b_n \sin(nt_k) = 0$$

*with $b_n = -na_n$.*

*Proof.* In order to find the distances at which local maxima and minima of (9) occur, we need to calculate the zeros of the first derivative. Based on (11), this derivative is given as

$$\frac{\mathrm{d}}{\mathrm{d}t}\sum_{n=0}^{N-1} a_n\cos(nt) = -\sum_{n=1}^{N-1} na_n\sin(nt) = \sum_{n=1}^{N-1} b_n\sin(nt), \tag{14}$$

with coefficients $b_n = -na_n$. With the definitions of $t$, $\ell$, and $\tilde{\ell}$, the critical distances $d_k$ can be calculated according to (13). □

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$$\underline{P_{\mathrm{S}}}(d,\omega_0,\omega_1,\ldots,\omega_{N-1};h_{\mathrm{Tx}},h_{\mathrm{Rx}},P_{\mathrm{T}}) = \frac{P_{\mathrm{T}}}{N}\left(\frac{c}{2}\right)^2\left[\sum_{i=0}^{N-1}\frac{1}{\omega_i^2}\left(\frac{1}{\ell^2}+\frac{1}{\tilde{\ell}^2}\right) - \frac{2}{\ell\tilde{\ell}}\sqrt{\sum_{i=0}^{N-1}\frac{1}{\omega_i^4} + 2\sum_{i=0}^{N-1}\sum_{j<i}\frac{\cos\left((\omega_i-\omega_j)\frac{\tilde{\ell}-\ell}{c}\right)}{\omega_i^2\omega_j^2}}\,\right] \tag{7}$$

**Algorithm 1** Procedure to Calculate the Zeros of (14)

1: **function** CALCULATECRITICALDISTANCES($\boldsymbol{\omega}$, $\Delta\omega$, $d_{\text{min}}$, $d_{\text{max}}$, $h_{\text{Tx}}$, $h_{\text{Rx}}$)
2: Calculate coefficients $a_n$ and $b_n$ // According to (12)
3: Construct Fourier-Frobenius matrix $\boldsymbol{B}$ // [26, Eq. (9)]
4: $z_k = \text{EIGENVALUES}(\boldsymbol{B})$
5: $t_k = \arg z_k + 2\pi m - \mathrm{j}\log|z_k|,\ m \in \mathbb{Z}$ // [26, Eq. (10)]
6: $\Delta\ell_k = ct_k/\Delta\omega$
7: $d_k = \sqrt{(4h_{\text{Tx}}^2 - \Delta\ell_k^2)(4h_{\text{Rx}}^2 - \Delta\ell_k^2)}/(2\Delta\ell_k)$
8: **return** $d_k \in [d_{\text{min}}, d_{\text{max}}]$
9: **end function**

**Proposition 2.** *The zeros of* (14) *can be efficiently calculated numerically by computing the eigenvalues of the Fourier-Frobenius matrix according to [26, Thm. 2].*

A summary of how the critical distances $d_k$ can be calculated is presented in Algorithm 1. An implementation is made publicly available at [27].

By combining Proposition 1, Lemma 2, and Proposition 2, we can derive the following theorem, which allows us to calculate the worst-case receive power in the distance interval.

**Theorem 1** (Worst-Case Receive Power)**.** *For the considered two-ray ground reflection system using frequencies $\omega_0, \omega_1, \dots, \omega_{N-1}$ in parallel, the worst-case receive power within the distance interval $[d_{\text{min}}, d_{\text{max}}]$ is given by*

$$\min_{d\in[d_{\text{min}},d_{\text{max}}]} \underline{P_{\text{S}}} = \min\left\{\underline{P_{\text{S}}}(d_{\text{min}}), \underline{P_{\text{S}}}(d_{\text{max}}), \underline{P_{\text{S}}}(d_k) \;\middle|\; d_k \in (d_{\text{min}}, d_{\text{max}})\right\} \quad (15)$$

*where $\underline{P_{\text{S}}}$ is given according to* (7) *and $d_k \in (d_{\text{min}}, d_{\text{max}})$ can be calculated according to Algorithm 1.*

*Remark* 1. While Algorithm 1 also returns the distances at which local maxima occur, it is not necessary to filter them out. While this could be done using the second derivative of (11), this computational effort can also be spent on calculating the lower bound $\underline{P_{\text{S}}}(d_k)$ at these distances. When taking the minimum of all values in (15), the local maxima are automatically discarded. Alternatively, one could evaluate the second derivative only for the first critical distance since local minima and maxima alternate.

*Example* 1. The application of Theorem 1 is illustrated by the following numerical example. We consider a communication system with parameters $h_{\text{Tx}} = 10\,\text{m}$, $h_{\text{Rx}} = 1.5\,\text{m}$, $f_0 = 8\,\text{GHz}$, $f_1 = 8.8\,\text{GHz}$, and $f_2 = 9.6\,\text{GHz}$, i.e., $\Delta f = 800\,\text{MHz}$ and $N = 3$. The distance $d$ between transmitter and receiver is between $d_{\text{min}} = 35\,\text{m}$ and $d_{\text{max}} = 85\,\text{m}$. Using Algorithm 1, the critical distances $d_k$ where local maxima and minima occur are calculated to $d_1 = 38.7\,\text{m}$, $d_2 = 47.0\,\text{m}$, $d_3 = 52.4\,\text{m}$, $d_4 = 59.1\,\text{m}$, and $d_5 = 79.4\,\text{m}$. Local minima are found at distances $d_1$, $d_3$, and $d_5$. The receive power $P_{\text{S}}$ together with its lower bound $\underline{P_{\text{S}}}$ and the corresponding values of $d_k$ are shown in Figure 2. The global minimum within the distance interval $[d_{\text{min}}, d_{\text{max}}]$ is calculated according to (15). It is achieved at $d_5$ with a value of around $\underline{P_{\text{S}}}(d_5) = -136\,\text{dB}$. The value of the actual receive power at $d_5$ is also $P_{\text{S}}(d_5) = -136\,\text{dB}$, illustrating the tightness of the bound.

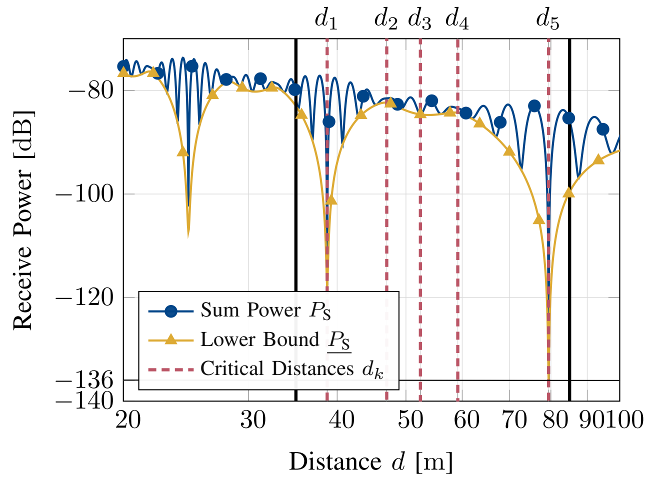


Figure 2. Sum power $P_{\text{S}}$ and its lower bound $\underline{P_{\text{S}}}$ for parameters $h_{\text{Tx}} = 10\,\text{m}$, $h_{\text{Rx}} = 1.5\,\text{m}$, $f_0 = 8\,\text{GHz}$, $f_1 = 8.8\,\text{GHz}$, and $f_2 = 9.6\,\text{GHz}$. The dashed lines indicate the critical distances $d_k$ between $d_{\text{min}} = 35\,\text{m}$ and $d_{\text{max}} = 85\,\text{m}$. The global minimum of $\underline{P_{\text{S}}}$ in $[d_{\text{min}}, d_{\text{max}}]$ is around $-136\,\text{dB}$. (Example 1)

*Remark* 2. It should be noted that it is not necessary to use only consecutive frequencies. The Fourier series approach from Lemma 2 can still be used with non-consecutive frequencies by setting the corresponding coefficients to zero. Consider an example which uses three frequencies $f_0 = 5\,\text{GHz}$, $f_1 = 6\,\text{GHz}$, and $f_2 = 8\,\text{GHz}$. While those frequencies are not uniformly spaced, we can use $\Delta f = 1\,\text{GHz}$ as an underlying frequency spacing and set the coefficient corresponding to $f = 7\,\text{GHz}$ to zero. This leads to valid Fourier coefficients $a_n$ and $b_n$, and Algorithm 1 can still be applied using this adapted calculation of $a_n$ and $b_n$ in Line 2.

## IV. Worst-Case Design for Selection Combining

After considering the worst-case power for the system when the receiver employs MRC, we now shift our focus to SC. SC is a diversity combining technique, in which the receiver only uses the strongest component [24]. In our case, this corresponds to only using the frequency among the $N$ subcarriers that achieves the highest receive power, i.e., the total receive power at distance $d$ is given as

$$P_{\text{SC}}(d, \omega_0, \dots, \omega_{N-1}) = \max_i P_{\text{R},i}(d, \omega_i)\,.$$

One advantage is its simplicity as it requires less complex synchronization compared to MRC.

In order to derive a worst-case design for SC, we establish the following relation between the total receive power $P_{\text{SC}}$ for SC and the total receive power $P_{\text{S}}$ for MRC, which we have considered in the previous section.

**Theorem 2** (Worst-Case Receive Power Selection Combining)**.** *Consider the previously described two-ray ground reflection system that is using $N$ frequencies in parallel. The receiver employs SC. For this system, the total receive power $P_{\mathrm{SC}}$ can be lower bounded using the sum power $P_{\mathrm{S}}$ and its lower bound $\underline{P_{\mathrm{S}}}$ by the following relation*

$$P_{\mathrm{SC}}(d) \geq \frac{1}{N} P_{\mathrm{S}}(d) \geq \frac{1}{N} \underline{P_{\mathrm{S}}}(d)\,. \tag{16}$$

*Proof.* As a base for the proof, we use Hölder's inequality [28, Thm. 13]

$$\|fg\|_1 \leq \|f\|_p \|g\|_q\,. \tag{17}$$

with $p, q \in [1, \infty]$, $\frac{1}{p} + \frac{1}{q} = 1$, and $f, g \in \mathbb{R}^N$. Setting $p = \infty$ and $q = 1$ yields

$$\sum_{i=1}^{N} |f_i g_i| \leq \max_i |f_i| \cdot \sum_{i=1}^{N} |g_i|\,, \tag{18}$$

where we have additionally used the expressions for the maximum norm $\|f\|_\infty$ and Manhattan norm $\|g\|_1$. Next, we set $f = (P_{\mathrm{R},0}, P_{\mathrm{R},2}, \dots, P_{\mathrm{R},N-1})^\mathsf{T}$ and $g = (1, 1, \dots, 1)^\mathsf{T}$, for which by Hölder's inequality the following holds

$$\sum_{i=0}^{N-1} P_{\mathrm{R},i} \leq N \max_i P_{\mathrm{R},i}\,. \tag{19}$$

Rearranging this expression and combining it with the definitions of the sum power $P_{\mathrm{S}}$ in the case of MRC, its lower bound $\underline{P_{\mathrm{S}}}$, and $P_{\mathrm{SC}}$ in the case of SC yields (16). □

The result of Theorem 2 is important because it can be used to establish the usefulness of frequency assignment schemes for both MRC and SC simultaneously. This connection will be discussed further in the following section.

## V. Frequency Assignment for Worst-Case Design

Using Theorem 1 from Section III, we can calculate the worst-case receive power for a given set of assigned frequencies. In this section, we compare different approaches to assign frequencies such that the worst-case receive power is maximized.

In this section, we consider the scenario that the frequency spacing $\Delta f$ cannot be freely changed, but a fixed set of $N$ available frequencies is given. For this set, we still assume an equal spacing according to (8). However, we now aim to assign $K$ out of the $N$ frequencies to the user, $0 < K \leq N$, such that the worst-case receive power is maximized.

Since this is a non-linear combinatorial optimization problem, we compare different numerical solution algorithms. However, due to the complexity of the problem, it should be noted that it is infeasible to find the global optimum for practical values of $K$ and $N$ as there are $\binom{N}{K}$ ways to select $K$ out of the $N$ frequencies.

Since Theorem 1 applies specifically to MRC at the receiver, we establish in the following corollary that the frequency allocation schemes for worst-case design of MRC systems are also valid for SC. Therefore, all results presented in this section apply to both MRC and SC (up to a scaling factor).

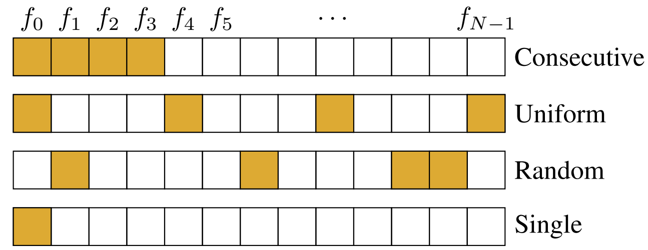


Figure 3. Exemplary illustration of the different frequency assignment schemes. In this case, $K = 4$ out of $N$ frequencies are assigned to the user. In the consecutive scheme, the first $K$ frequencies are assigned. In the random scheme, $K$ frequencies are randomly selected, and in the uniform scheme, the $K$ frequencies are uniformly spaced over all $N$ available frequencies. As an additional baseline comparison, we use the case that only the single frequency $f_0$ is used.

**Corollary 1.** *Frequency assignment schemes for worst-case design of MRC systems are also applicable for SC at the receiver. While the same frequency assignment can be used, the final receive power $P_{\mathrm{SC}}$ for SC is scaled by the factor $1/K$ compared to the sum power $P_{\mathrm{S}}$ for MRC.*

*Proof.* From Theorem 2, we know that $1/K \cdot \underline{P_{\mathrm{S}}}$ is a lower bound on the receive power $P_{\mathrm{SC}}$ for SC. A frequency assignment that maximizes $\underline{P_{\mathrm{S}}}$ therefore also maximizes a lower bound of $P_{\mathrm{SC}}$ as the constant $1/K$ is not part of the optimization variable and only scales the final solution. □

### A. Frequency Assignment Schemes

In the following, we compare four different frequency assignment algorithms. An illustration of the different baseline schemes is shown in Figure 3. As a comparison, we also calculate the worst-case receive power for the single frequency case, when only $f_0$ is used.

1) *Greedy Algorithm:* The first assignment algorithm is a greedy algorithm, which performs $K$ iterations. In each step, the frequency, which maximizes the worst-case receive power when added, is added to the set of assigned frequencies. A detailed description is given as Algorithm 2.
2) *Random Assignment:* As a simple comparison, we use a random allocation, where $K$ out of the $N$ available frequencies are randomly selected with uniform probability.

**Algorithm 2** Greedy Frequency Assignment Algorithm

```
function GreedyAssign(ω, Δω, d_min, d_max, h_Tx, h_Rx)
    A = ∅                          // Initialize assigned frequencies A
    k = 1                          // Initialize iteration counter
    while k ≤ K do
        f^(k) = argmax_{f_i} min_{d∈[d_min,d_max]} P_S(d, A ∪ {f_i})
        A = A ∪ {f^(k)}            // Add best frequency
        k = k + 1                  // Increment counter
    end while
    return A
end function
```

Table I

WORST-CASE RECEIVE POWER FOR DIFFERENT FREQUENCY ASSIGNMENT ALGORITHMS FOR EXEMPLARY SYSTEM PARAMETERS. THE ALGORITHMS ASSIGN $K$ OUT OF $N$ FREQUENCIES FROM THE FREQUENCY SET $\{f_0 + i\Delta f \mid i = 0, 1, \ldots, N-1\}$. THE FIXED PARAMETERS FOR ALL EVALUATED SCENARIOS ARE $f_0 = 26.5\,\text{GHz}$, $h_{\text{Tx}} = 10\,\text{m}$, AND $h_{\text{Rx}} = 1.5\,\text{m}$. FOR COMPARISON, THE SCENARIO WITH ONLY THE SINGLE FREQUENCY $f_0$ IS SHOWN.

| $N$ | $\Delta f$[MHz] | $K$ | $d_{\min}$[m] | $d_{\max}$[m] | Greedy [dB] | Random [dB] | Consecutive [dB] | Uniform [dB] | Single [dB] |
|---|---|---|---|---|---|---|---|---|---|
| 300 | 10 | 16 | 35 | 40 | −90.5 | −91.8 | −96.6 | −90.9 | −128.0 |
| 300 | 10 | 16 | 90 | 100 | −98.4 | −99.6 | −111.8 | −98.5 | −150.9 |
| 300 | 10 | 16 | 150 | 200 | −104.3 | −106.1 | −123.8 | −105.4 | −168.0 |
| 300 | 10 | 8 | 150 | 200 | −104.3 | −106.6 | −129.8 | −105.4 | −168.0 |
| 300 | 10 | 32 | 150 | 200 | −104.3 | −105.7 | −117.8 | −105.4 | −168.0 |
| 60 | 50 | 16 | 90 | 100 | −98.4 | −99.4 | −99.3 | −98.4 | −150.9 |
| 400 | 1 | 16 | 90 | 100 | −99.7 | −104.6 | −131.7 | −104.2 | −150.9 |
| 3333 | 0.12 | 8 | 90 | 100 | −99.7 | −105.2 | −150.2 | −104.2 | −150.9 |

3) *Consecutive Frequencies:* Another simple approach is to assign a block of consecutive frequencies to the user. For this baseline scheme, the user is assigned the first $K$ frequencies, i.e., $f_0, f_1, \ldots, f_{K-1}$.
4) *Uniform Frequencies:* As a final comparison, we assign frequencies, which are uniformly spaced over all $N$ available frequencies, i.e., every $\lceil N/K \rceil$-th frequency is assigned to the user.

### B. Numerical Evaluation

We have evaluated all four frequency assignment schemes in various numerical simulations. For the frequency band, we have used the n257 band from the 5G NR frequency range 2 ($26.5\,\text{GHz}$ to $29.5\,\text{GHz}$) [29]. The results for all parameter constellations can be found in Table I. For each algorithm, the worst-case receive power $\min_{d\in[d_{\min}, d_{\max}]} \underline{P_{\text{S}}}$ is shown, i.e., higher values correspond to a better performance. For comparison, we additionally show the values for the case that only a single frequency $f_0$ is used. For the random assignment, the average of $250$ runs for each parameter combination is reported. The source code to reproduce all of the results can be found in [27].

First, it can be seen that the greedy algorithm outperforms the other assignment schemes in every evaluated scenario. When the overall frequency band, from which the frequencies can be assigned, is wide, the uniform scheme performs close to the greedy algorithm. In Table I, this can be seen from the first lines, where the overall frequency band is the full $3\,\text{GHz}$ and the performance of the advantage of the greedy algorithm over the uniform scheme is only up to around $1\,\text{dB}$.

However, in the case that the frequency band is narrow, the greedy algorithm significantly outperforms all schemes. In Table I, this corresponds to the last two lines where the overall frequency range is only a single channel with a bandwidth of $400\,\text{MHz}$. In those cases, the greedy algorithm demonstrates an improvement of more than $4\,\text{dB}$ over the uniform scheme and more than $30\,\text{dB}$ over the consecutive frequency assignment. This performance improvement comes at the cost of a higher complexity. While all baseline algorithms do not require any computation of the worst-case power to generate the set of assigned frequencies, the greedy algorithm with its $K$ iterations needs $N + (N-1) + \cdots + (N-K+1) = \frac{1}{2}(K(2N-K+1))$ calculations of the worst-case power according to (15). However, for practical values of $N$ and $K$, this is an acceptable level of required computations.

Furthermore, it can be seen that the results for $K = 8$, $K = 16$, and $K = 32$ are identical for the greedy assignment when all other parameters are fixed to $N = 300$, $\Delta f = 10\,\text{MHz}$, $d_{\min} = 150\,\text{m}$, and $d_{\max} = 200\,\text{m}$. Thus, adding more frequencies does not necessarily improve the worst-case receive power. It would therefore be sufficient to only assign $K = 8$ carriers without degrading the worst-case performance and leaving more frequencies available for other users and simultaneously requiring less computations for the greedy assignment.

Finally, it should be noted that all evaluated frequency assignment schemes significantly improve the performance over using only the single frequency $f_0$. Even when only $K = 8$ consecutive frequencies are used in parallel, a worst-case improvement of almost $40\,\text{dB}$ is achieved for the scenario with $N = 300$, $\Delta f = 10\,\text{MHz}$, $d_{\min} = 150\,\text{m}$, and $d_{\max} = 200\,\text{m}$. Using the greedy algorithm, this improvement is further increased to more than $60\,\text{dB}$ over using only a single frequency.

The results reported in Table I correspond to the worst-case sum power for MRC from Theorem 1. According to Corollary 1, the worst-case receive power values for SC are shifted by $-10\log_{10}(K)\,\text{dB}$. For instance, the receive power for the greedy assignment would be $-108.7\,\text{dB}$ in the last line of Table I for SC at the receiver.

## VI. CONCLUSION

In this work, we have revisited the classical two-ray model and provided a new design approach using frequency diversity for scenarios with only partial channel state information, such that a certain QoS can always be guaranteed despite the uncertainty about the channel. We have derived the worst-case receive power for a given set of frequencies and compared multiple algorithms to assign frequencies such that the worst-case power is maximized. Additionally, we have shown that our approach is applicable to both MRC and SC at the receiver.

Besides being able to guarantee a certain QoS level, even in the worst case, the proposed approach is resource efficient. In particular, our results show that adding more frequencies has a diminishing value. Thus, good performance can be achieved by assigning only a small number of less than ten frequencies to a single user, leaving more frequencies to be used by other users. Based on this, the optimization of the frequency assignment to multiple users arises as an extension for future work. Apart from maximizing the worst-case receive power of multiple users, other problems remain open. This includes the extension to more than two multi-path components and the optimization of the frequency spacing $\Delta f$.

## References


[1] J. Park et al., "Extreme ultra-reliable and low-latency communication," *Nat. Electron.*, vol. 5, no. 3, pp. 133–141, Mar. 2022. DOI: 10.1038/s41928-022-00728-8.

[2] M. Bennis, M. Debbah, and H. V. Poor, "Ultrareliable and low-latency wireless communication: Tail, risk, and scale," *Proc. IEEE*, vol. 106, no. 10, pp. 1834–1853, Oct. 2018. DOI: 10.1109/JPROC.2018.2867029.

[3] F. Haber and M. Noorchashm, "Negatively correlated branches in frequency diversity systems to overcome multipath fading," *IEEE Trans. Commun.*, vol. 22, no. 2, pp. 180–190, Feb. 1974. DOI: 10.1109/TCOM.1974.1092173.

[4] K.-L. Besser and E. A. Jorswieck, "Reliability bounds for dependent fading wireless channels," *IEEE Trans. Wirel. Commun.*, vol. 19, no. 9, pp. 5833–5845, Sep. 2020. DOI: 10.1109/TWC.2020.2997332.

[5] K.-L. Besser, P.-H. Lin, and E. A. Jorswieck, "On fading channel dependency structures with a positive zero-outage capacity," *IEEE Trans. Commun.*, vol. 69, no. 10, pp. 6561–6574, Oct. 2021. DOI: 10.1109/TCOMM.2021.3097755.

[6] Y. Hou et al., "A copula-based approach to performance analysis of fluid antenna system with multiple fixed transmit antennas," *IEEE Wirel. Commun. Lett.*, vol. 13, no. 2, pp. 501–504, Feb. 2024. DOI: 10.1109/lwc.2023.3333554.

[7] E. Jorswieck, M. Weber, P. Schlegel, and K.-L. Besser, "On massive antenna channel models with dependent fading: Theory and experiments," in *Proc. 2024 19th Int. Symp. Wirel. Commun. Syst. (ISWCS)*, IEEE, Jul. 2024. DOI: 10.1109/iswcs61526.2024.10639145.

[8] R. J. Weiler, M. Peter, W. Keusgen, A. Kortke, and M. Wisotzki, "Millimeter-wave channel sounding of outdoor ground reflections," in *Proc. 2015 IEEE Radio Wirel. Symp. (RWS)*, IEEE, Jan. 2015, pp. 95–97. DOI: 10.1109/RWS.2015.7129712.

[9] J. Naganawa, K. Morioka, J. Honda, N. Kanada, N. Yonemoto, and Y. Sumiya, "Antenna configuration mitigating ground reflection fading on airport surface for AeroMACS," in *Proc. 2017 IEEE Conf. Antenna Meas.& Appl. (CAMA)*, IEEE, Dec. 2017. DOI: 10.1109/cama.2017.8273487.

[10] D. W. Matolak and R. Sun, "Air-ground channel characterization for unmanned aircraft systems: The over-freshwater setting," in *Proc. 2014 Integr. Commun. Navig. Surveill. Conf. (ICNS)*, IEEE, Apr. 2014. DOI: 10.1109/icnsurv.2014.6819996.

[11] D. W. Matolak and R. Sun, "Air-ground channel characterization for unmanned aircraft systems—Part I: Methods, measurements, and models for over-water settings," *IEEE Trans. Veh. Technol.*, vol. 66, no. 1, pp. 26–44, Jan. 2017. DOI: 10.1109/tvt.2016.2530306.

[12] C.-C. Chiu, A.-H. Tsai, H.-P. Lin, C.-Y. Lee, and L.-C. Wang, "Channel modeling of air-to-ground signal measurement with two-ray ground-reflection model for UAV communication systems," in *Proc. 2021 30th Wirel. Opt. Commun. Conf. (WOCC)*, IEEE, Oct. 2021. DOI: 10.1109/wocc53213.2021.9603250.

[13] E. N. Papasotiriou, J. Kokkoniemi, A.-A. A. Boulogeorgos, J. Lehtomäki, A. Alexiou, and M. Juntti, "A new look to 275 to 400 GHz band: Channel model and performance evaluation," in *Proc. 2018 IEEE 29th Annu. Int. Symp. Pers. Indoor Mob. Radio Commun. (PIMRC)*, IEEE, 2018. DOI: 10.1109/PIMRC.2018.8580934.

[14] K. Guan et al., "On millimeter wave and THz mobile radio channel for smart rail mobility," *IEEE Trans. Veh. Technol.*, vol. 66, no. 7, pp. 5658–5674, Jul. 2017. DOI: 10.1109/TVT.2016.2624504.

[15] W. Khawaja, O. Ozdemir, F. Erden, E. Ozturk, and I. Guvenc, "Multiple ray received power modelling for mmWave indoor and outdoor scenarios," *IET Microw. Antennas& Propag.*, vol. 14, no. 14, pp. 1825–1836, Nov. 2020. DOI: 10.1049/iet-map.2020.0046.

[16] E. Zöchmann, K. Guan, and M. Rupp, "Two-ray models in mmWave communications," in *Proc. 2017 IEEE 18th Int. Work. Signal Process. Adv. Wirel. Commun. (SPAWC)*, IEEE, Jul. 2017. DOI: 10.1109/SPAWC.2017.8227681.

[17] L. Rapaport, A. Etinger, B. Litvak, G. Pinhasi, and Y. Pinhasi, "Quasi optical multi-ray model for wireless communication link in millimeter wavelengths," *MATEC Web Conf.*, vol. 210, 03006, 2018. DOI: 10.1051/matecconf/201821003006.

[18] S. Jaeckel, L. Raschkowski, S. Wu, L. Thiele, and W. Keusgen, "An explicit ground reflection model for mm-Wave channels," in *Proc. 2017 IEEE Wirel. Commun. Netw. Conf. Work.*, IEEE, Mar. 2017. DOI: 10.1109/wcncw.2017.7919093.

[19] G. Kwon, A. Conti, H. Park, and M. Z. Win, "Joint communication and localization in millimeter wave networks," *IEEE J. Sel. Top. Signal Process.*, vol. 15, no. 6, pp. 1439–1454, 2021. DOI: 10.1109/JSTSP.2021.3113115.

[20] D. Capriglione, D. Casinelli, and L. Ferrigno, "Use of frequency diversity to improve the performance of RSSI-based distance measurements," in *Proc. 2015 IEEE Int. Work. Meas.& Netw. (M&N)*, IEEE, Oct. 2015. DOI: 10.1109/iwmn.2015.7322973.

[21] K.-L. Besser, E. A. Jorswieck, and J. P. Coon, "Multi-user frequency assignment for ultra-reliable mmWave two-ray channels," in *Proc. 20th Int. Symp. Model. Optim. Mob. Ad hoc Wirel. Netw. (WiOpt)*, IEEE, Sep. 2022, pp. 283–290. DOI: 10.23919/WiOpt56218.2022.9930571.

[22] K.-L. Besser, E. A. Jorswieck, and J. P. Coon, "An efficient frequency diversity scheme for ultra-reliable communications in two-path fading channels," *J. Commun. Netw.*, vol. 26, no. 4, pp. 409–423, Aug. 2024. DOI: 10.23919/JCN.2024.000031.

[23] T. S. Rappaport, *Wireless Communications: Principles and Practice*, 2nd ed. Prentice Hall, 2002.

[24] D. Brennan, "Linear diversity combining techniques," *Proc. IRE*, vol. 47, no. 6, pp. 1075–1102, Jun. 1959. DOI: 10.1109/JRPROC.1959.287136.

[25] R. N. Bracewell, *The Fourier Transform and Its Applications* (McGraw-Hill Series in Electrical and Computer Engineering: Circuits and Systems), 3rd ed. McGraw Hill, 2000.

[26] J. P. Boyd, "Computing the zeros, maxima and inflection points of Chebyshev, Legendre and Fourier series: Solving transcendental equations by spectral interpolation and polynomial rootfinding," *J. Eng. Math.*, vol. 56, pp. 203–219, Nov. 2006. DOI: 10.1007/s10665-006-9087-5.

[27] K.-L. Besser. "Frequency assignment for guaranteed QoS in two-ray models with limited location information, Supplementary material. "[Online]. Available: https://github.com/klb2/two-ray-freq-assignment.

[28] G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities* (Cambridge Mathematical Library). Cambridge University Press, 1934.

[29] *5G NR: User equipment (UE) radio transmission and reception*, Part 2: Range 2 Standalone, ETSI TS 138 101-2: NR, version 17.8.0, 3GPP, Jan. 11, 2023.