Timely Trajectory Reconstruction in Finite Buffer Remote Tracking Systems

Sunjung Kang, Vishrant Tripathi, and Christopher G. Brinton

***Abstract*—Remote tracking systems play a critical role in applications such as IoT, monitoring, surveillance and healthcare. In such systems, maintaining both real-time state awareness (for online decision making) and accurate reconstruction of historical trajectories (for offline post-processing) are essential. While the Age of Information (AoI) metric has been extensively studied as a measure of freshness, it does not capture the accuracy with which past trajectories can be reconstructed. In this work, we investigate reconstruction error as a complementary metric to AoI, addressing the trade-off between timely updates and historical accuracy. Specifically, we consider three policies, each prioritizing different aspects of information management: Keep-Old, Keep-Fresh, and our proposed Inter-arrival-Aware dropping policy. We compare these policies in terms of impact on both AoI and reconstruction error in a remote tracking system with a finite buffer. Through theoretical analysis and numerical simulations of queueing behavior, we demonstrate that while the Keep-Fresh policy minimizes AoI, it does not necessarily minimize reconstruction accuracy. In contrast, our proposed Inter-arrival-Aware dropping policy dynamically adjusts packet retention decisions based on generation times, achieving a balance between AoI and reconstruction error. Our results provide key insights into the design of efficient update policies for resource-constrained IoT networks.**

I. Introduction

With the emergence of the Internet of Things (IoT), remote tracking systems have been gaining much attention in monitoring for environmental, agricultural, urban and personal healthcare applications [1]. These systems rely on sensors transmitting time-sensitive data to remote monitors for real-time tracking and analysis of objects and their trajectories. Maintaining the freshness and accuracy of updates is crucial for ensuring effective system performance in such applications.

The Age of Information (AoI) has been widely studied as a measure of information freshness, quantifying the time elapsed since the last update was generated [2], [3]. The AoI has become a cornerstone for analyzing timeliness in applications where real-time updates are critical. However, many applications also require accurate trajectory reconstruction alongside real-time monitoring. For instance, in a surveillance application, real-time monitoring enables immediate threat detection, while trajectory reconstruction helps analyze long-term movement patterns. Similarly, in healthcare, wearable devices provide real-time alerts while reconstructing activity trajectories for long-term fitness tracking and personalized recommendations. To capture these dual objectives, we consider employing reconstruction error as a complementary metric to AoI.

Balancing freshness and reconstruction accuracy, however, is particularly challenging in resource-limited IoT environments. The transmitter's constrained buffer necessitates strategic packet retention and discarding. Prior studies have shown that age-optimal policies retain only the freshest updates, requiring minimal buffering [4]. However, this approach discards older packets, which are critical for trajectory reconstruction.

In this paper, we investigate how packet-dropping policies and finite buffer sizes impact both the AoI and the reconstruction accuracy in remote tracking systems. We compare three policies: (i) Keep-Old, which prioritizes historical packets; (ii) Keep-Fresh, which minimizes AoI by discarding older updates; and (iii) our Inter-arrival-Aware policy, which dynamically balances both objectives based on system conditions.

A. Related Works

A key challenge in AoI research is optimizing sampling and scheduling to minimize information staleness while adhering to system constraints such as energy, bandwidth, and communication delays [2]–[18]. Many works assume that updates can be generated at will, focusing on optimal transmission policies under different conditions, including random access, energy constraints, and heterogeneous channels [2], [3], [5]–[7]. Others consider random update arrivals, analyzing queuing strategies to minimize AoI [4], [8], [9]. Packet replacement policies have been proposed to improve both the AoI and the peak age of information (PAoI) by discarding outdated packets [10]. While these approaches have provided significant advancements in age-optimal sampling and packet management, they often overlook the importance of historical data, which can be critical for applications requiring accurate reconstruction of past trajectories. A recent work introduces a randomized update policy balancing current and past state information, combining LCFS and FCFS queuing disciplines for improved tracking performance [19]. However, this work assumes an infinite buffer, overlooking the challenges of packet management under resource constraints.

In multi-source networks, scheduling plays a critical role in AoI minimization. Centralized strategies, such as greedy, Max-Weight, and Whittle's Index policies, have been studied for unreliable channels [11]–[13]. Joint sampling and scheduling designs further reduce AoI [14]. Other studies explore scheduling for correlated sources, emphasizing inference ac-

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curacy beyond freshness [15], [16]. Decentralized approaches like Fresh-CSMA, asynchronous sleep-wake scheduling, and reinforcement learning-based data collection aim to optimize scheduling in dynamic environments [17], [18], [20]. These works share a common focus on minimizing AoI but often overlook historical information, as seen in single-source AoI optimization, where resource constraints make balancing timely updates and past data retention more challenging.

Research in remote estimation has explored various methodologies to optimize state estimation accuracy across different contexts and applications. Event-triggered sampling policies and age-aware estimation frameworks have been studied for minimizing estimation error in different network conditions [21]–[24]. Multi-source scenarios further introduce resource-sharing constraints, with centralized and decentralized scheduling strategies proposed to balance estimation accuracy and communication costs [25], [26]. Recent works explore Whittle index-based updating and one-bit policies for efficient estimation in random access systems [27], [28]. These works focus on accurately estimating the current state of the source, whereas our work considers reconstructing past states, that is, estimating missing information over time rather than only the present state.

B. Contributions

In this paper, we consider a remote tracking system where packets are randomly generated, queued, and transmitted from a source to a receiver. Unlike conventional AoI-based policies that focus solely on maintaining information freshness, we aim to balance two objectives: (i) minimizing AoI for real-time monitoring, and (ii) reducing reconstruction error for accurate trajectory reconstruction. Unlike remote estimation, which minimizes immediate state errors, our approach captures cumulative historical accuracy in order to improve the reconstruction of past states.

Our contributions can be summarized as follows:

- We formulate the problem of balancing real-time monitoring and trajectory reconstruction in remote tracking systems. Using reconstruction error alongside AoI, we capture the trade-off between ensuring timely updates for responsiveness and retaining older data for accurate historical reconstruction.
- We analyze the average peak age and reconstruction error under two fixed packet-dropping policies: (i) Keep-Old, which prioritizes historical data, and (ii) Keep-Fresh, which retains the most recent packets. Our results show that Keep-Old generally performs worse in both metrics, with Keep-Fresh providing better overall performance.
- To further minimize reconstruction accuracy while maintaining freshness, we introduce (iii) the Inter-arrival-Aware dropping policy, which dynamically selects packets based on their generation times. By considering temporal diversity, this policy improves reconstruction accuracy without significantly compromising on age performance.
- Finally, through numerical experiments, we evaluate how different dropping policies and buffer sizes influence AoI and reconstruction error. Our results provide insights into designing efficient update management strategies under resource constraints.

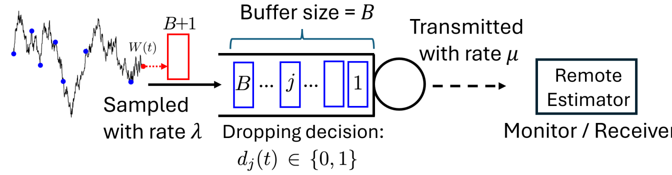


Fig. 1: System model.

Due to page limitations, all proofs have been deferred to our online technical report [29].

II. System Model and Problem Formulation

We consider a network where a sensor generates packets containing the source's state and stores them in a finite buffer queue before transmission to an estimator, as shown in Fig. 1. Packets in the queue get served under the Last-Come-First-Serve (LCFS) queuing discipline. The receiver uses these packets for both real-time tracking and historical reconstruction. The transmitter must manage buffer constraints and limited transmissions, making packet-dropping decisions based on freshness and historical importance.

A. System Model

The source's state $W(t)$ evolves according to a Wiener process. Update packets are generated according to a Poisson process with rate $\lambda > 0$. Let t_j denote the generation time of packet j. Update packet j contains the state $W(t_j)$ at its generation time t_j and is stored in a finite buffer of size B. We consider a non-preemptive LCFS queuing system, where packets in service are completed without interruption.

When the buffer is full upon a new packet's generation, the transmitter decides whether to drop the new packet or replace an existing packet. Let $d_j(t) \in \{0, 1\}$ denote the dropping decision at time t, where $j = 1, ..., B, B+1$. The value $d_j(t) = 1$ indicates that the j^{th} packet is selected for dropping. The indices $j = 1$, B and $B+1$ correspond to the packet at the head of the buffer, at the tail of the buffer and the new packet, respectively. If $d_{B+1}(t) = 1$, the new packet is dropped, and the buffer remains unchanged. Conversely, if $d_j(t) = 1$ for $j \in \{1, ..., B\}$, the selected packet currently in the buffer is dropped, and the new packet is stored at the buffer's tail. We will design policies for which the dropping decision takes into account both the freshness of the information and the historical importance of the packet for reconstructing the past trajectory.

At the receiver, the received packets are used to reconstruct the source's historical trajectory. However, packet drops and a limited sampling rate create information gaps, introducing uncertainty in offline reconstruction over a long horizon. Thus, the receiver must estimate the state of the source during periods for which no direct observations are available.

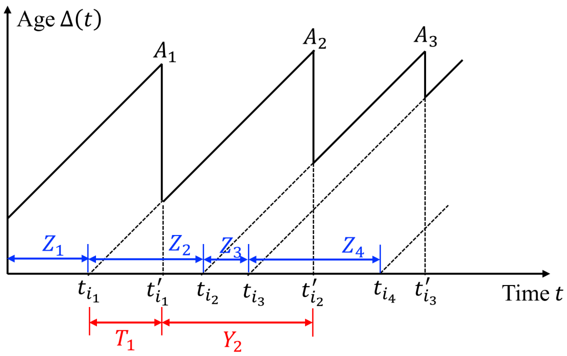


Fig. 2: Sample path of age $\Delta(t)$, where packet 2 is delivered after packet 3, making it stale and thus not reducing the age. Consequently $i_2 = 3$.

B. Age of Information

The packets serve a dual role: tracking the current state of the system in real time and reconstructing past states for historical analysis. The *Age of Information (AoI)* $\Delta_j(t)$ for update packet j is a key measure of how fresh a packet is, where $\Delta_j(t) = t - t_j$. A low AoI indicates that the packet is more relevant for real-time tracking of the current state of the source. The AoI $\Delta(t)$ for the system at time t is defined as the age of the freshest delivered packet:

$$\Delta(t) = t - \max_{\{i: t'_i \leq t\}} t_i, \tag{1}$$

where t'_i is the delivery time of update packet i. We call that packet j is fresh at time t if $\Delta_j(t) < \Delta(t)$.

We note that not all generated packets are delivered in the generation order due to the finite buffer size and LCFS queueing discipline. Let i_k denote the index of the k^{th} fresh packet. Further, we let A_k denote the k^{th} peak age, which is the maximum value of the age immediately before the k^{th} fresh packet is received at the receiver, i.e., $A_k = \Delta((t'_{i_k})^-)$, where $f(t^-) := \lim_{s \to t^-} f(s)$.

Fig. 2 shows a sample path of age evolution $\Delta(t)$. Let $T_{i_k} := t'_{i_k} - t_{i_k}$ denote the system time of packet i_k, capturing the total time from generation to the delivery of the k^{th} fresh packet. Let $Y_k := t'_{i_k} - t'_{i_{k-1}}$ denote the inter-delivery time, which is the time interval between the deliveries of packets i_{k-1} and i_k. Then, the k^{th} peak age A_k can be expressed as $A_k = T_{i_{k-1}} + Y_k$. The time-average peak age is then given by

$$\bar{A} = \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} A_k = \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} (T_{i_{k-1}} + Y_k). \tag{2}$$

Due to the ergodicity of the process of packet generations and deliveries, the time-average peak age can be expressed as

$$\bar{A} = \mathbb{E}[T_{i_{k-1}}] + \mathbb{E}[Y_k]. \tag{3}$$

C. Reconstruction Error

In addition to real-time tracking, the receiver aims to reconstruct the historical trajectory of the source's state using the received packets. However, due to packet drops and the limited sampling rate, there is often missing information between the times at which packets are generated. Given the received packets generated at times t_i and t_{i+1}, the receiver estimates $W(t)$ for $t \in (t_i, t_{i+1})$ using the Linear Minimum Mean Square Error (LMMSE) estimator:

$$\hat{W}(t) = W(t_i) + \frac{t - t_i}{t_{i+1} - t_i}(W(t_{i+1}) - W(t_i)). \tag{4}$$

Let $\tilde{T}$ denote the latest delivery time before time t, i.e., $\tilde{T} = \max\{t'_i \ : \ t'_i \leq T\}$. The reconstruction error $\text{RE}(T)$ over $[0, T]$ is defined by

$$\text{RE}(T) := \tfrac{1}{\tilde{T}} \int_0^{\tilde{T}} (W(s) - \hat{W}(s))^2 \, ds. \tag{5}$$

For a Wiener process, the reconstruction error simplifies to

$$\text{RE}(T) = \tfrac{n(\tilde{T})}{\tilde{T}} \tfrac{1}{n(\tilde{T})} \sum_{i=1}^{n(\tilde{T})} \tfrac{(t_{i+1} - t_i)^2}{6}, \tag{6}$$

where $n(T)$ is the number of packets delivered by time T.

As $T \to \infty$, we have $\frac{n(\tilde{T})}{\tilde{T}} \to \lambda_{\text{eff}} = \lambda(1 - P_L)$, where λ_{eff} denote the effective arrival rate, and P_L is the packet loss probability. Let Z_i be the inter-generation time between $(i-1)^{th}$ and i^{th} update packets, i.e., $Z_i = t_i - t_{i-1}$ with $t_0 = 0$. Due to the ergodicity of the process of packet generations and deliveries, we have

$$\lim_{T \to \infty} \frac{1}{n(\tilde{T})} \sum_{i=1}^{n(\tilde{T})} (t_{i+1} - t_i)^2 = \mathbb{E}[Z_i^2]. \tag{7}$$

Hence, as $T \to \infty$, the average reconstruction error can be obtained as

$$\overline{\text{RE}} := \lim_{T \to \infty} \text{RE}(T) = \tfrac{\lambda_{\text{eff}} \mathbb{E}[Z_i^2]}{6}. \tag{8}$$

Let π_i be the steady state probability of queue length being i for $i \in \{0, 1, ..., B+1\}$, which is given by

$$\pi_i = \tfrac{\rho^i}{\sum_{j=0}^{B+1} \rho^j} \quad \text{for } i = 0, 1, \cdots, B+1, \tag{9}$$

where $\rho = \frac{\lambda}{\mu}$ is the traffic intensity. Note that, in an $M/M/1/B+1$ queue, the packet loss probability P_L is equivalent to the probability that the system is busy, i.e., $P_L = \pi_{B+1}$. Thus, the effective arrival rate is given by

$$\lambda_{\text{eff}} = \lambda(1 - \pi_{B+1}) = \tfrac{\lambda(1-\rho^{B+1})}{1-\rho^{B+2}}. \tag{10}$$

The expected inter-arrival time $\mathbb{E}[Z_i]$ is independent of the dropping policy, which is given by $\frac{1}{\lambda_{\text{eff}}}$. However, the second moment $\mathbb{E}[Z_i^2]$ depends on the dropping policy, affecting queueing dynamics and, consequently, reconstruction error.

III. $M/M/1/2$ Queuing Systems

In this section, we analyze an $M/M/1/2$ queue, where both inter-arrival times and services times follow a memoryless, exponential distribution (referring to M). The system consists of a single server and one waiting slot, focusing on the Keep-Old and Keep-Fresh policies. While prior studies have focused on the AoI as a performance measure, we extend this analysis to include reconstruction error. In a non-preemptive $M/M/1/2$ system, all delivered packets are fresh ($i_k = k$), as the buffer holds at most one packet, ensuring that the transmitted packet is always the freshest available.

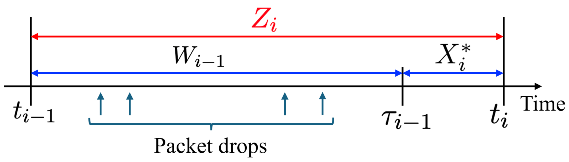


Fig. 3: Sample path of packet arrivals and departures under the Keep-Old policy.

A. Keep-Old Policy

The Keep-Old policy retains older packets in the buffer, discarding new arrivals when full ($d_1(t) = 0$ and $d_2(t) = 1$). Under this policy, the average peak age $\bar{A}$ is analyzed in [4]:

$$\bar{A}_{\text{Keep-Old}} = \frac{1}{\lambda} + \frac{3}{\mu} - \frac{2}{\lambda+\mu}. \tag{11}$$

which asymptotically approaches $\frac{3}{\mu}$ as $\lambda \to \infty$.

As discussed in Section II-C, the reconstruction error depends on the second moment $\mathbf{E}[Z_i^2]$ of the inter-arrival times between successfully delivered packets. The inter-arrival time Z_i consists of two independent components: the waiting time W_{i-1}, which depends on whether the packet finds the system busy upon arrival, and the service-to-arrival time X_i^*, the interval between the service start of packet $i-1$ and the arrival of packet i (see Fig. 3). This leads to the following expression:

$$\mathbb{E}[Z_i^2] = \mathbb{E}[W_{i-1}^2] + 2\mathbb{E}[W_{i-1}]\mathbb{E}[X_i^*] + \mathbb{E}[(X_i^*)^2]. \tag{12}$$

Using memoryless properties of inter-arrival times and service times and the PASTA property, we can obtain

$$\mathbb{E}[Z_i^2] = \frac{2}{\mu^2} + \frac{2}{\lambda^2}. \tag{13}$$

Combining with (10), we present the following lemma.

Lemma 3.1: *The long-term average reconstruction error under the Keep-Old policy in an M/M/1/2 queuing system is given by*

$$\overline{RE}_{\textit{Keep-Old}} = \frac{(\lambda+\mu)(\lambda^2+\mu^2)}{3\lambda\mu(\lambda^2+\lambda\mu+\mu^2)}. \tag{14}$$

From this lemma, we can see that the average reconstruction error asymptotically approaches $\frac{1}{3\mu}$. We provide a detailed proof in our technical report [29].

B. Keep-Fresh Policy

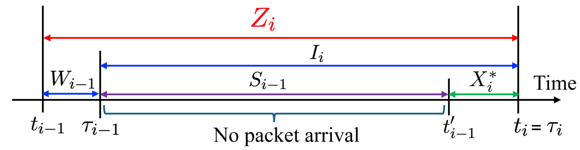
The Keep-Fresh policy replaces the existing packet in the buffer when a new arrival occurs, i.e, $d_1(t) = 1$ and $d_2(t) = 0$, ensuring that only the freshest updates are retained. This policy minimizes AoI, making it ideal for real-time applications. The average peak AoI under this policy is given by [4]

$$\bar{A}_{\text{Keep-Fresh}} = \frac{1}{\mu} + \frac{\lambda}{(\lambda+\mu)^2} + \frac{1}{\lambda} + \frac{1}{\mu}\frac{\lambda}{\lambda+\mu}, \tag{15}$$

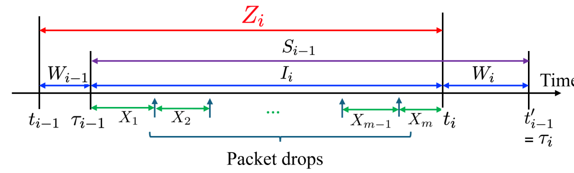
which asymptotically approaches $\frac{2}{\mu}$ as $\lambda \to \infty$.

Under the Keep-Fresh policy, the inter-arrival time Z_i consists of the waiting time W_{i-1} and the interval I_i between the service start of the previous packet and the arrival of the next one as shown in Fig. 4. The second moment of Z_i is given by

$$\mathbb{E}[Z_i^2] = \mathbb{E}[W_{i-1}^2] + 2\mathbb{E}[W_{i-1}]\mathbb{E}[I_i] + \mathbb{E}[I_i^2], \tag{16}$$



(a) No arrival occurs during the service time of packet i_{k-1}.



(b) Arrivals occur during the service time of packet i_{k-1}.

Fig. 4: Sample path of packet arrivals and departures under the Keep-Fresh policy.

where W_{i-1} and I_i are independent due to the memoryless property of inter-arrival and service times.

Unlike Keep-Old, where packets arriving at a full buffer are always dropped, Keep-Fresh allows replacement. This creates new dynamics in queueing behavior, particularly in how waiting times are distributed. Using the no-arrival probability during service and the memoryless property of exponential distributions, we derive the expected values

$$\mathbb{E}[W_{i-1}] = \frac{\lambda}{(\lambda+\mu)^2}, \quad \mathbb{E}[W_{i-1}^2] = \frac{2\lambda}{(\lambda+\mu)^3}, \tag{17}$$

$$\mathbb{E}[I_i] = \frac{\mu(2\lambda+\mu)}{\lambda(\lambda+\mu)^2} + \frac{\lambda}{\mu(\lambda+\mu)}, \tag{18}$$

$$\mathbb{E}[I_i^2] = \frac{2\mu(3\lambda^2+3\lambda\mu+\mu^2)}{\lambda^2(\lambda+\mu)^3} + \frac{2\lambda}{\mu^2(\lambda+\mu)}. \tag{19}$$

The detailed proof is in [29]. Combining (16), (17), (18) and (19), we present the following lemma.

Lemma 3.2: *The average reconstruction error under the Keep-Fresh policy in an M/M/1/2 queuing system is given by*

$$\overline{RE}_{\textit{Keep-Fresh}} = \frac{\lambda_{\textit{eff}}\mathbb{E}[Z_i^2]}{6}, \tag{20}$$

where $\lambda_{\textit{eff}} = \frac{\lambda(\mu^2+\lambda\mu)}{\mu^2+\lambda\mu+\lambda^2}$ *and* $\mathbb{E}[Z_i^2] = \frac{2(\lambda^2+\mu^2)}{\lambda^2\mu^2} - \frac{2\lambda(2\lambda+\mu)}{(\lambda+\mu)^4}$.

From this lemma, we can observe that the average reconstruction error asymptotically approaches $\frac{1}{3\mu}$. *Thus, despite achieving superior age performance, the Keep-Fresh policy also provides better reconstruction error in moderate-traffic scenarios. However, in heavy-traffic conditions ($\lambda \to \infty$), both policies achieve the same asymptotic reconstruction error.*

C. Inter-arrival-Aware Dropping Policy

The Keep-Fresh policy minimizes age but can degrade reconstruction accuracy by discarding historically valuable packets. To address this, we propose an Inter-arrival-Aware dropping policy, which dynamically balances freshness and historical importance based on packet generation times.

Given the generation times of the in-service packet (t_s), buffered packet (t_b), and new arrival (t_n), the policy evaluates temporal gaps and follows the decision rule:

$$(d_1(t_\text{n}), d_2(t_\text{n})) = \begin{cases} (1,0), & \text{if } t_\text{b} - t_\text{s} < t_\text{n} - t_\text{b} \\ (0,1), & \text{otherwise.} \end{cases} \tag{21}$$

This prioritizes fresher packets when the buffered packet is outdated while preserving historical information if the new arrival adds little value.

Direct analysis of the Inter-arrival-Aware dropping policy is intractable due to the continuous-valued generation times, which result in a high-dimensional Markov chain. To gain insight, we consider the system under heavy traffic ($\lambda \to \infty$). In this regime, we normalize time within a service interval $[\tau_i, \tau_i + S]$, where $S \sim \exp(\mu)$, so that the interval becomes $[0, 1]$. By the order-statistics property of Poisson arrivals, conditioned on there being k arrivals during the service period, their epochs (after scaling by S) are distributed as the order statistics of k i.i.d. Uniform$[0, 1]$ random variables.

Under this normalization, in the heavy-traffic regime, given that the in-service packet has a waiting time W_{i-1}, the waiting time W_i of the buffered packet is approximated by

$$W_i = 1 - 2^{\alpha} W_{i-1}, \tag{22}$$

where $\alpha = \arg\min_{\alpha\in\{0,1,2,\ldots\}}\{2^{\alpha}W_{i-1} > 0.5\}$. We refer Appendix [29] for the detailed proof. Numerical experiments indicate that the invariant distribution of W on $(0, 1)$ has mean approximately 0.375; consequently, the average waiting time of the buffered packet in real time is approximately $\frac{0.375}{\mu}$. Using this result, we present the following lemma.

Lemma 3.3: *The long-term average peak age $\bar{A}_{IaA}$ for an $M/M/1/2$ queueing system under the Inter-arrival-Aware dropping policy approximately approach as follows*

$$\bar{A}_{IaA} \to \frac{2.375}{\mu} \text{ as } \lambda \to \infty. \tag{23}$$

Moreover, numerical evaluation suggests that the reconstruction error under the Inter-arrival-Aware dropping policy asymptotically approaches $\overline{RE}_{\text{IaA}} \to\approx \frac{1}{0.362\mu}$ as $\lambda \to \infty$.

Comparing the asymptotic behaviors of three dropping policies, we can see that (1) the Keep-Old policy performs worst in terms of both age and reconstruction error, (2) the Keep-Fresh policy outperforms others in terms of age performance and (3) the Inter-arrival-Aware dropping policy outperforms others in terms of reconstruction error.

IV. $M/M/1/B+1$ Queueing Systems

We extend the analysis of the $M/M/1/2$ system to a general $M/M/1/B+1$ system with a buffer capacity of B. The server is non-preemptive and follows an LCFS discipline. Unlike in $M/M/1/2$, where all delivered packets are fresh, for $B > 2$, some delivered packets may be stale, leading to $i_k \neq k$, where i_k is the index of the k^{th} fresh packet.

A. Keep-Old Policy

From (3), the average peak age can be expressed as $\bar{A} = \mathbb{E}[A_k] = \mathbb{E}[T_{i_{k-1}}] + \mathbb{E}[Y_k]$. The expected system time is given by $\mathbb{E}[T_{i_{k-1}}] = \mathbb{E}[T_{i_k}] = \mathbb{E}[W_{i_k}] + \mathbb{E}[S_{i_k}]$, with $\mathbb{E}[S_{i_k}] = \frac{1}{\mu}$ since service times are exponentially distributed with rate μ.

The expected waiting time $\mathbb{E}[W_{i_k}]$ depends on the system state at arrival. If the system is empty, the packet is served immediately. If the buffer is full, the waiting time corresponds to the remaining service time of the in-service packet, which follows an exponential distribution with mean $1/\mu$. When the system contains $1 \le n \le B-1$ packets, a newly arriving packet remains fresh only if no further arrivals occur before the in-service packet departs, which happens with probability $\frac{\mu}{\lambda+\mu}$. Accounting for these cases, we obtain:

$$\mathbb{E}[W_{i_k}] = \frac{\pi_B}{\mu} + \sum_{n=1}^{B-1} \frac{\pi_n \mu}{\lambda+\mu}\frac{1}{\lambda+\mu}. \tag{24}$$

To compute the expected inter-departure time $\mathbb{E}[Y_k]$, we analyze the probability of different arrival scenarios, considering both arrivals that occur during an in-service packet and buffer occupancy states. The full derivation is in [29], leading to

$$\begin{aligned}\mathbb{E}[Y_k] = \pi_0 \cdot \frac{\lambda^2+\lambda\mu+\mu^2}{\lambda\mu(\lambda+\mu)} + \left(\pi_B + \frac{\mu}{\lambda+\mu}\sum_{n=1}^{B-1}\pi_n\right) \\ \cdot \left[\sum_{j=0}^{n-1}\left(\frac{\mu}{\lambda+\mu}\right)^j \frac{\lambda}{\lambda+\mu}\frac{j+1}{\mu} + \left(\frac{\mu}{\lambda+\mu}\right)^n\left(\frac{n}{\mu}+\frac{1}{\lambda}\right)\right].\end{aligned} \tag{25}$$

Combining and rearranging $\mathbb{E}[S_k] = \frac{1}{\mu}$, (24), (25) and (9), we present the following lemma.

Lemma 4.1: *The long-term average peak age $\bar{A}_{Keep\text{-}Old}(B)$ for an M/M/1/B+1 queueing system under the Keep-Old policy with a LCFS discipline are given by*

$$\begin{aligned}\bar{A}_{Keep\text{-}Old}(B) = \frac{1}{\mu} + \frac{1}{C_1}\left[\frac{1}{\lambda} + \frac{1}{\mu-\lambda}\left(1+\frac{\lambda\mu}{(\lambda+\mu)^2}\right)\right. \\ \left. + \left(\frac{2}{\mu} - \frac{1}{\mu-\lambda}\left(1+\frac{\mu^2}{(\lambda+\mu)^2}\right)\right)\rho^B\right],\end{aligned} \tag{26}$$

where $C_1 = 1 + \frac{\lambda\mu}{\mu^2-\lambda^2} - \frac{\lambda^2}{\mu^2-\lambda^2}\rho^B$ and $\rho = \frac{\lambda}{\mu}$.

From this lemma, we can observe the long-term average peak age under the Keep-Old policy exhibits specific asymptotic behaviors. As $\lambda \to \infty$ with a fixed buffer size $B \ge 1$, the average peak age approaches $\frac{3}{\mu}$. For given $\lambda \ge \mu$, as $B \to \infty$, it approaches $\frac{3}{\mu} + \frac{1}{\lambda+\mu}$. In the low-traffic regime ($\lambda < \mu$), as $B \to \infty$, the peak age converges to $\frac{1}{\mu} + \frac{\mu(2\lambda^2+2\lambda\mu+\mu^2)}{\lambda(\lambda+\mu)(\mu^2+\lambda\mu-\lambda^2)}$.

For reconstruction error, we analyze the second moment $\mathbb{E}[Z_i^2]$ of the inter-arrival time, considering cases where a fresh packet arrives when the buffer is full or not. Using probability-weighted expectations, we have

$$\mathbb{E}[Z^2_{\text{Keep-Old}}] = 2\pi_B\left(\frac{1}{\mu^2} + \frac{1}{\lambda\mu} + \frac{1}{\lambda^2}\right) + \frac{2}{\lambda^2}\sum_{n=0}^{B-1}\pi_n. \tag{27}$$

The full derivation is in [29]. Rearranging this with (9), we present the following lemma.

Lemma 4.2: *The long-term average reconstruction error $\overline{RE}_{K\text{-}O}(B)$ for an M/M/1/B+1 queueing system under the Keep-Old policy with a LCFS discipline are given by*

$$\overline{RE}_{Keep\text{-}Old}(B) = \frac{\lambda_{eff}\mathbb{E}[Z^2_{K\text{-}O}]}{6}, \tag{28}$$

where $\lambda_{eff} = \frac{\lambda(1-\rho^{B+1})}{1-\rho^{B+2}}$, $\mathbb{E}[Z^2_{K\text{-}O}] = \frac{2}{C_2(\mu-\lambda)}\left(\frac{\mu}{\lambda^2} - \frac{\lambda}{\mu^2}\rho^B\right)$ and $C_2 = \frac{\mu}{\mu-\lambda} - \frac{\lambda}{\mu-\lambda}\rho^B$.

For low traffic ($\lambda < \mu$), reconstruction error approaches $\frac{1}{3\lambda}$ as $B \to \infty$. In high traffic ($\lambda > \mu$), it converges to $1/(3\mu)$, showing service rate dominance. For fixed B, as $\lambda \to \infty$, it stabilizes at $\frac{1}{3\mu}$.

B. Keep-Fresh Policy

The expected service time $\mathbb{E}[S_{i_k}] = \frac{1}{\mu}$ remains policy-independent. However, under the Keep-Fresh policy, a packet remains fresh only if no new arrivals occur during the in-service packet's remaining service time, which occurs with probability $\frac{\mu}{\lambda+\mu}$. Then, the expected waiting time is given by

$$\mathbb{E}[W_{i_k}] = \sum_{n=1}^{B+1} \frac{\mu\pi_n}{\lambda+\mu}\frac{1}{\lambda+\mu}. \tag{29}$$

For the inter-departure time $\mathbb{E}[Y_k]$, the expectation remains the same as in the Keep-Old policy when the system is empty, but differs when the buffer is non-empty due to different queueing dynamics. However, a similar analytical approach can be applied with slight modifications. The expected inter-departure time is given by (30), with detailed derivations provided in [29]. Combining and rearranging $\mathbb{E}[S_{i_k}] = \frac{1}{\mu}$, (29), (30) and (9), we present the following lemma.

Lemma 4.3: *The long-term average peak age $\bar{A}_{\text{Keep-Fresh}}(B)$ for an M/M/1/B+1 queueing system under the Keep-Fresh policy with a LCFS discipline are given by*

$$\begin{aligned}\bar{A}_{\text{Keep-Fresh}}(B) = \frac{1}{\mu} + \frac{1}{C_1}\Bigg[\frac{1}{\lambda} + \frac{1}{\mu-\lambda}\left(1+\frac{\lambda\mu}{(\lambda+\mu)^2}\right)\\ + \left(\frac{1}{\mu} - \frac{1}{\mu-\lambda}\left(1+\frac{\lambda^2}{(\lambda+\mu)^2}\right)\right)\rho^B\Bigg],\end{aligned} \tag{31}$$

where $C_1 = 1 + \frac{\lambda\mu}{\mu^2-\lambda^2} - \frac{\lambda^2}{\mu^2-\lambda^2}\rho^B$ and $\rho = \frac{\lambda}{\mu}$.

As $\lambda \to \infty$ for a given $B \geq 1$, the average peak age under the Keep-Fresh policy approaches $\frac{2}{\mu}$. For $\lambda \geq \mu$, as $B \to \infty$, it converges to $\frac{2}{\mu} + \frac{1}{\lambda} + \frac{1}{\lambda+\mu}$. When $\lambda < \mu$, $\bar{A}_{\text{Keep-Fresh}}(B)$ matches the Keep-Old policy, indicating that under low traffic, both policies achieve the same asymptotic peak age as B grows. However, for $\lambda \geq \mu$, Keep-Fresh maintains an advantage by prioritizing fresher updates.

For reconstruction error, we analyze the second moment $\mathbb{E}[Z_i^2]$ of inter-arrival times by considering cases where the buffer is full or not. When not full, inter-arrival time follows an exponential distribution with mean $\frac{1}{\lambda}$. When full, it depends on whether a new packet arrives before the in-service packet departs, incorporating both the remaining service time and the next arrival time. The detailed derivation is in [29].

Lemma 4.4: *The long-term average reconstruction error $\overline{RE}_{\text{Keep-Fresh}}(B)$ for an M/M/1/B+1 queueing system under the Keep-Fresh policy with a LCFS discipline are given by*

$$\overline{RE}_{\text{Keep-Fresh}}(B) = \frac{\lambda_{\text{eff}}\mathbb{E}[Z^2_{\text{K-F}}]}{6}, \tag{32}$$

where $\lambda_{\text{eff}} = \frac{\lambda(1-\rho^{B+1})}{1-\rho^{B+2}}$ and

$$\mathbb{E}[Z^2_{\text{K-F}}] = \frac{1}{C_2}\left(\frac{2\mu}{(\mu-\lambda)\lambda^2} + \left(\mathcal{Z}_1 + \mathcal{Z}_2 - \frac{2\mu^2}{(\mu-\lambda)\lambda^3}\right)\rho^B\right),$$

where

$$\mathcal{Z}_1 = \frac{2\mu^3(6\lambda^2+4\lambda\mu+\mu^2)+2\lambda^3(\lambda^2+3\lambda\mu+3\mu^2)}{\lambda^2\mu^2(\lambda+\mu)^3},$$

$$\mathcal{Z}_2 = \frac{2\mu^2(3\lambda^2+3\lambda\mu+\mu^2)}{\lambda^3(\lambda+\mu)^3} + \frac{2}{\mu(\lambda+\mu)} \text{ and } C_2 = \frac{\mu}{\mu-\lambda} - \frac{\lambda}{\mu-\lambda}\rho^B.$$

For $\lambda < \mu$, the reconstruction error asymptotically matches the Keep-Old policy, approaching $\frac{1}{3\lambda}$ as $B \to \infty$. When $\lambda > \mu$, it converges to a lower value than the Keep-Old policy, given by $\frac{\lambda-\mu}{\lambda}(\mathcal{Z}_1 + \mathcal{Z}_2 + \frac{2\mu^2}{(\lambda-\mu)\lambda^3})$. For fixed B, as $\lambda \to \infty$, it approaches $\frac{1}{3\mu}$, again aligning with the Keep-Old policy.

C. Inter-arrival-Aware Dropping Policy

The Inter-arrival-Aware dropping policy dynamically evaluates inter-arrival time gaps to determine which packet should be dropped when the buffer is full, aiming to preserve packets that maximize temporal diversity. It considers the timing of all packets in the system, including those in service, buffered, and newly arriving, ensuring an informed dropping decision.

Let $\mathcal{S}$, $\mathcal{B}$ and $\mathcal{D}$ denote packets in service, the buffer, and delivered, respectively. The generation time of the i^{th} buffered packet is denoted as t_{b_i} for $i = 1, \ldots, B$, and $t_{\text{b}_{B+1}}$ denotes the newly arriving packet. For each packet i, the most recent preceding packet is indexed as $\text{b}_i^* = \max\{j < i : j \in \mathcal{S} \cup \mathcal{B} \cup \mathcal{D}\}$, and the inter-arrival gap is given by $\Delta_i = t_{\text{b}_i} - t_{\text{b}_i^*}$.

When the buffer is full, the packet with the smallest inter-arrival gap is dropped, ensuring diverse temporal representation. The dropping decision is given by

$$j^* = \arg\min_{j\in\{1,\ldots,B+1\}} \Delta_j. \tag{33}$$

Thus, $d_{j^*}(t_{\text{b}_{B+1}}) = 1$, while $d_j(t_{\text{b}_{B+1}}) = 0$ for $j \neq j^*$.

A theoretical analysis of this policy is challenging due to interdependent dropping decisions and dynamic arrivals. Instead, we evaluate its performance numerically in Section V, examining its impact under varying traffic conditions.

V. Numerical Results

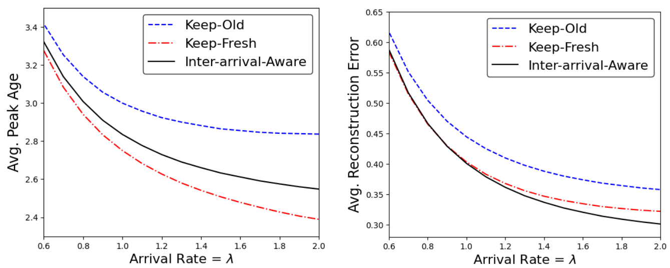
In this section, we present numerical results to validate the theoretical analysis and gain further insights into the performance of the proposed policies under various system configurations. The simulations are designed to illustrate the average peak age and reconstruction error metrics, highlighting the impact of buffer size, arrival rates, and service rates.

A. Single-Buffer Queueing System

We first consider an $M/M/1/2$ system with a fixed service rate $\mu = 1$. Fig. 5 shows that the Keep-Fresh policy achieves the lowest average peak age by prioritizing the freshest packets, whereas the Inter-arrival-Aware policy minimizes reconstruction error by dynamically selecting which packets to drop. At $\lambda = 2$, Keep-Fresh reduces peak age by 6.64% compared to Inter-Arrival-Aware, while Inter-Arrival-Aware improves reconstruction accuracy by 6.46% over Keep-Fresh. As λ increases, Keep-Fresh maintains its advantage in age performance, whereas Inter-Arrival-Aware continues to achieve superior reconstruction accuracy.

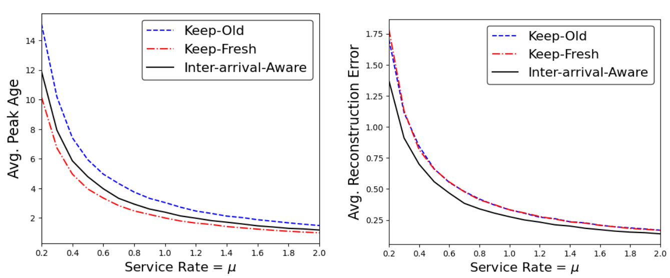
Under heavy traffic conditions ($\lambda = 10^3$), Fig. 6 demonstrates that increasing μ reduces both peak age and reconstruction error across all policies. Faster service enables fresher updates, benefiting Keep-Fresh in terms of age and allowing Inter-arrival-Aware to retain more informative packets for reconstruction. Keep-Fresh achieves a 15.79% lower peak

$$
\mathbb{E}[Y_k] = \pi_0 \left(\frac{\lambda}{(\lambda+\mu)\mu} + \frac{1}{\lambda} \right) + \frac{\mu}{\lambda+\mu} \sum_{n=1}^{B-1} \pi_n \left(\sum_{j=0}^{n-1} \left(\frac{\mu}{\lambda+\mu} \right)^j \frac{\lambda}{\lambda+\mu} \frac{j+1}{\mu} + \left(\frac{\mu}{\lambda+\mu} \right)^n \left(\frac{n}{\mu} + \frac{1}{\lambda} \right) \right)
$$
$$
+ \frac{\mu}{\lambda+\mu} (\pi_B + \pi_{B+1}) \left(\sum_{j=0}^{B-1} \left(\frac{\mu}{\lambda+\mu} \right)^j \frac{\lambda}{\lambda+\mu} \frac{j+1}{\mu} + \left(\frac{\mu}{\lambda+\mu} \right)^B \left(\frac{B}{\mu} + \frac{1}{\lambda} \right) \right). \tag{30}
$$



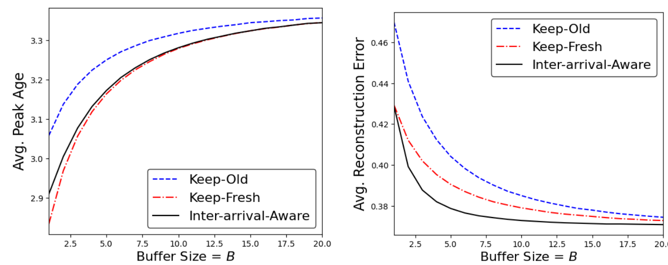
(a) Average peak age. (b) Average reconstruction error.

Fig. 5: Comparison of three packet-dropping policies in an M/M/1/2 queueing system with service rate is $\mu = 1$.



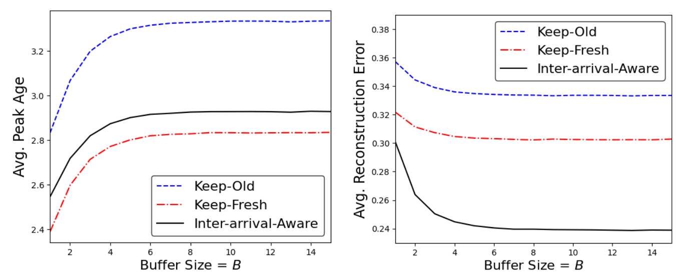
(a) Average peak age. (b) Average reconstruction error.

Fig. 6: Comparison of three packet-dropping policies in an $M/M/1/2$ queueing system under heavy traffic ($\lambda = 10^3$).



(a) Average peak age. (b) Average reconstruction error.

Fig. 7: Comparison of three packet-dropping policies in an $M/M/1/B+1$ queueing system with $\lambda = 0.9$ and $\mu = 1$.



(a) Average peak age. (b) Average reconstruction error.

Fig. 8: Comparison of three packet-dropping policies in an $M/M/1/B+1$ queueing system with $\lambda = 2$ and $\mu = 1$.

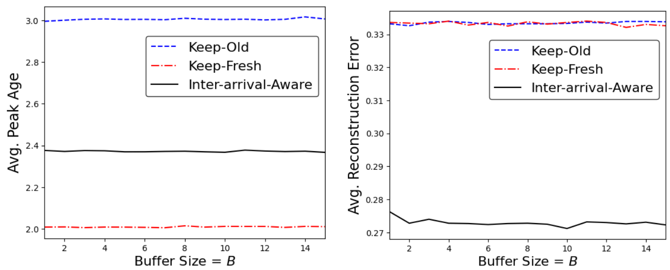
age than Inter-Arrival-Aware, ensuring fresher updates, while Inter-Arrival-Aware reduces reconstruction error by 16.92% compared to Keep-Fresh, effectively preserving historical information for improved reconstruction accuracy.

To further assess the robustness of our policies, we extended our results to $M/G/1/2$ and $G/M/1/2$ systems as well, where G refers to a general distribution that can model various scenarios such as log-normal service times, Erlang arrivals, and Pareto inter-arrival distributions. These results, provided in our online technical report [29], confirm the observations here: Keep-Fresh remains optimal for minimizing peak age, while Inter-arrival-Aware outperforms in reconstruction accuracy, especially under bursty traffic patterns.

B. B-Buffer Queueing System

We next examine the impact of buffer size B on system performance in an $M/M/1/B+1$ queue. Figs. 7, 8 and 9 illustrate the evolution of peak age and reconstruction error under different traffic conditions. At low traffic ($\lambda = 0.9$), packet drops are infrequent, leading to minimal performance differences between policies. As B increases, peak age initially rises since staled packets accumulate in the buffer, increasing the chance of transmitting older packets instead of fresher ones. However, as B grows further, this effect diminishes. Meanwhile, reconstruction error steadily decreases as a larger buffer retains more packets, improving state estimation by reducing information loss.

In higher traffic ($\lambda = 2$), buffer overflows occur more frequently, making the choice of dropping policy more significant. Keep-Fresh offers the best age performance, whereas Inter-arrival-Aware minimizes reconstruction error. Under extreme congestion ($\lambda = 10^3$), buffer occupancy changes rapidly, and the choice of retained packets has a substantial impact. These results collectively demonstrate that the optimal packet-dropping policy depends on the system objective. Keep-Fresh is preferred when minimizing information latency, while Inter-arrival-Aware is beneficial when preserving historical state information for reconstruction.



(a) Average peak age. (b) Average reconstruction error.

Fig. 9: Comparison of three packet-dropping policies in an $M/M/1/B+1$ queueing system with $\lambda = 10^3$ and $\mu = 1$.

VI. Conclusion

In this paper, we investigated the trade-off between real-time monitoring and historical trajectory reconstruction in remote tracking systems. While age-optimal policies minimize AoI by prioritizing freshness, they often discard older packets critical for accurate trajectory reconstruction. To address this, we used reconstruction error as a complementary metric to AoI and analyzed the impact of different packet-dropping policies. Our results showed that the Keep-Fresh policy achieves the lowest AoI but does not necessarily optimize reconstruction accuracy. To balance these objectives, we proposed the Inter-arrival-Aware dropping policy, which dynamically selects packets based on their generation times. Numerical evaluations demonstrated that this policy improves reconstruction accuracy while maintaining reasonable freshness. These findings provide a framework for efficient information management in remote tracking systems, particularly in IoT applications requiring both real-time updates and historical analysis. Future work could explore multi-source networks and learning-based adaptive policies for further optimization.

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