Wireless Market Competition with Time/Frequency Prioritized Spectrum Sharing

Yaxuan Liu, Qixuan Zai and Randall Berry
Department of Electrical and Computer Engineering
Northwestern University, Evanston, IL 60208, USA
Email: {YaxuanLiu2023@u,qixuanzai2028@u,rberry@}.northwestern.edu

***Abstract*—Spectrum sharing plays an important role in meeting the demand for the 6G and advanced wireless communications. This paper considers a framework of time-frequency prioritized spectrum sharing, in which a given band of spectrum is shared both in time and frequency with certain service providers (SPs) having priority access to given time-frequency partitions and other partitions available for use by all SPs. We investigate the competition between the two SPs in such a setting under both Bertrand and Cournot competition models, where SPs compete by either announcing prices or quantities, respectively. We calculate a market equilibrium for both competition models and analyze the impact of the available bandwidth and the spectrum sharing parameters on the revenues of the SPs, the customer surplus, and the social welfare.**

***Index Terms*—spectrum sharing, market modeling, Bertrand competition, Cournot competition**

I. Introduction

Spectrum sharing has been put forward as a needed approach for meeting the projected demands for wireless services into 6G and beyond [1]. Many approaches to spectrum sharing can be viewed as partitioning a band of spectrum in a given location both in time and frequency, where certain users may have priority to access a given time-frequency partition. For example, in the recent CBRS system deployed in the U.S., the available bandwidth is partitioned into 10 MHz channels, some of which are licensed to specific service providers (SP) who obtain a Priority Access License (PAL) [2]. These SPs then temporally share the given band with incumbent users. Other portions of this band are available for any provider to use as Generalized Authorized Access (GAA) users. Time/frequency prioritization can also be provided through the choice of different technologies by different users of shared spectrum. For example, in unlicensed spectrum, a protocol like LTE-U [3] may give users adopting it an effective priority to a portion of the spectrum at certain times over users adopting a different protocol like WiFi [4].

We consider a model for time-frequency prioritized spectrum sharing that is motivated by such scenarios. Our goal is to study the market interaction of SPs utilizing such spectrum, that are competing for a common pool of customers. This approach is related to the body of work studying market competition and spectrum sharing including [5]–[15]. In particular, we consider models as in [5]–[8], [15], where firms compete for a common pool of customers who are sensitive to both the price they pay for service and the congestion of the service they are using. Here, this congestion depends on the prioritized time-frequency sharing.

We focus on a model with two SPs, each with an exclusive proprietary band of spectrum and prioritized access to a portion of a shared band. Additionally, we assume that both SPs can use the shared band at any time/frequency where no SP has priority access.[1] Prior work in this area has focused on one of two different models for competition. For example, [6]–[8] consider a Bertrand competition model where SPs compete by announcing prices, while [5], [15] consider Cournot competition, where SPs compete by announcing quantities of customers to serve. Here, we consider both forms of competition and compare their outcomes.

Our main results are as follows:

- In Bertrand competition, prioritized sharing can improve the revenues of the SPs, but it harms the customer surplus. It may improve social welfare with a small shared bandwidth, but hurt it when the shared bandwidth is large.
- In Cournot competition, prioritized sharing may improve the revenue of a SP with a small shared bandwidth, but harms it when the shared bandwidth is large enough. It also hurts the customer surplus and social welfare.
- Cournot competition is more likely to achieve a higher social welfare than Bertrand competition when the prioritized sharing parameters are set so that it does not deviate so much from the case without prioritized sharing, or when the licensed bandwidths are relatively large and the shared bandwidth lies in a particular range.
- When the shared bandwidth is large enough, the customer surplus and social welfare can be enlarged by setting any three bandwidths to be larger, but the revenue of a SP can only be improved if its licensed bandwidth is adjusted properly.

This research was supported in part by NSF grants CNS-2148183, ECCS-2331106.

[1]This generalizes model in previous work. For example, work in [5] considers a case where a SP may have priority to some frequencies, but does not model temporal priority. Likewise, the work in [8] considers a special case where only one SP has priority to a given time-frequency band, while any other SP must only use the shared portion of the band.

ISBN 978-3-903176-73-7 © 2025 IFIP

II. System Model

In this article, we consider a market similar to that in [4], but with two incumbent SPs and no entrants. Both the SPs may use the prioritized sharing technology, which is a duty-cycle based technology. Each SP has its own licensed band of spectrum, so it can provide its licensed service on this band, and they also have access to a single shared band of spectrum, which can be shared using either prioritized sharing or shared without any priority. The bandwidths of the licensed bands of SPs 1 and 2 are respectively denoted by B_1 and B_2, and the shared bandwidth is denoted by W. When prioritized sharing is adopted, a SP has exclusive access to a portion of the shared spectrum for a fixed fraction of time. Portions of the spectrum in time and/or frequency that are not reserved for priority use can be used by both SPs, e.g. similar to unlicensed access with WiFi. Hence, a SP can offer both a priority and a non-priority service. We assume that the prioritized sharing modes of both SPs are the same at all time, which take a percentage α of time to be 'ON'. We also use β as the percentage of the shared spectrum occupied by the prioritized sharing during its 'ON' mode, where each SP takes a percentage of $\beta/2$ of the unlicensed spectrum. When $\alpha = 0$ or $\beta = 0$, it is equivalent to the case without prioritized sharing.

The SPs are assumed to compete for a common pool of infinitesimal customers. Besides the Bertrand competition model considered in [4], where each SP announces the prices for both its licensed and unlicensed services, and gets an amount of customers determined by the market, we also consider another competition scheme, that is, Cournot competition [5], where the strategy of each SP is related to the amounts of customers served by its licensed and unlicensed services, and the prices are given by a mechanism to ensure the compatibility between the prices and the user amounts. For SP i ($i = 1, 2$), its licensed service announces a price denoted by $p_{i,l}$ and achieves a user amount denoted by $x_{i,l}$. For the unlicensed service, its announced price and user amount are $p_{i,u}$ and $x_{i,u}$, respectively, and thus the total revenue it achieves from both services is $p_{i,l}x_{i,l} + p_{i,u}x_{i,u}$.

As in [8], [16], a service is characterized by a congestion cost denoted by $g(X, Y)$, which is assumed to be increasing in the total customer mass X on the spectrum band and decreasing in the bandwidth Y. In this article, we assume $g(X, Y) = X/Y$, which is proportional to the customer mass per unit bandwidth. When the SPs apply prioritized sharing, the congestion that the customers experience will vary across the duty cycle. We assume that customers are sensitive to the average congestion across the duty cycle, which is a reasonable assumption since over the time-scale that customers select SPs and services they will receive service over many duty cycles. The average congestion of customers served by the licensed service of SP i ($i = 1, 2$) is given by

$$\hat{g}_{i,l}\left(x_{i,l}\right) = \alpha \frac{x_{i,l}}{B_i + \beta W/2} + (1-\alpha)\frac{x_{i,l}}{B_i} = \frac{x_{i,l}}{B_{ei}}, \quad (1)$$

where $B_i + \beta W/2$ is the total bandwidth of the licensed service when prioritized sharing is 'ON,' and

$$B_{ei} = B_i + \frac{\alpha\beta W}{2 + (1-\alpha)\beta W/B_i}. \quad (2)$$

We call B_{ei} the *equivalent licensed bandwidth* of SP i, where 'equivalent' means the congestion $\hat{g}_{i,l}\left(x_{i,l}\right)$ given in (1) is the same as in a setting where the licensed bandwidth of the same SP is B_{ei} and prioritized sharing is not adopted. The average congestion experienced by customers of the unlicensed service is

$$\hat{g}_u\left(X_u\right) = \alpha \frac{X_u}{(1-\beta)W} + (1-\alpha)\frac{X_u}{W} = \frac{X_u}{W_e}, \quad (3)$$

where $X_u = x_{1,u} + x_{2,u}$ is the total amount of users served by non-prioritized sharing. Note that unlicensed users of either SP share a same spectrum, so they experience the same congestion. $(1-\beta)W$ is the shared bandwidth that is not occupied by prioritized sharing during its 'ON' mode and thus available for non-prioritized sharing, and

$$W_e = W\left(1 - \frac{\alpha\beta}{1-\beta(1-\alpha)}\right) \quad (4)$$

is called *equivalent shared bandwidth*. As α or β goes up, prioritized sharing deviates more from the case without prioritized sharing, and thus the equivalent licensed bandwidths B_{e1} and B_{e2} increase, while the equivalent shared bandwidth W_e decreases. It is also not difficult to verify that $B_{e1} + W_e$, $B_{e2} + W_e$, and the total equivalent bandwidth $B_{e1} + B_{e2} + W_e$ decrease in β, and therefore prioritized sharing results in a lower total equivalent bandwidth compared with the case without prioritized sharing.

As in [6], [7], we assume that customers are only interested in the services with the lowest *delivered price*, which is given by the sum of the announced price and the average congestion cost of the service. Hence, the delivered price of the licensed service provided by SP i is $p_{i,l} + \hat{g}_{i,l}\left(x_{i,l}\right)$, and the delivered price of the unlicensed service provided by the same SP is $p_{i,u} + \hat{g}_u\left(x_{1,u} + x_{2,u}\right)$. We assume that customers are characterized by a linear inverse demand function $P(q) = 1 - q$, which indicates the delivered price at which a mass of q customers are willing to pay for service. The announced prices of the SPs and the demands of services are assumed to satisfy the Wardrop equilibrium conditions [17]. In our model, the conditions are

$$\begin{aligned}
&p_{i,l} + \hat{g}_{i,l}\left(x_{i,l}\right) = P(X), && \text{for } x_{i,l} > 0,\ i = 1, 2, && \text{(5a)}\\
&p_{i,l} + \hat{g}_{i,l}\left(x_{i,l}\right) \geq P(X), && \text{for } i = 1, 2, && \text{(5b)}\\
&p_{i,u} + \hat{g}_u(x_{1,u} + x_{2,u}) = P(X), && \text{for } x_{i,u} > 0,\ i = 1, 2, && \text{(5c)}\\
&p_{i,u} + \hat{g}_u(x_{1,u} + x_{2,u}) \geq P(X), && \text{for } i = 1, 2, && \text{(5d)}
\end{aligned}$$

where $X = \sum_{i=1}^{2}(x_{i,l} + x_{i,u})$ is the total amount of users that are served. The conditions imply that at the Wardrop equilibrium, all the services serving a positive amount of customers will end up with the same delivered price, which is given by the inverse demand function, and all the services with no customers have a delivered price equal to or higher than that. A Nash equilibrium of the game is one in which the customers are in a Wardrop equilibrium and no SP i can improve its revenue $p_{i,l}x_{i,l} + p_{i,u}x_{i,u}$ by changing its competition strategy (anticipating the impact this will have on the Wardrop equilibrium). At an equilibrium, the customer surplus is defined as the difference between the delivered price each customer pays and the amount it is willing to pay, integrated over all the customers, i.e.,

$$CS = \int_0^X P(x) - P(X)\,dx = \frac{1}{2}X^2.$$

The social welfare of the market is the sum of consumer welfare and the SPs' revenues:

$$SW = CS + \sum_{i=1}^{2}(p_{i,l}x_{i,l} + p_{i,u}x_{i,u}).$$

III. Equilibrium Points

In this section, we analyze the Nash equilibria of the model introduced in Section II. Two competition schemes, Bertrand competition and Cournot competition, are analyzed, where the competition strategies of the SPs are announced prices and planned user amounts, respectively.

A. Bertrand Competition

We first consider Bertrand competition, where both the SPs announce equilibrium prices of their services in order to maximize their respective revenues, and the user amount of each service is determined by the Wardrop equilibrium condition given in (5). We first show how the announced prices $p_{1,l}, p_{2,l}, p_{1,u}, p_{2,u}$ affect the user amounts $x_{1,l}, x_{2,l}, x_{1,u}, x_{2,u}$, and then analyze the competition between the two SPs, characterized by the Nash equilibrium under Bertrand competition.

According to (5c) and (5d), if the two SPs announce different prices for the unlicensed services, that is, $p_{1,u} \neq p_{2,u}$, then the SP with a higher price would end up with a zero unlicensed user amount, that is, $x_{1,u} = 0$ if $p_{1,u} > p_{2,u}$, or $x_{2,u} = 0$ if $x_{2,u} > x_{1,u}$; otherwise, if the announced prices are the same, then from the customers' point of view, there is no difference between two SPs because they occupy the same band. According to the Wardrop equilibrium conditions (5), the congestion expressions (1), (3), and the inverse demand function $P(q) = 1 - q$, given the fixed announced prices $p_{1,l}, p_{2,l}, p_{1,u}, p_{2,u}$, the final inverse demand is

$$P(X) = \frac{1 + (\kappa_{1,l}B_{e1}p_{1,l} + \kappa_{2,l}B_{e2}p_{2,l} + \kappa_u W_e p_u)}{1 + (\kappa_{1,l}B_{e1} + \kappa_{2,l}B_{e2} + \kappa_u W_e)},$$

where $p_u = \min(p_{1,u}, p_{2,u})$, and each of $\kappa_{1,l}, \kappa_{2,l}, \kappa_u$ is either 1 if the respective equation (5a) or (5c) can be satisfied (with positive or zero user amount), or 0 otherwise. We have $\kappa_k = 1$ if and only if $p_k \leq P(X)$. The licensed user amounts are

$$x_{i,l} = \begin{cases} B_{ei}(P(X) - p_{i,l}), & \kappa_{i,l} = 1, \\ 0, & \kappa_{i,l} = 0 \end{cases} \quad (i = 1, 2), \quad (6)$$

and the total unlicensed user amount is

$$X_u = x_{1,u} + x_{2,u} = \begin{cases} W_e(P(X) - p_u), & \kappa_u = 1, \\ 0, & \kappa_u = 0. \end{cases} \quad (7)$$

It is guaranteed that the selection of $\kappa_{1,l}, \kappa_{2,l}, \kappa_u$ is unique, and so is the distribution of customer demand.

We then move on to Bertrand competition between the SPs. Similar to the result given in [4, Lemma 4.1], the unlicensed services provided by both SPs are free of charge because if any SP announces a positive price, then the other SP can announce a slightly lower price to grab the whole market. Hence, we have $p_{1,u} = p_{2,u} = 0$, and the strategy for SP j is to adjust $p_{j,l}$ to maximize its revenue $p_{j,l}x_{j,l}$ subject to the constraints given in (5) (or equivalently, given in (6) and (7)). Since a positive revenue is attainable for SP j (for example, by announcing a positive but low enough price $p_{j,l}$ and getting a positive number of customers), under a Nash equilibrium, the licensed service of any SP should serve a positive number of customers, so the equation (5a) is guaranteed by both $i = 1$ and 2, or equivalently, we have $\kappa_{1,l} = \kappa_{2,l} = 1$ in (6). Based on the above analysis, the announced prices of different services in equilibrium are given by

$$p_{1,l}^b = \frac{(2 + 2W_e + 2B_{e1} + B_{e2})}{4(1+W_e+B_{e1})(1+W_e+B_{e2}) - B_{e1}B_{e2}}, \quad (8)$$

$$p_{2,l}^b = \frac{(2 + 2W_e + 2B_{e2} + B_{e1})}{4(1+W_e+B_{e1})(1+W_e+B_{e2}) - B_{e1}B_{e2}},$$

$$p_{1,u}^b = p_{2,u}^b = 0,$$

and the respective user amounts are given in (9)-(11). Here, we use the superscript "b" to denote the Bertrand equilibrium.

Based on the announced prices and user amounts given above, the following results can be proved, which show the impacts of prioritized sharing and bandwidths:

Lemma 1: In Bertrand competition, when B_{e1} and B_{e2} increase, and the total equivalent bandwidth $B_{e1} + B_{e2} + W_e$ decreases, the announced price, user amount, and revenue for licensed service provided by any SP increase, but the unlicensed user amount X_u^b and the total user amount (and thus the customer surplus) decreases.

This is natural because a larger equivalent bandwidth can relieve the congestion on the band, allowing the SP to get more customers and announce a higher price, but it occupies the shared band, making the equivalent shared bandwidth and the total equivalent bandwidth smaller, so that the unlicensed service becomes less valuable due to higher congestion, motivating its customers to switch to licensed service or leave the market. A special case of Lemma 1 is given below.

Corollary 2: In Bertrand competition, the revenues of both SPs increase in β, but the user amount of unlicensed services and the customer surplus decrease in β. Particularly, prioritized sharing increases the revenues of both SPs, but it reduces the user amount of unlicensed services and thus harms the customer surplus.

The prioritized sharing may improve or harm social welfare, which depends on other parameters. The effects of prioritized sharing on the social welfare are investigated in Section IV.

$$x_{1,l}^{b}=\frac{B_{e1}\left(1+W_e+B_{e2}\right)\left(2+2W_e+2B_{e1}+B_{e2}\right)}{\left(1+W_e+B_{e1}+B_{e2}\right)\left[4\left(1+W_e+B_{e1}\right)\left(1+W_e+B_{e2}\right)-B_{e1}B_{e2}\right]}, \tag{9}$$

$$x_{2,l}^{b}=\frac{B_{e2}\left(1+W_e+B_{e1}\right)\left(2+2W_e+2B_{e2}+B_{e1}\right)}{\left(1+W_e+B_{e1}+B_{e2}\right)\left[4\left(1+W_e+B_{e1}\right)\left(1+W_e+B_{e2}\right)-B_{e1}B_{e2}\right]}, \tag{10}$$

$$X_u^b=x_{1,u}+x_{2,u}=W_e\frac{2\left(2+2W_e+B_{e1}+B_{e2}\right)\left(1+W_e+B_{e1}+B_{e2}\right)+B_{e1}B_{e2}}{\left(1+W_e+B_{e1}+B_{e2}\right)\left[4\left(1+W_e+B_{e1}\right)\left(1+W_e+B_{e2}\right)-B_{e1}B_{e2}\right]}. \tag{11}$$

Lemma 3: In Bertrand competition, the revenue R_i^b of SP i, as a function of B_{ei}, $B_{e,-i}$ [2], W_e, decreases in $B_{e,-i}$ and W_e. There exists $0 \le b_1 \le b_2$ such that, as B_{ei} increases, R_i^b increases when $B_{ei} < b_1$, decreases when $b_1 < B_{ei} < b_2$ (if $b_2 > b_1$), and increases when $B_{ei} > b_2$.

It is natural for R_i^b to decrease in $B_{e,-i}$ and W_e since the congestion on bands that SP i cannot get a positive profit decreases, motivating customers served by the licensed service of SP i to shift to their services. The variation of R_i^b as B_{ei} increases is a little bit more complicated. In fact, as B_{ei} increases, the licensed service price $p_{-i,l}^b$ of its opponent decreases, making SP i drop its price to retain its customers, so its revenue R_i^b may decrease even though its licensed user amount $x_{i,l}^b$ increases.

Corollary 4: If the licensed bandwidth of a SP is large enough in Bertrand competition, its revenue increases in its licensed bandwidth.

Corollary 5: The revenue of one SP is decreasing in the licensed bandwidth of its opponent in Bertrand competition.

Lemma 6: The total user amount in Bertrand competition, as a function of B_{e1}, B_{e2}, W_e, increases when either of the equivalent bandwidths B_{e1}, B_{e2}, W_e increases, resulting in the increase of the customer surplus.

Corollary 7: The customer surplus in Bertrand competition increases when either of the actual bandwidths B_1, B_2, W increases.

Lemma 8: The social welfare in Bertrand competition, as a function of B_{e1}, B_{e2}, W_e, increases in B_{e1} and B_{e2}. There exists a $w \ge 0$ such that the social welfare decreases in W_e when $W_e < w$ (if $w > 0$) but increases in W_e when $W_e > w$.

Corollary 9: The social welfare in Bertrand competition increases in the licensed bandwidth of any SP, i.e., B_1 and B_2.

B. Cournot Competition

We then consider Cournot competition, where both the SPs compete by planning the user amounts of their services. Service prices are then determined according to (5). Note that the announced prices are the results of customer amounts, and when the user amount of a service is zero, whether the delivered price is equal to the inverse demand or strictly higher than that has no effect on the revenue of any SP and thus does not affect the equilibrium. For example, if $x_{i,l} = 0$ for some SP i, then $p_{i,l}$ can be any value greater than or equal to $P(X) - \hat{g}_{i,l}(x_{i,l})$. Therefore, we assume that the delivered prices are always equal to the inverse demand function, i.e., the equations in (5) are always satisfied no matter if the user amount is positive.

For SP j ($j = 1, 2$), its revenue maximization can be expressed as

$$\begin{aligned}
\max_{x_{j,l},x_{j,u}} \quad & p_{j,l}x_{j,l}+p_{j,u}x_{j,u} \\
\text{subject to} \quad & p_{i,l}=P\left(X\right)-\hat{g}_{i,l}\left(x_{i,l}\right), \quad i=1,2, \\
& p_{i,u}=P\left(X\right)-\hat{g}_{u}\left(x_{1,u}+x_{2,u}\right), \quad i=1,2, \\
& x_{1,l},x_{2,l},x_{1,u},x_{2,u}\ge 0.
\end{aligned} \tag{12}$$

After some mathematical manipulations, the solution to (12) is given by

$$x_{j,u}=\max\left\{0,\frac{W_e-x_{-j,l}W_e}{2\left(1+B_{ej}+W_e\right)}-\frac{x_{-j,u}}{2}\right\},$$

$$x_{j,l}=\frac{1-\left(2x_{j,u}+x_{-j,l}+x_{-j,u}\right)}{2\left(1+1/B_{ej}\right)}.$$

Upon combining the expressions of $x_{j,l}$ and $x_{j,u}$ for both $j = 1, 2$, the Nash equilibrium can be given in (13)-(14), where we use the superscript "c" to denote the Cournot equilibrium. Combined with the conditions in (12), the announced prices of licensed services are given in (15), and that of unlicensed services provided by any SP is given by

$$p_u^c=\begin{cases} p_{2,l}^c, & B_{e1}>2\left(1+B_{e2}+W_e\right), \\ p_{1,l}^c, & B_{e2}>2\left(1+B_{e1}+W_e\right), \\ 1/3, & \text{otherwise.}\end{cases} \tag{16}$$

According to (13)-(16), for $i = 1, 2$, if the equivalent licensed bandwidth B_{ei} of SP i is much larger than W_e and that of its opponent, so that $B_{e1} \ge 2(B_{e2} + W_e + 1)$, then SP i does not need to provide unlicensed service to maximize its profit, and the whole shared band can be occupied by SP $-i$. Thus, the spectral allocation and pricing model is the same as the case where the equivalent bandwidths of SPs i and $-i$ are B_{ei} and $B_{e,-i} + W_e$, respectively, and the equivalent shared bandwidth is zero. Otherwise, if neither B_{e1} nor B_{e2} is significantly large, then the unlicensed service price is always $1/3$ regardless of the equivalent bandwidths. We can also arrive at the following results, showing the impacts of prioritized sharing and bandwidths on the market outcomes:

Lemma 10: In Cournot competition, when B_{e1} and B_{e2} increase, and the total equivalent bandwidth $B_{e1} + B_{e2} + W_e$ decreases, the user amount $x_{1,l}^c$, $x_{2,l}^c$ served by licensed service of any SP increases, while the user amount $x_{1,u}^c$, $x_{2,u}^c$ served

[2] In this paper, we use $-i$ to denote SP $j \neq i$.

$$x_{i,l}^c = \begin{cases} \frac{(2+B_{e,-i}+W_e)}{4(1+1/B_{ei})(1+B_{e,-i}+W_e)-(W_e+B_{e,-i})}, & B_{ei} > 2\left(1+B_{e,-i}+W_e\right), \\ \frac{B_{ei}(1+2/B_{e,-i})}{4(1+1/B_{e,-i})(1+B_{ei}+W_e)-(W_e+B_{ei})}, & B_{e,-i} > 2\left(1+B_{ei}+W_e\right), \\ \frac{B_{ei}(2+B_{e,-i}+2W_e)}{4(1+B_{ei}+W_e)(1+B_{e,-i}+W_e)-B_{ei}B_{e,-i}}, & \text{otherwise} \end{cases} \tag{13}$$

$$x_{i,u}^c = \begin{cases} 0, & B_{ei} > 2\left(1+B_{e,-i}+W_e\right), \\ \frac{W_e(1+2/B_{e,-i})}{4(1+1/B_{e,-i})(1+B_{ei}+W_e)-(W_e+B_{ei})}, & B_{e,-i} > 2\left(1+B_{ei}+W_e\right), \\ 2W_e \frac{2(1+B_{e,-i}+W_e)-B_{ei}}{3[4(1+B_{e,-i}+W_e)(1+B_{ei}+W_e)-B_{e,-i}B_{ei}]}, & \text{otherwise} \end{cases} \tag{14}$$

$$p_{i,l}^c = \begin{cases} \frac{(B_{ei}+1)(B_{e,-i}+W_e+2)}{4B_{ei}+4B_{e,-i}+4W_e+3B_{ei}B_{e,-i}+3B_{ei}W_e+4}, & B_{ei} > 2\left(1+B_{e,-i}+W_e\right), \\ \frac{(B_{e,-i}+2)(B_{ei}+W_e+1)}{4B_{e,-i}+4B_{ei}+4W_e+3B_{e,-i}B_{ei}+3B_{e,-i}W_e+4}, & B_{e,-i} > 2\left(1+B_{ei}+W_e\right), \\ \frac{6B_{ei}+3B_{e,-i}+10W_e+4W_e^2+3B_{ei}B_{e,-i}+4B_{ei}W_e+4B_{e,-i}W_e+6}{3(4B_{ei}+4B_{e,-i}+8W_e+4W_e^2+3B_{ei}B_{e,-i}+4B_{ei}W_e+4B_{e,-i}W_e+4)}, & \text{otherwise,} \end{cases} \tag{15}$$

by unlicensed service of any SP decreases (or remains zero). As a result, the total customer amount decreases, indicating a lower customer surplus.

This is a natural result, as a larger bandwidth is more likely to serve more customers.

Corollary 11: In Cournot competition, the user amount served by the licensed service of any SP increases in β, but that served by the unlicensed service of any SP decreases in β. The customer surplus also decreases in β. Particularly, prioritized sharing increases the licensed user amounts but decreases the unlicensed user amounts, thus lowering the customer surplus.

Whether the prioritized sharing improves or harms the revenue of a SP depends on other parameters, which is investigated in Section IV.

Lemma 12: In Cournot competition, the customer surplus and social welfare, as functions of B_{e1}, B_{e2}, W_e, are increasing. A larger equivalent bandwidth B_{e1}, B_{e2}, or W_e results in a higher customer surplus and a higher welfare.

Corollary 13: A larger actual bandwidth B_1, B_2, or W results in a higher customer surplus and a higher welfare.

Lemma 14: In Cournot competition, the revenue R_i^c of a SP i, as a function of B_{ei}, $B_{e,-i}$, W_e, increases in B_{ei} and decreases in $B_{e,-i}$. R_i^c decreases in W_e when $B_{ei} > 2\left(1+B_{e,-i}+W_e\right)$ (so that $x_{i,u}^c = 0$) but increases in W_e otherwise.

Note that in Cournot competition, the revenue of a SP consists of two parts, including that obtained in the licensed service and that from the unlicensed service. When W_e is large enough, a bigger W_e can lower the congestion on the shared band and attract more customers for both SPs, so the trend of R_i^c as W_e increases is different from that in Bertrand competition scenario.

Corollary 15: The revenue of a SP in Cournot competition increases in its licensed bandwidth and decreases in its opponent's licensed bandwidth.

IV. Numerical Results

In this section, numerical results are presented to show the effects of bandwidths and parameters α, β on the revenues of both SPs, customer surplus, and social welfare. The announced prices and user amounts of services are calculated in both Bertrand and Cournot competition settings according to their respective expressions given in Section III.

A. Fixed Actual Bandwidths

We first show the effects of α and β on the market outcomes given fixed actual bandwidths B_1, B_2, W. When $B_1 = 1$, $B_2 = 2$, $W = 1$, under Bertrand competition, the revenues of both SPs and customer and social surpluses as functions of α are shown in Fig. 1. Note that $\alpha = 0$ and $\beta = 0$ are

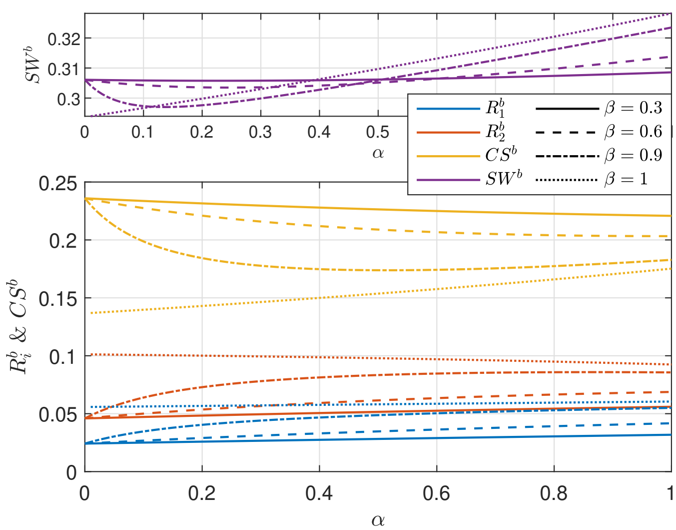

Fig. 1. Market outcomes as functions of α in Bertrand competition with $B_1 = 1$, $B_2 = 2$, $W = 1$, and different β.

equivalent, both representing the scenario without prioritized sharing, so the outcomes achieved by $\beta = 0$ are the same as those under the case $\alpha = 0$. It can be observed that prioritized sharing improves the revenues of both SPs but hurts the customer surplus, which is consistent with Corollary 2. Given a fixed $\alpha > 0$, a bigger β results in higher revenues of both SPs and a lower customer surplus. Given a fixed $\beta > 0$, when α is relatively small, the revenues increase in α, and the customer surplus decreases in α. This may be because a greater α results in larger equivalent licensed bandwidths B_{ei} of both SPs and smaller equivalent shared bandwidth W_e, which is beneficial for the SPs to get higher profits. However, when α is close to one, the revenue may be decreasing in α in some cases, possibly because the competition between the two SPs becomes more intense, e.g., the revenue of SP 2 when $\beta = 0.9$. For the customer surplus, it decreases in α when α is small or β is not very large, but when $\beta = 0.9$, the customer surplus may increase in α when α is large. Nevertheless, for any pair of (α, β), the customer surplus does not exceed its maximum level achieved without prioritized sharing. Whether the prioritized sharing technique improves the social welfare depends on the values of α and β, and Fig. 1 indicates that prioritized sharing is more likely to improve the social welfare with a higher value of α, and the highest social welfare is achieved at $\alpha = \beta = 1$. This may be because when α is large, the equivalent licensed bandwidths B_{ei} are much larger than the respective B_i, improving the revenues a lot, but the loss of the total equivalent bandwidth $B_{e1} + B_{e2} + W_e$ compared with the case without prioritized sharing is small, indicating that the customer welfare is not reduced so much.

In Cournot competition, the market outcomes are shown in Fig. 2. Unlike Bertrand competition, where a SP can only

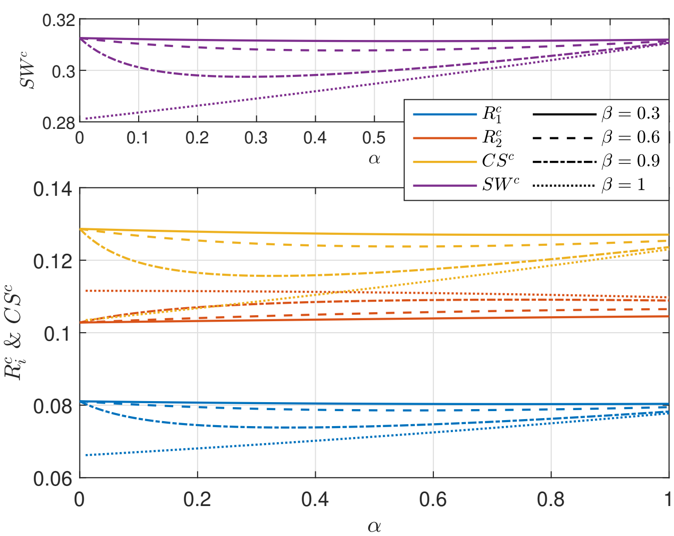

Fig. 2. Market outcomes as functions of α in Bertrand competition with $B_1 = 1$, $B_2 = 2$, $W = 1$, and different β.

get profit from its licensed service, it is not guaranteed that the prioritized sharing is beneficial to a SP. When prioritized sharing is applied, SP 1 with a smaller actual bandwidth gains a lower revenue, but SP 2 with a larger actual bandwidth achieves a higher revenue. The prioritized sharing technique harms the customer surplus, and a bigger β results in a lower customer surplus, which is similar to Bertrand competition and has been proved in Corollary 11. Prioritized sharing also hurts the social welfare, as shown on the top of Fig. 2.

It is easy to verify from Figs. 1 and 2 that, compared with Bertrand competition, Cournot improves the revenues of SPs but hurts the custom welfare. The effect of social welfare is not so straightforward. According to Fig. 3, where the social welfares of both competitions are plotted, it can be seen that

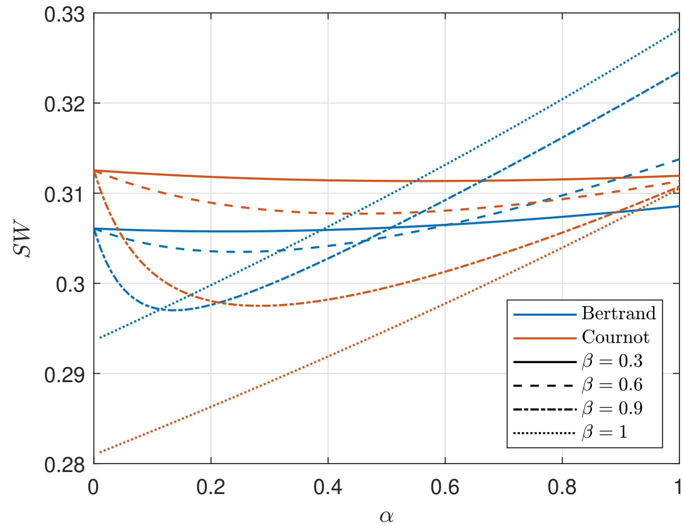

Fig. 3. Social welfares as functions of α in both Bertrand and Cournot competition, with $B_1 = 1$, $B_2 = 2$, $W = 1$, and different β.

Cournot competition is more likely to achieve a higher welfare when α and β do not deviate too far from the case without prioritized sharing, in other words, when α and β are not too large. However, this trend is only shown for the case of $B_1 = 1$, $B_2 = 2$, $W = 1$ and may not hold for other sets of bandwidths.

B. Fixed α and β

In this subsection, we show the impacts of the bandwidths B_1, B_2, W both with and without prioritized sharing. Given $B_1 = 1$, $\alpha = \beta = 0.5$, the revenues of both SPs are shown in Fig. 4, where $\alpha\beta = 0$ refers to the scenarios without prioritized sharing. In this example, each SP gets a higher revenue

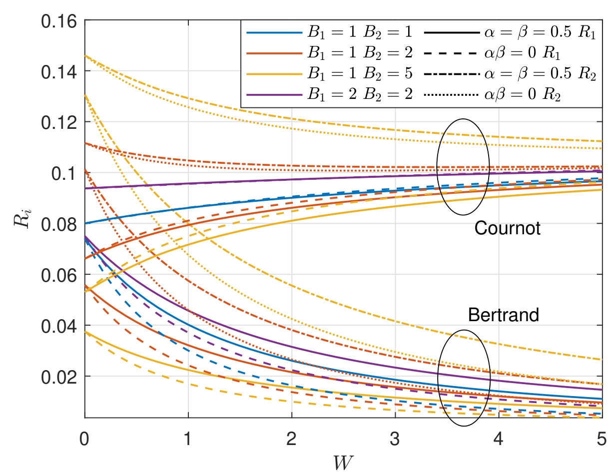

Fig. 4. Revenues as functions of W in Bertrand and Cournot competitions. For the cases where $B_1 = B_2$, only R_1 is shown since $R_2 = R_1$.

with a larger bandwidth of itself or a smaller bandwidth of its opponent, as attractive in Corollary 4, Corollary 5, and Corollary 15. In Bertrand competition, the revenues of both

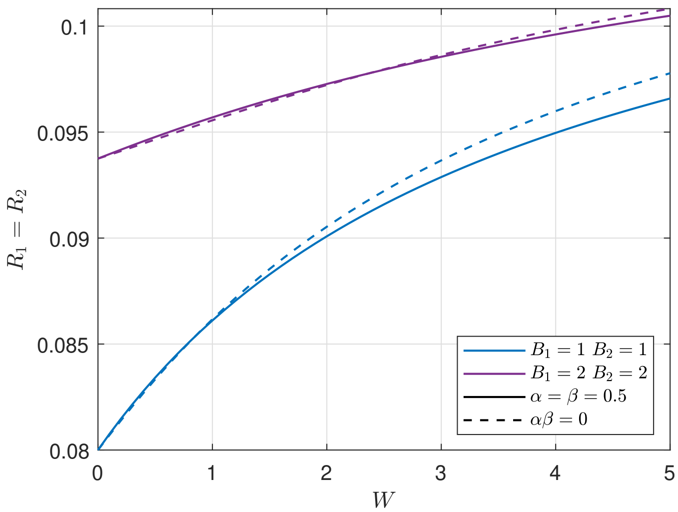

Fig. 5. Zoomed-in version of Fig. 4, but only for revenues under $B_1 = B_2 = 1$ or 2 and Cournot competition.

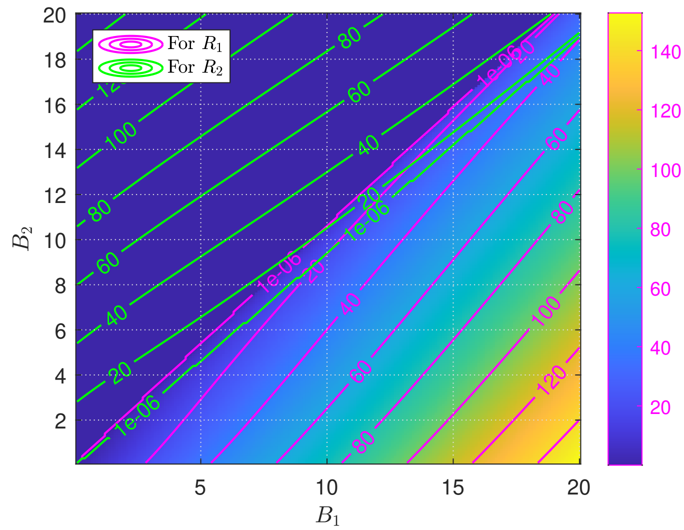

Fig. 6. Maximum W to allow prioritized sharing to improve the revenue of one of the SPs in Cournot competition, given $\alpha = \beta = 0.5$ and B_1, B_2.

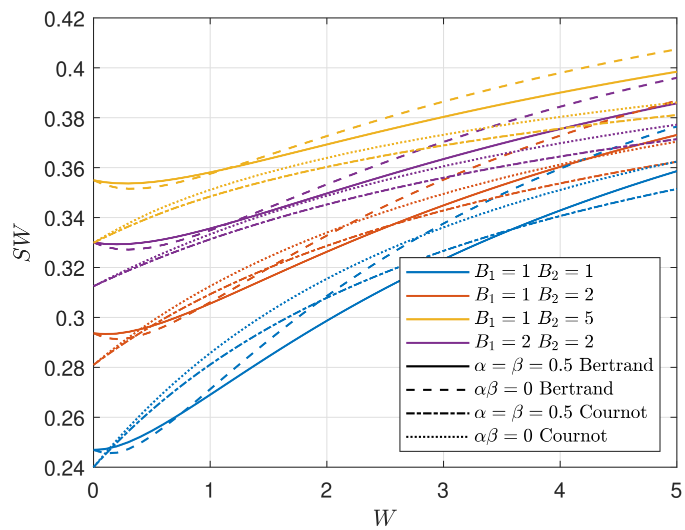

Fig. 7. Social welfares as functions of W in Bertrand and Cournot competitions.

SPs are also decreasing in the shared bandwidth W, probably because the free unlicensed access service would be more welcomed with lower congestion, compared with licensed services. By improving the equivalent licensed bandwidths and suppressing the equivalent shared bandwidths, prioritized sharing improves the revenues of the SPs, which is consistent with the result given in Corollary 2. Such trends are not the case in Cournot competition, where the use of prioritized sharing or a higher W may increase or decrease the revenue of a SP since the unlicensed service is not free of charge. In our example, the revenue R_1 of SP 1 with a lower bandwidth is increasing in W when W is between 0 and 5, but R_2 decreases in W when $B_1 = 1$, $B_2 = 2$ or 5, and $W \leq 2.5$. However, when W is high enough, the revenues of both SPs increase in W since a larger W allows the SPs to suppress the congestion and serve more customers, and Cournot competition allows the SPs to make use of this benefit rather than falling into a price war. As for the effects of prioritized sharing, when $B_1 = 1$, $B_2 = 2$ or 5, prioritized sharing lowers the revenue R_1 of SP 1 with its bandwidth lower than its opponent, but improves the revenue R_2 of SP 2. When $B_1 = B_2 = 1$ or 2, prioritized sharing improves the revenues of the SPs when W is small but hurts them when W is large, as shown in Fig. 5. The maximum threshold of W that allows prioritized sharing to improve the revenue of each SP, as a function of B_1 and B_2, is shown in Fig. 6. It can be observed that the threshold of W for R_i increases in B_i and decreases in B_{-i}. We also find that such thresholds decrease and increase as β goes from 0 to 1, and increase in α.

The social welfare under the same set of parameters is shown in Fig. 7. The welfare is increasing in W in Bertrand competition when W is larger than 0.5, and in Cournot competition for all W as proved in Corollary 13. A larger licensed bandwidth of any SP yields a higher welfare, which is consistent with Corollary 9 and Corollary 13. In Bertrand competition, prioritized sharing is beneficial for the social welfare when the shared bandwidth W is very small, but harms the social welfare when W is large. The maximum threshold of W that allows prioritized sharing to improve the social welfare is shown in Fig. 8. It can be observed that this threshold generally increases in the licensed bandwidth B_i of SP i, especially when the ratio B_i/B_{-i} is not too large. Under the current setting of $\alpha = \beta = 0.5$, this threshold is always lower than either B_1 or B_2. We also observe that such a threshold shows a rising trend as α increases, and may decrease and then increase in β as β goes from 0 to 1. In Cournot competition, prioritized sharing always hurts the social welfare. Moreover, under the cases $(B_1, B_2) = (1, 5)$ and $(2, 2)$, Bertrand competition achieves a higher welfare than Cournot competition, which also holds when W is very small or sufficiently large in the other two cases of

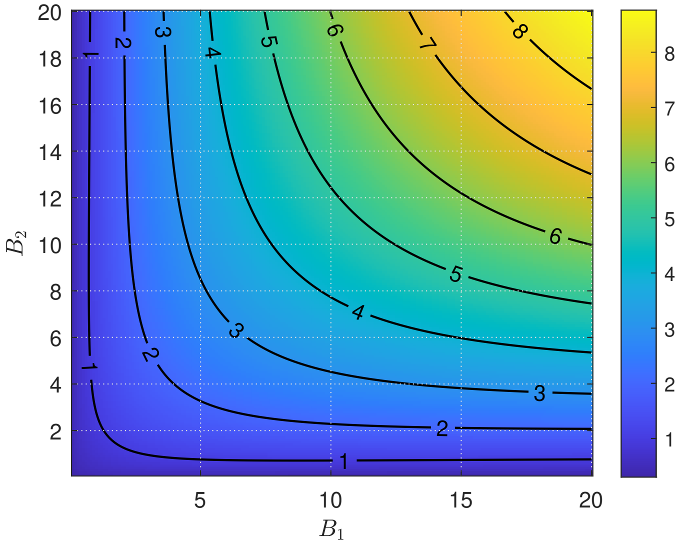

Fig. 8. Maximum W to allow prioritized sharing to improve the social welfare in Bertrand competition, given $\alpha = \beta = 0.5$ and B_1, B_2.

licensed bandwidths. This suggests that Cournot competition may be beneficial to the social welfare only when the licensed bandwidths are not very large and the shared bandwidth lies in a particular range.

V. Conclusion

In this article, we considered competition in a wireless market, where two SPs compete using time-frequency prioritized sharing of a common band of spectrum as well as their own proprietary spectrum. We investigated two competition schemes, Bertrand and Cournot competitions, and analyzed the effects of the prioritized sharing technology, which allows the licensed services to occupy a portion of the shared band in a duty-cycle mode. We derived the equilibrium announced prices and user amounts of each SP under both Bertrand and Cournot competitions. Our results show that prioritized sharing can improve the revenues of the SPs in Bertrand competition, or in Cournot competition when the shared bandwidth is relatively small. Prioritized sharing also harms the customer surplus, and may be harmful to the social welfare in Cournot competition, but it might benefit the social welfare when the shared bandwidth is small. In addition, Cournot competition is beneficial for the SPs but harmful to the customers, and it is more likely to achieve a higher social welfare than Bertrand competition when the prioritized sharing parameters are set so that it does not deviate so much from the case without prioritized sharing, or when the licensed bandwidths are relatively large and the shared bandwidth lies in a particular range.

Possible future work includes considering more general settings such as the case where there are entrants (as in [4]) or more than two incumbent SPs in the market. Other possible directions include considering dynamic settings or heterogeneous customers.

References

[1] The White House, "National spectrum strategy," 2024.

[2] R. Berry, T. W. Hazlett, M. Honig, and J. N. Laneman, "Evaluating the CBRS experiment," in *Proceedings of TPRC 51*, 2023.

[3] B. Bojović, L. Giupponi, Z. Ali, and M. Miozzo, "Evaluating unlicensed LTE technologies: LAA vs LTE-U," *IEEE Access*, vol. 7, pp. 89 714–89 751, 2019.

[4] X. Wang and R. A. Berry, "Market competition between LTE-U and WiFi," *IEEE Trans. Netw. Sci. Eng.*, vol. 8, no. 1, pp. 765–779, Jan. 2021.

[5] R. Berry, M. Honig, T. Nguyen, V. Subramanian, and R. Vohra, "The value of sharing intermittent spectrum," *Management Science*, vol. 66, no. 11, pp. 5242–5264, May. 2020.

[6] T. Nguyen, H. Zhou, R. A. Berry, M. L. Honig, and R. Vohra, "The impact of additional unlicensed spectrum on wireless services competition," in *Proc. IEEE Int. Symp. Dyn. Spectr. Access Netw. (DySPAN)*, May 2011, pp. 146–155.

[7] ——, "The cost of free spectrum," *Operations Research*, vol. 64, no. 6, pp. 1217–1229, 2016.

[8] X. Wang and R. A. Berry, "The impact of bundling licensed and unlicensed wireless service," in *Proc. IEEE Conf. Comput. Commun (INFOCOM)*, April 2018, pp. 1394–1402.

[9] P. Maillé, B. Tuffin, and J.-M. Vigne, "Competition between wireless service providers sharing a radio resource," in *NETWORKING 2012*, 2012.

[10] J. Jia and Q. Zhang, "Competitions and dynamics of duopoly wireless service providers in dynamic spectrum market," in *Proc. of ACM MobiHoc*, 2008.

[11] H. Inaltekin, T. Wexler, and S. B. Wicker, "A duopoly pricing game for wireless IP services," in *Proc. 4th Annu. IEEE Commun. Soc. Conf. Sensor Mesh Ad Hoc Commun. Netw.*, 2007, pp. 600–609.

[12] G. Adams and C. S. Yoo, "Braess's paradox in wireless broadband?: Toward a principled basis for allocating licensed and unlicensed spectrum," *U of Penn, Inst for Law & Econ Research Paper*, Aug. 2023.

[13] E. Kavurmacioglu, M. Alanyali, and D. Starobinski, "Competition in secondary spectrum markets: Price war or market sharing?" in *Proc. IEEE Int. Symp. Dyn. Spectr. Access Netw. (DySPAN)*, 2012, pp. 440–451.

[14] G. Saha and A. A. Abouzeid, "Optimal spectrum partitioning and licensing in tiered access under stochastic market models," *IEEE/ACM Trans. Netw.*, vol. 29, no. 5, pp. 1948–1961, 2021.

[15] K. Mu and R. Berry, "Market impacts of pooling intermittent spectrum," in *Proc. IEEE Int. Symp. Dyn. Spectr. Access Netw. (DySPAN)*, 2024, pp. 189–196.

[16] P. Maillé and B. Tuffin, "Price war with partial spectrum sharing for competitive wireless service providers," in *Proc. IEEE Glob. Telecommun. Conf. (GLOBECOM)*, Nov. 2009, pp. 1–6.

[17] J. G. Wardrop, "Some theoretical aspect of road traffic research," *Proceedings of the Institution of Civil Engineers*, vol. 1, no. 3, pp. 325–362, 1952.