Toward Reliable and Timely Communications: Maximizing the Information Output Rate with Aging Information

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***Abstract*—For delay-sensitive applications, codewords become outdated with a diminishing amount of information as they age. In this paper, the above information aging process is incorporated into the characterization of information output rate, a combined measure of information timeliness and reliability. The analysis shows that by optimally tuning the information encoding rate of codewords, the maximum information output rate may significantly drop when the information ages fast, indicating a high penalty caused by the information aging process.**

***Index Terms*—Maximum information output rate, information aging function, age of codewords**

I. Introduction

Communication systems aim to deliver information in the most reliable and timely manner. While reliable communications has been the primary focus for decades, timely communications is gaining increasing momentum owing to the growing prevalence of delay-sensitive applications [1].

For reliable communications, the ultimate limit of information encoding/transmission rate of codewords is well characterized by the information-theoretical capacity. Yet delay, which plays a central role in timely communications, has been largely ignored in the classical capacity notion [2]. In wireless communication networks, the end-to-end delay of a codeword could be caused by many factors, such as the waiting-to-be-served time at the data buffer and the transmission/retransmission time, which are random in nature due to fluctuating channel conditions and interference from other transmitters. To integrate delay with information-theoretical capacity, an analytical framework was recently proposed in [3] from the queueing-theoretical perspective. By modeling the information flow as a queue of codewords, the information output rate was proposed as a measure of effectiveness of information delivery that is jointly determined by the information encoding rate and mean access delay of codewords. To maximize the information output rate, the information encoding rate should be carefully chosen, which could be far below the information-theoretical capacity when the delay constraint is stringent.

A key assumption adopted in [3] is that the amount of information carried by each codeword does not change with time: It is only determined by the information encoding rate, which is given when a codeword is generated. In practice, however, codewords could become outdated. For instance, for automatic operation, a control signal may become obsolete if it exceeds a certain deadline. For real-time monitoring, the measurement could be less relevant as time grows due to the change of status of the monitoring target. In those scenarios, even though a codeword is correctly/reliably received eventually, the information carried by the codeword is diminished as it ages. For timely communications, it is important to take such an information aging process into account, and evaluate its effect on the maximum information output rate.

A. *Information Aging versus Age of Information*

Note that the above information aging process should be distinguished from the Age of Information (AoI) [4]–[8], which has been widely adopted in the literature as a measure of information timeliness [9]–[12]. Specifically, AoI evaluates how synchronized the receiver is with the transmitter. It is measured at the receiver side, and updated when a packet/codeword is successfully received as its delay (i.e., the time difference from generation to successful reception).

The information aging process is, however, related to the age of each *codeword*. Intuitively, a codeword starts to age when it is generated (or arrives at the codeword queue), and reaches the highest age when it is successfully transmitted. Different from the AoI, the age of codewords is measured at the transmitter side. More importantly, when the codeword queue is empty, the age of codewords is zero, yet the AoI would continue to increase. The implication behind a large AoI in this case is that if the receiver has not received any codewords for a while, then it is not synchronized with the transmitter. It indicates that the *last* successfully received codeword has been outdated. That, nevertheless, does not foretell whether/how outdated the next to-be-received codeword would be.

In fact, how outdated a codeword is only depends on its own age, and how fast the amount of information decays with its age. In this paper, the latter is evaluated by the information aging function, which is introduced as a non-increasing function of a codeword's age. For each codeword, the amount of information that can be successfully delivered is jointly determined by the initial amount of information upon generation (or the information encoding rate), the information aging function, and its delay.

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B. Maximizing the Information Output Rate with Aging Information

With aging information, the maximum information output rate crucially depends on the information aging function and the distribution of delay of codewords. For demonstration, we take the example of Rayleigh fading channels with no channel state information at the transmitter side, and derive the maximum information output rate with Bernoulli arrivals of codewords under two representative information aging functions. The analysis shows that to maximize the information output rate, different from the non-aging case where the information input rate should be sufficiently large to push the codeword queue to the saturated condition, with aging information, the codeword queue should remain in the unsaturated condition, with the information encoding rate of codewords optimally set according to the information input rate. The maximum information output rate could be significantly lower than that with non-aging information when the information ages fast, indicating a high penalty caused by the information aging process.

The information output rate is a combined measure of information timeliness and reliability, to optimize which requires a balance between them. To see that, we further compare the maximum information output rates with the retry limit $L = 1$ and $L = \infty$. In the former case, a codeword would be dropped once it fails, and in the latter case, codewords are removed from the queue only when they are successfully transmitted. With $L = 1$, even though the information is delivered in the most timely manner, part of codewords are dropped and never reach the receiver side. The analysis shows that whether the benefit in information timeliness outweighs the loss in reliability crucially depends on the information input rate and the information aging function. When the information input rate is below a certain threshold, which increases with a slower information aging rate, the codewords should not be dropped as $L = \infty$ achieves a higher maximum information output rate than $L = 1$.

The remainder of this paper is organized as follows. Section II presents the system model, where key assumptions and definitions are outlined. The maximum information output rates in fading channels with two representative information aging functions are characterized in Section III. Conclusions are summarized in Section IV.

II. Queue of Codewords with Aging Information

Consider a single user with information bits assembled into codewords. Assume that each codeword lasts for one time slot. Codewords are stored in a data buffer and removed only when they are successfully transmitted.

For the queue of codewords, assume a stationary arrival process, and that the interarrival times between codewords $\{X_i\}$ are independent. Codewords are served on a first-come-first-served basis. Let $A(t)$ denote the number of codewords arrived by time slot t, $t = 1, 2, \ldots$ which can be modeled as a renewal process as $A(t) = \max\{m : \sum_{i=1}^{m} X_i \leq t\}$. Let $\tilde{R}_i^A$ (in the unit of bit/Hz) denote the amount of information normalized by the bandwidth carried by the i-th codeword upon arrival (i.e., being generated), $i = 1, 2, \ldots$. The total amount of information arrived by time t is then $\tilde{R}_{in}(t) = \sum_{i=1}^{A(t)} \tilde{R}_i^A$, which is a reward renewal process. The information input rate (in the unit of bit/s/Hz), R_{in}, defined as the long-term average number of input information bits per unit time per unit bandwidth, can be written as

$$R_{in} \triangleq \frac{1}{\sigma} \lim_{t\to\infty} \frac{\tilde{R}_{in}(t)}{t} = \lambda \cdot \frac{\mathrm{E}[\tilde{R}_i^A]}{\sigma}, \qquad (1)$$

where $\lambda \triangleq \lim_{t\to\infty} \frac{1}{t} A(t) = \frac{1}{\mathrm{E}[X]}$ is the arrival rate of codewords, i.e., the long-term average number of arrived codewords packets per time slot, and σ is the slot length.

For the service process, let $S(t)$ denote the number of codewords being served by time slot t, $t = 1, 2, \ldots$, which can also be modeled as a renewal process as $S(t) = \max\{m : \sum_{i=1}^{m} Z_i \leq t\}$, where $Z_i = D_i + Y_i$, with D_i and Y_i denoting the service time of the i-th codeword, and the idle time of the queue before its service, respectively. Let $\tilde{R}_i^S$ (in the unit of bit/Hz) denote the amount of information normalized by the bandwidth carried by the i-th codeword upon completion of service (i.e., being successfully transmitted), $i = 1, 2, \ldots$. The total amount of information successfully delivered by time t is then $\tilde{R}_{out}(t) = \sum_{i=1}^{S(t)} \tilde{R}_i^S$, which is a reward renewal process. The information output rate (in the unit of bit/s/Hz), R_{out}, defined as the long-term average number of successfully transmitted information bits per unit time per unit bandwidth, can be written as

$$R_{out} \triangleq \frac{1}{\sigma} \lim_{t\to\infty} \frac{\tilde{R}_{out}(t)}{t} = \lambda_{out} \cdot \frac{\mathrm{E}[\tilde{R}_i^S]}{\sigma}, \qquad (2)$$

where $\lambda_{out} \triangleq \lim_{t\to\infty} \frac{1}{t} S(t) = \frac{1}{\mathrm{E}[Z]}$ is the throughput of the codeword queue, i.e., the long-term average number of successfully transmitted codewords per time slot. λ_{out} depends on whether the queue is saturated or not. When it is saturated, that is, the queue is always non-empty, we have $Y_i = 0$ for all $i = 1, 2, \ldots$, and thus

$$\lambda_{out} = \frac{1}{\mathrm{E}[D]} = \mu, \qquad (3)$$

where μ is the service rate, i.e., the long-term average number of served codewords per time slot given that the queue is non-empty. On the other hand, if the queue is unsaturated, the probability of the queue being non-empty is given by $\frac{\lambda}{\mu} = \frac{\mathrm{E}[D]}{\mathrm{E}[Z]}$, and we have

$$\lambda_{out} = \frac{\lambda}{\mu E[D]} = \lambda. \qquad (4)$$

By combining (3) and (4) and noting that $\lambda < \mu$ when the queue is unsaturated, the throughput can further be written as $\lambda_{out} = \min\{\lambda, \mu\}$.

By comparing (1) with (2), we can see that the information output rate R_{out} is equal to the information input rate R_{in} only when the codeword queue is unsaturated, i.e., $\lambda_{out} = \lambda$, and the amount of information carried by each packet remains unchanged with time, i.e., $\tilde{R}_i^S = \tilde{R}_i^A$.

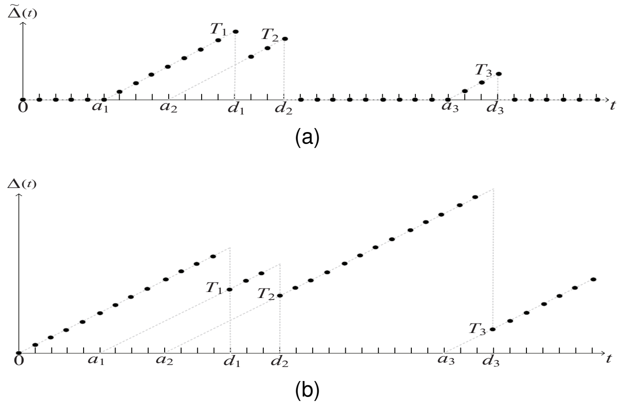

Fig. 1. A sample path of (a) age of codewords $\tilde{\Delta}(t)$ and (b) AoI $\Delta(t)$. a_i and d_i denote the time of arrival and time of service completion for the i-th codeword, respectively, and $T_i = d_i - a_i$ denotes the queueing delay of the i-th codeword, $i = 1, 2, \ldots$.

A. Age of Codewords

In practice, information may become outdated as time goes by. To take the information aging into account, assume that the amount of information carried by each codeword depends on the codeword's *age*. Intuitively, a codeword starts to age when it arrives at the queue. Define the age of a codeword as the amount of time it stays in the queue. Apparently its age upon completion of service is its queueing delay, i.e., the amount of time from arriving till being successfully transmitted.

Specifically, let $\tilde{\Delta}_i(t)$ denote the age of the i-th codeword at time slot t:

$$\tilde{\Delta}_i(t) \triangleq \begin{cases} t - a_i & a_i \leq t \leq d_i \\ 0 & otherwise, \end{cases} \tag{5}$$

where a_i and d_i denote the time of arrival and time of service completion for the i-th codeword, respectively, $i = 1, 2, \ldots$. The age of the head-of-line (HOL) codeword at time t can then be written as $\tilde{\Delta}(t) = \max_{i \in \{1,2,\ldots\}} \tilde{\Delta}_i(t)$. Fig. 1a presents a sample path of $\tilde{\Delta}(t)$. For comparison, the AoI $\Delta(t)$ [4] is illustrated in Fig. 1b. It can be seen that although $\tilde{\Delta}(d_i) = \Delta(d_i) = T_i$, where $T_i = d_i - a_i$ denotes the queueing delay (i.e., waiting time plus service time) of the i-th codeword, $i = 1, 2, \ldots$, $\tilde{\Delta}(t)$ is much smaller than $\Delta(t)$. Moreover, the age of codewords $\tilde{\Delta}(t)$ is measured at the transmitter side, rather than the receiver side. If the queue at the transmitter is empty, i.e., there is no HOL codeword, $\tilde{\Delta}(t) = 0$, while the AoI $\Delta(t)$ would continue to increase.[1]

B. Information Aging Function

Age of codewords $\tilde{\Delta}(t)$ indicates how timely the information delivery is. For given $\tilde{\Delta}(t)$, how much information can be delivered eventually also depends on how fast the information becomes outdated. Further introduce the information aging function, denoted by $g(t)$, which is a non-increasing function of a codeword's age: $g(t) \leq g(t-1)$ for $t = 1, 2, \ldots$ and $g(0) = 1$.

Specifically, for the i-th codeword, $i = 1, 2, \ldots$, the amount of information it carries is R_i^A upon arrival (i.e., being generated). It starts to decrease as its age grows, with the decreasing rate of $g(\tilde{\Delta}_i(t))$. Upon completion of service (i.e., being successfully transmitted), the amount of information it carries, $\tilde{R}_i^S$, can be written as

$$\tilde{R}_i^S = \tilde{R}_i^A \cdot g(\tilde{\Delta}_i(d_i)) = \tilde{R}_i^A \cdot g(T_i), \tag{6}$$

$i = 1, 2, \ldots$.

The information aging function $g(t)$ depends on the applications. For instance, for real-time applications such as control information in industrial automation, a codeword would become useless when it exceeds a certain deadline. In that case, the information aging function can be set as $g(t) = 0$ for $t > D_0$, where D_0 is the hard delay constraint. For channel measurement, the correlation between the samples and the real-time channel gain gradually decreases as time goes by. $g(t)$ can then be set as the normalized autocorrelation function of the channel gain process. When $g(t) = 1$ for all t, the information carried by each codeword does not depend on its age.

C. Maximum Information Output Rate

By combining (6) with (2), the information output rate can be written as

$$R_{out} = \mathrm{E}[R_i^A \cdot g(T_i)] \cdot \lambda_{out}, \tag{7}$$

where $R_i^A = \frac{\tilde{R}_i^A}{\sigma}$ denotes the information encoding rate of codewords (in the unit of bit/s/Hz), which does not change with time. Eq. (7) shows that the information output rate is a combined measure of information timeliness and reliability: It is jointly determined by the queueing delay and throughput of codewords, which measure how timely and reliably the information delivery is, respectively.

Both the delay and throughput of the codeword queue depend on whether the queue is saturated or not. By combining (7) with (1), (3) and (4), we have

$$R_{out} = \begin{cases} \frac{\mathrm{E}[R_i^A \cdot g(T_i)]}{\mathrm{E}[R_i^A]} \cdot R_{in} & R_{in} < \mathrm{E}[R_i^A] \cdot \mu \\ \mathrm{E}[R_i^A \cdot g(T_i)] \cdot \mu & R_{in} \geq \mathrm{E}[R_i^A] \cdot \mu. \end{cases} \tag{8}$$

Note that when the queue becomes saturated, the queueing delay of each codeword increases with time unboundedly. If the information aging function $g(t) = 0$ as $t \to \infty$, then the information output rate R_{out} drops to zero when $R_{in} \geq \mathrm{E}[R_i^A] \cdot \mu$.

Given the information aging function $g(t)$ and information input rate R_{in}, the information encoding rate of codewords $\{R_i^A\}$ can further be optimized to maximize the information output rate. Let C denote the maximum information output rate:

$$C = \max_{\{R_i^A\}} R_{out}. \tag{9}$$

[1]Note that a large AoI $\Delta(t)$ could be caused by infrequent transmissions, with which the information may still be delivered in a timely manner. To see that, consider a scenario where control signaling packets are generated infrequently, but each can always be successfully transmitted in one time slot. In this case, age of packets $\tilde{\Delta}(t) \leq 1$ for all t, but the AoI $\Delta(t)$ is large as the queue is idle for most of the time.

In [3], the maximum information output rate in fading channels has been characterized in various scenarios by assuming non-aging information (i.e., $g(t) = 1$). In the following, we will further demonstrate the effect of information aging on the maximum information output rate.

III. Maximum Information Output Rate in Fading Channels with Aging Information

Consider a block fading channel, that is, the channel gain remains fixed within one block, but varies from block to block. Assume that the length of each time slot is equal to the time duration of one block, and the blocklength is sufficiently large for averaging out the additive noise, i.e., the maximum achievable rate in each block is determined by the additive-White-Gaussian-noise (AWGN) channel capacity. Assume that the receiver always has the channel state information (CSI), that is, the realization of fading gain in the current time slot, but the CSI at the transmitter side (CSIT) is optional. Assume that the mean transmission signal-to-noise ratio (SNR) is ρ.

For a HOL codeword, let p_s denote the probability of a successful transmission in each time slot. Without CSIT, each codeword has to be transmitted with constant power and constant information encoding rate, i.e., $R_i^A = R_0$, $i = 1, 2, \ldots$. If the information encoding rate R_0 of a codeword exceeds the maximum rate that can be supported by the channel, then an outage event occurs, and the codeword needs to be retransmitted in the next slot. p_s can then be written as $p_s = \Pr\left\{\log_2(1+\rho|h|^2) \geq R_0\right\}$, which is indeed the probability of no outage. With CSIT, on the other hand, the transmission power can be adjusted according to the channel gain, and the information encoding rate R_i^A can be set as $R_i^A = \log_2(1 + SNR(h_i)|h_i|^2)$ when $SNR(h_i) > 0$. If the channel condition is not sufficiently good, however, the transmission power may be set to zero, in which case the codeword transmission will be suspended. p_s can then be written as $p_s = \Pr\left\{SNR(h) > 0\right\}$.

Note that the service rate of the codeword queue $\mu = p_s$. For non-aging information, i.e., $g(t) = 1$ for $t = 1, 2, \ldots$, the information output rate can be simplified from (8) as

$$R_{out}^{non-aging} = \begin{cases} R_{in} & R_{in} < \mathrm{E}[R_i^A] \cdot p_s \\ \mathrm{E}[R_i^A] \cdot p_s & R_{in} \geq \mathrm{E}[R_i^A] \cdot p_s. \end{cases} \tag{10}$$

It can be seen from (10) that to maximize the information output rate, the codeword queue should be pushed to the *saturated* condition, and the maximum information output rate is given by[2]

$$C^{non-aging} = \max_{\{R_i^A\}} \ \mathrm{E}[R_i^A] \cdot p_s. \tag{11}$$

[2]It has been demonstrated in [3] that the maximum information output rate $C^{non-aging}$ is closely related to the channel capacity, which dictates the highest information *encoding* rate of codewords. Without CSIT, $C^{non-aging}$ can be written as $C^{non-aging} = \max_{0 \leq \epsilon \leq 1} R_\epsilon \cdot (1-\epsilon)$, where R_ϵ is the ϵ-outage capacity. With CSIT, $C^{non-aging}$ is equal to the ergodic capacity with the optimal waterfilling power allocation [13].

For aging information, however, the information output rate is also determined by the queueing delay of codewords and the information aging function $g(t)$. If $g(t) \neq 0$ only for $t < \infty$, then the information output rate $R_{out} \neq 0$ only when the codeword queue is *unsaturated*. According to (8)-(9), the maximum information output rate can be written as

$$C = \begin{cases} R_{in} \cdot \max_{\{R_i^A\}} \frac{\mathrm{E}[R_i^A \cdot g(T_i)]}{\mathrm{E}[R_i^A]} & R_{in} < C^{non-aging} \\ 0 & R_{in} \geq C^{non-aging}. \end{cases} \tag{12}$$

It can be clearly seen from (12) that with aging information, the maximum information output rate closely depends on the information aging function $g(t)$ and the distribution of the queueing delay of codewords T.

In the following, we will take two examples to demonstrate the effect of information aging function on the maximum information output rate. Without loss of generality, assume Bernoulli arrivals of the codeword queue, that is, in each time slot, the probability of a new codeword arriving is λ. The queueing delay of each codeword then follows the Geometric distribution with parameter $\tilde{p} = \frac{p_s - \lambda}{1-\lambda}$. Moreover, we limit the discussion to Rayleigh fading channels with no CSIT, in which case each codeword is transmitted with constant SNR ρ and constant information encoding rate R_0, and the probability of successful transmission of HOL codewords is

$$p_s = \exp\left\{-\frac{2^{R_0}-1}{\rho}\right\}. \tag{13}$$

With non-aging information, the maximum information output rate without CSIT in Rayleigh fading channels has been given in [3] as

$$C^{NAR} = \exp\left\{-\frac{e^{\mathbb{W}_0(\rho)}-1}{\rho}\right\} \mathbb{W}_0(\rho) \log_2 e, \tag{14}$$

achieved when the information encoding rate is set to

$$R^{*,NAR} = \mathbb{W}_0(\rho) \log_2 e, \tag{15}$$

where $\mathbb{W}_0(\cdot)$ is the principal branch of the Lambert W function.

A. With a Hard Delay Constraint

Let us first assume that there is a hard delay constraint D_0. That is, the codewords become useless if they stay in the queue for more than D_0 time slots. In that case, the information aging function can be set as

$$g(t) = \begin{cases} 1 & 1 \leq t \leq D_0 \\ 0 & t > D_0, \end{cases} \tag{16}$$

for $D_0 \geq 1$. With a finite delay constraint $D_0 < \infty$, we have

$$\mathrm{E}_T[g(T)] = 1 - \left(1 - \frac{R_0 \cdot p_s - R_{in}}{R_0 - R_{in}}\right)^{D_0}. \tag{17}$$

It can be seen from (12) that with a constant information encoding rate R_0, the optimal information encoding rate to maximize the information output rate can be written as

$$R^* = \arg\max_{R_0 > 0} \ \mathrm{E}_T[g(T)], \tag{18}$$

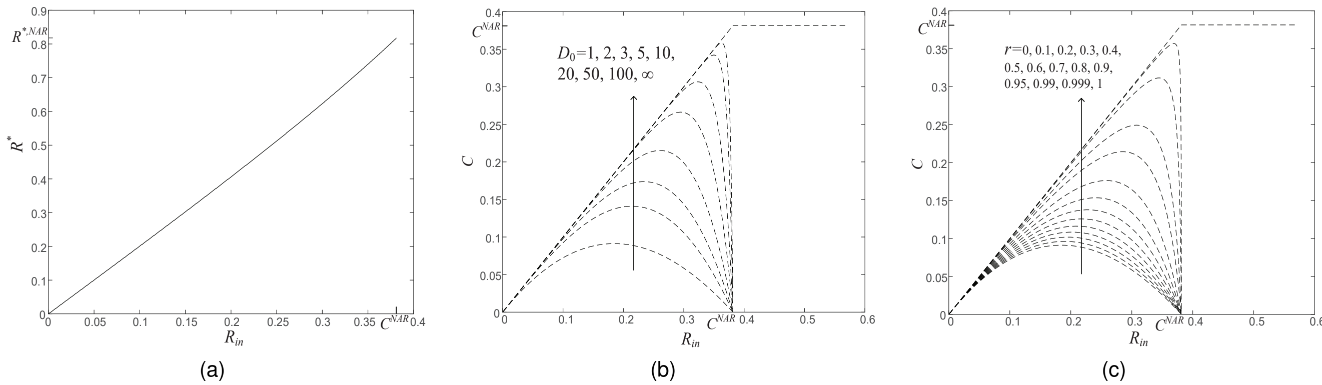

Fig. 2. (a) Optimal information encoding rate R^* (bit/s/Hz) and (b)(c) maximum information output rate C (bit/s/Hz) versus information input rate R_{in} (bit/s/Hz). $\rho = 0$ dB. Information aging function $g(t)$ is given in (16) in (b) and (21) in (c), respectively.

for $R_{in} < C^{non-aging}$. For Rayleigh fading channels with no CSIT, it can be obtained from (13) and (17) that the optimal information encoding rate R^* is the root of the following equation of $R_0 > 0$:

$$\exp\left\{-\frac{2^{R_0}-1}{\rho}\right\}\cdot\left(\frac{R_0 2^{R_0}\ln 2}{\rho}\cdot(R_0-R_{in})+R_{in}\right)=R_{in}, \quad (19)$$

for $R_{in} < C^{NAR}$. It can be seen from (19) that R^* is independent of the delay constraint D_0, but should be carefully adjusted based on the information input rate R_{in}. As $R_{in} \to C^{NAR}$, $R^* \to R^{*,NAR}$. Fig. 2a shows that the optimal information encoding rate R^* almost linearly increases with the information input rate R_{in}.

The maximum information output rate can further be obtained from (12) and (17) as

$$C=\begin{cases} R_{in}\cdot\left[1-\left(1-\frac{R^*\exp\left\{-\frac{2^{R^*}-1}{\rho}\right\}-R_{in}}{R^*-R_{in}}\right)^{D_0}\right] & R_{in}<C^{NAR} \\ 0 & R_{in}\geq C^{NAR} \end{cases} \quad (20)$$

Fig. 2b demonstrates the effect of delay constraint D_0 on the maximum information output rate C. It can be seen that for given information input rate $R_{in} < C^{NAR}$, the smaller the delay constraint D_0, the lower the maximum information output rate C. C could be much lower than C^{NAR}, indicating a high penalty caused by the information aging process.

Note that with a finite D_0, $C = 0$ if the information input rate R_{in} reaches or exceeds C^{NAR} because the delay becomes infinite. With $D_0 = \infty$, the information does not age with time. We then have $C = C^{NAR}$ for $R_{in} \geq C^{NAR}$.

B. With an Exponentially Decreasing Aging Function

Now assume that the amount of information carried by each codeword exponentially decreases as its age grows. Set the information aging function as

$$g(t) = r^{t-1}, \qquad t = 1, 2, \ldots \quad (21)$$

for $0 \leq r \leq 1$. The smaller r, the faster the amount of information decreases with time.

With $0 \leq r < 1$, we have

$$\mathrm{E}_T[g(T)]=\frac{R_0\cdot p_s-R_{in}}{R_0-R_{in}-r(R_0-R_0\cdot p_s)}=\frac{1}{r+(1-r)\cdot\frac{R_0-R_{in}}{R_0\cdot p_s-R_{in}}}. \quad (22)$$

It can be seen from (22) and (17) that both information aging functions lead to the same optimal information encoding rate R^*, which only depends on the information input rate R_{in}. For Rayleigh fading channels with no CSIT, R^* is the root of (19), illustrated in Fig. 2a. The maximum information output rate can be obtained from (12) and (22) as

$$C=\begin{cases} R_{in}\cdot\frac{R^*\exp\left\{-\frac{2^{R^*}-1}{\rho}\right\}-R_{in}}{R^*\left(1-r+r\exp\left\{-\frac{2^{R^*}-1}{\rho}\right\}\right)-R_{in}} & R_{in}<C^{NAR} \\ 0 & R_{in}\geq C^{NAR}. \end{cases} \quad (23)$$

As Fig. 2c shows, for given information input rate $R_{in} < C^{NAR}$, a smaller r indicates that the information ages faster, and thus leads to a lower maximum information output rate. As $r \to 1$, the information does not age, and the maximum information output rate reaches C^{NAR} for $R_{in} \geq C^{NAR}$.

C. Discussion: To Drop or Not to Drop

Note that in the above analysis, it is assumed that the codewords stay in the buffer until they are successfully transmitted. In practice, a retry limit L is often imposed, with which a codeword could be dropped if it fails for L times. Intuitively, to optimize the delay/AoI performance, the retry limit should be set to $L = 1$, with which every codeword, regardless of being successful or failed, only stays in the buffer for one time slot.

Despite the minimum delay, lots of codewords may be dropped with $L = 1$. Compared with the non-dropping case, i.e., $L = \infty$, gains may not necessarily be achieved in terms of the maximum information output rate. For non-aging information, it has been shown in [3] that the maximum information output rate $C^{non-aging}$ is independent of the retry limit L. To achieve $C^{non-aging}$, the information input rate R_{in} should be sufficiently large to push the queue to the saturated condition, i.e., $R_{in} \geq C^{non-aging}$.

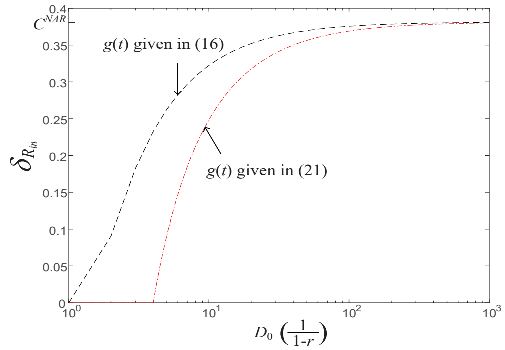

Fig. 3. $\delta_{R_{in}}$ versus D_0 with $g(t)$ given in (16) and $\delta_{R_{in}}$ versus $1/(1-r)$ with $g(t)$ given in (21).

With aging information, to see which one leads to a higher maximum information output rate, $L = 1$ or $L = \infty$, recall that the throughput of the codeword queue with $L = 1$ is given by [3]

$$\lambda_{out}^{L=1} = \begin{cases} \lambda \cdot p_s & \lambda < 1 \\ p_s & \lambda \geq 1, \end{cases} \tag{24}$$

and the queueing delay of each codeword with $L = 1$ is 1 time slot. With a constant information encoding rate R_0, the information output rate can then be written as

$$R_{out}^{L=1} = \begin{cases} R_{in} \cdot p_s & R_{in} < R_0 \\ R_0 \cdot p_s & R_{in} \geq R_0. \end{cases} \tag{25}$$

For Rayleigh fading channels with no CSIT, the optimal information encoding rate is

$$R^{*,L=1} = \begin{cases} R_{in} & R_{in} < R^{*,NAR} \\ R^{*,NAR} & R_{in} \geq R^{*,NAR}, \end{cases} \tag{26}$$

and the maximum information output rate is

$$C^{L=1} = \begin{cases} R_{in} \cdot \exp\left\{-\frac{2^{R_{in}}-1}{\rho}\right\} & R_{in} < R^{*,NAR} \\ C^{NAR} & R_{in} \geq R^{*,NAR}. \end{cases} \tag{27}$$

By comparing (27) with (20) and (23), it can be seen that $L = 1$ leads to a higher maximum information output rate only when the information input rate R_{in} is sufficiently high. Let $\delta_{R_{in}}$ denote the threshold of R_{in}. That is, $C^{L=1} > C^{L=\infty}$ if and only if $R_{in} > \delta_{R_{in}}$. Fig. 3 illustrates $\delta_{R_{in}}$ with the two information aging functions given in (16) and (21). It can be seen from Fig. 3 that $\delta_{R_{in}}$ decreases as the information ages faster, e.g., with a smaller D_0 or r. $\delta_{R_{in}} = 0$ when $D_0 = 1$ or $r = 0$, i.e., a codeword expires after one time slot, in which case $L = 1$ is always better. On the other hand, as $D_0 \to \infty$ or $r \to 1$, $\delta_{R_{in}} \to C^{NAR}$, indicating that $L = \infty$ leads to a higher maximum information output rate as long as the information input rate $R_{in} < C^{NAR}$.

Intuitively, with $L = 1$, the information is delivered in the most timely manner as $T_i = 1$ for all codewords. It is, nevertheless, achieved at the cost of reliable communications, as part of the codewords are dropped. With $L = \infty$, on the other hand, all the codewords can be successfully delivered eventually, though with high delay when the information input rate R_{in} is large. With R_{in} below the threshold $\delta_{R_{in}}$, $L = \infty$ achieves a larger maximum information output rate, indicating a better balance between information timeliness and reliability. Moreover, for aging information, the effect of delay on the information output rate crucially depends on the information aging function. The slower information aging rate, the less stringent requirement on information timeliness, and thus the wider range of R_{in} for $L = \infty$ to outperform $L = 1$.

IV. Conclusion

For timely communications, the delay experienced by each codeword may have a significant impact on the amount of information it can successfully deliver. Taking the information aging process into account, in this paper, the information aging function is introduced to describe how fast the information becomes outdated with time. To demonstrate how to optimize the information output rate with aging information, two representative information aging functions are considered, with which the maximum information output rate in fading channels without CSIT is characterized. The rate comparison of the retry limit $L = 1$ and $L = \infty$ further shows that the delay/AoI-optimal strategy $L = 1$ may not always lead to a higher maximum information output rate. $L = \infty$ strikes a better balance between information timeliness and reliability when the information input rate is below a certain threshold that increases with a slower information aging rate.

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