Cosine Phase Based Representation of Signals for Remote Monitoring in Multiuser Wireless Networks

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***Abstract*—In this paper, we investigate a novel representation of a band-limited and continuous-time signal using a sequence of impulses or events. To begin with, we evaluate the mean squared error between the original signal and its reconstructed version assuming that: (i) the signal samples are transmitted at the Shannon channel capacity; (ii) the signal representation using the cosine phase method is done before its transmission. The proposed approach is then compared with an existing approach based on the signal transmission of quantized samples. Finally, we provide some representative examples through Monte Carlo simulations, consistently revealing the superiority of the proposed signal representation approach over the existing one regarding transmission bandwidth and peak-to-peak value of the original signal.**

***Index Terms*—Event-driven transmission, continuous-time signals, mean-square error, wireless sensor networks.**

I. Introduction

A wireless sensor network (WSN) is a distributed network that typically consists of a fairly large number of autonomous battery-powered wireless nodes with limited energy-capabilities [1]. The data monitoring and reporting are important functions of the WSNs, building up an uplink-dominated system where multiple sensor nodes share the same network resources to communicate to a single gateway. This class of wireless network can be classified under the name massive machine-type communications (mMTC).

In this context, time-based strategies that perform sensing, processing, and communication functions as part of the monitoring and reporting schemes are known to bring challenges due to the very large number of users that share network resources while individually constrained by their energy limits [2], [3]. An answer to this is to employ signal processing and communication based on more informative events [4]. In contrast to the time-based approach, the event-based paradigm requires aperiodic transmissions of sample data when a meaningful event occurs (e.g. a threshold violation). In many types of signals (e.g., with spiking like variations), event-based communication results in a reduction of superfluous transmissions while maintaining system performance, e.g., signal reconstruction [5]–[8].

The event-based paradigm, however, suffers from synchronization issues due to its aperiodic nature, that may severely limit packets' transmission and identification. Recently, in [9], we developed a new method called semantic–functional communication (SFC) tailored for multiuser alarm systems with event-driven transmissions of sensors' states. This approach is based on meaningful data transmission in respect to the alarm system functionality. In [10], SFC was also demonstrated to be effective as the basis of a wireless network supporting the operation of a swarm of unmanned aerial vehicle (UAV). The proposed solution was demonstrated to offer the best trade-off between performance in terms of the functionality of the system (remote sensing), energy efficiency and time delay.

A key aspect of [10] is that the functionality expected to the system only required the transmission of the state (i.e., alarm message). In this paper, our aim is to extend this approach in the case where the functional goal is to reliably reconstruct signals (i.e., time series) transmitted by different source nodes at a given sink (e.g., a gateway). In this case, we are moving from the transmission of discrete states towards telemetry of continuous signals in a multiuser scenario.

The main issue is that traditionally Fourier representation of signals assumes a periodic sampling and synchronization. For our proposal, this needs to be revisited, and thus, we are proposing a novel representation of a band-limited continuous-time signal via a sequence of pulses and cosine phase shifts (which can be associated to events in the SFC approach.) Such a signal is described by two-phase shifts for each term of its Fourier transform. Assuming that the signal samples are transmitted at the Shannon channel capacity, and that the signal representation using the cosine phase method is done before its transmission, we evaluate the mean squared error (MSE) between the original signal and its recovered version. Moreover, we compare the proposed approach with an existing one (used as benchmark) based on signal transmission of quantized samples. Finally, Monte Carlo simulations are provided to assess the performance of the proposed system

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and the benchmark approach.

The remainder of the paper is organized as follows: In Section II, the underlying problem statement is formulated. In Section III, a novel representation of signals is proposed, namely signal representation by cosine phase. Section IV describes the transmission configurations adopted in this work. In Section V, some representative examples are provided, consolidated by the final remarks in Section VI.

II. Problem statement

A scenario where several signals are to be reported to a gateway or network node is considered. We begin with some important definitions. A sink node is a node that combines all the received signals, while a network node[1] is a node that reports its measurements to the sink node. It is assumed that a sink node cannot transmit to the sensor node, and that all the retransmissions, acknowledgment messages, or update requests are not made available at the latter. The scope of the proposed work is limited to the sampling&transmission and reconstruction processes at the sensor nodes and sink node, respectively.

A. Sampling, transmission power, and bandwidth

To begin with, we review the impact of the sampling rate in conjunction with quantization on the MSE for the recovered signal. Let $x(t)$ be a band-limited, continuous time signal, i.e., $\exists\, W > 0$ such that $|X(\omega)| = 0$, $\forall\, |\omega|/\pi > W$ with $X(\omega)$ representing the Fourier transform of $x(t)$. Thus, samples of $x(t)$ spaced less than or equal to $1/W$ seconds apart are a sufficient representation of $x(t)$. Assuming a uniform sampling with a minimum rate, i.e., the maximum time duration $1/W$ that meets $|X(\omega)| = 0$, $\forall\, |\omega|/\pi > W$, we have the kth sample of $x(t)$ given by $x[k] = x(k/W)$. The continuous time signal can be recovered from $y(t) = \sum_k x[k]\mathrm{sinc}(Wt - k)$, with $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$.

For simplicity, we assume a uniform quantization with a fixed rate (fixed code length) [11]. Thus, let $Q : \mathbb{R} \to \mathbb{R}$ be the M-bin causal quantization function defined as

$$Q(x) = \begin{cases} \left\lceil \frac{x - x_{\min}}{\Delta} \right\rceil \Delta - \frac{\Delta}{2} + x_{\min}, & x > x_{\min} \\ \frac{\Delta}{2} + x_{\min}, & x = x_{\min} \end{cases}, \quad (1)$$

where $\Delta = (x_{\max} - x_{\min})/M$, $x_{\min} = \min\{x(t)\}$, $x_{\max} = \max\{x(t)\}$, and $\lceil\cdot\rceil$ rounds to the smallest integer greater than or equal to the argument. Assuming that the samples $x[k]$ are quantized by $Q(x)$ given in (1), the encoded signals are transmitted with the Shannon capacity $C = B\log_2(1 + P/(BN_0))$, where B is the maximum bandwidth available for signal transmission, P is the average transmit power and $N_0/2$ is the power spectral density of the additive white Gaussian noise (AWGN). For an arbitrarily low bit error rate, the following condition should be met

$$M^W \le \left(1 + \frac{P}{BN_0}\right)^B. \quad (2)$$

Moreover, we assess the MSE between $x(t)$ and its recovered counterpart obtained from the received signal $y(t) = \sum_k Q(x[k])\mathrm{sinc}(Wt - k)$ assuming an error-free transmission (i.e., the transmitted packets are received successfully). Given that $x[k] - Q[k]$ is uniformly distributed in the interval $[-\Delta/2, \Delta/2[$, the MSE between $x(t)$ and $y(t)$ can be written as

$$\mathrm{MSE}_{x,y} = \mathbb{E}\left[(x[k] - Q[k])^2\right] = \frac{(x_{\max} - x_{\min})^2}{12M^2}, \quad (3)$$

where $\mathbb{E}[\cdot]$ denotes the expectation operator.

III. Signal representation by cosine phase

Here, we propose a novel approach for a signal representation via a sequence of pulses. In a nutshell, the proposed method represents each pair of the Fourier transform terms of a zero-mean, periodic, and band-limited signal with the sum of phase-shifted cosines. Next, we convert each phase shift obtained by the proposed signal representation into a time impulse before transmission. In what follows, the proposed signal representation by the cosine phase method is described in detail.

Without losing any generality, it is assumed that each sensor node monitors a single signal. Let $x_s(t)$ and S be the signal sensed by the sth sensor node and the number of sensor nodes, respectively. Considering $x_s(t)$ as a zero-mean and τ–periodic signal with $\tau = 2\pi/\omega_0$, a natural representation of such a signal is given by the Fourier series as

$$x_s(t) = \sum_{n=1}^{N} \big(a_{n,s}\cos(n\omega_0 t) + b_{n,s}\sin(n\omega_0 t)\big), \quad (4)$$

where the summation has $N = \lfloor \pi W/\omega_0 \rfloor$ nonzero terms because $x_s(t)$ is band limited and $\lfloor\cdot\rfloor$ rounds to the largest integer less than or equal to the argument. ω_0 is the ordinary frequency. $a_{n,s}$ and $b_{n,s}$ are the coefficients of the Fourier transform. Since

$$\begin{aligned}\cos\big(n\omega_0(t - t_a)\big) + \cos\big(n\omega_0(t - t_b)\big) = \\ \big(\cos(nt_a\omega_0) + \cos(nt_b\omega_0)\big)\cos(n\omega_0 t) + \\ \big(\sin(nt_a\omega_0) + \sin(nt_b\omega_0)\big)\sin(n\omega_0 t),\end{aligned}$$

the expression in (4) can be further expressed as

$$x_s(t) = \sum_{n=1}^{N} \cos\big(n\omega_0(t - t_a^{(s,n)})\big) + \cos\big(n\omega_0(t - t_b^{(s,n)})\big), \quad (5)$$

[1]In this work, a network node is also referred to as a sensor node.

with $t_a^{(s,n)}$ and $t_a^{(s,n)}$ being the coefficients of the proposed representation by cosine phase. By solving the following set of equations

$$\begin{cases}\cos(nt_a^{(s,n)}\omega_0)+\cos(nt_b^{(s,n)}\omega_0)=a_{n,s}\\ \sin(nt_a^{(s,n)}\omega_0)+\sin(nt_b^{(s,n)}\omega_0)=b_{n,s}\end{cases}, \tag{6}$$

we can then obtain $t_a^{(s,n)}$ and $t_b^{(s,n)}$. In light of the above, we can now proceed with the quantization and transmission of the values of $t_a^{(s,n)}$ and $t_b^{(s,n)}$. We assume that the transmitter sends $t_a^{(s,n)}$ and $t_b^{(s,n)}$ at a rate lower than the channel capacity and $\hat{t}_z^{(s,n)}$ is the recovered version of $t_z^{(s,n)}$ at the receiver (it is worthwhile recalling that the decoding process can be error-free), with $z \in \{a, b\}$. By noting that

$$t_z^{(s,n)} \in \left[-\frac{\pi}{n\omega_0}, \frac{\pi}{n\omega_0}\right],$$

we can quantize $t_z^{(s,n)}$ using (1) as follows. Hereafter, we refer to the quantization interval Δ as l solely for the proposed signal representation by the cosine phase method. Thus, we have $l = 2\pi/(M_{\text{RbCP}}n\omega_0)$, with M_{RbCP}[2] being the number of bins, and for an arbitrarily low bit error rate, the following condition must be met

$$\begin{aligned}\frac{2N\log_2(M_{\text{RbCP}})}{\tau} &\le B\log_2\left(1+\frac{P}{BN_0}\right)\\ (M_{\text{RbCP}})^{\frac{2N}{\tau}} &\le \left(1+\frac{P}{BN_0}\right)^B.\end{aligned} \tag{7}$$

Note that every τ seconds, the calculation of $t_z^{(s,n)}$ and transmission process begin. Under these conditions, the following conclusions can be drawn.

First, the MSE of the recovered signal is given by

$$\text{MSE}_{\text{RbCP}} = \frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \mathbb{E}\left[(x_s(t)-\hat{x}_s(t))^2\right] dt, \tag{8}$$

where $\hat{x}_s(t)$ represents the signal recovered by applying the quantized value of $t_a^{(s,n)}$ and $t_b^{(s,n)}$ in (5).

Subsequently, using the Fourier series of $x_s(t)$ and $\hat{x}_s(t)$, it can be noted that

$$\frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\cos(kt\omega_0)\cos(nt\omega_0)\,dt = \begin{cases}0, & \text{if } k\neq n\\ \frac{1}{2}, & \text{otherwise}\end{cases},$$

$$\frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\sin(kt\omega_0)\sin(nt\omega_0)\,dt = \begin{cases}0, & \text{if } k\neq n\\ \frac{1}{2}, & \text{otherwise}\end{cases},$$

and

$$\frac{1}{\tau}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}}\cos(kt\omega_0)\sin(nt\omega_0)\,dt = 0,$$

[2]For ease of notation, we use RbCP to denote representation by cosine phase.

$\forall\, k \in \mathbb{Z}$, $n \in \mathbb{Z}$, and τ is an integer multiple of $\frac{2\pi}{\omega_0}$. After some mathematical manipulations, we get MSE_{RbCP}, which can be expressed as

$$\begin{aligned}\text{MSE}_{\text{RbCP}}{=}2N+\sum_{n=1}^{N}&\mathbb{E}\left[\cos\left(n\omega_0\left(\hat{t}_a^{(s,n)}-\hat{t}_b^{(s,n)}\right)\right)\right]+\\ &+\mathbb{E}\left[\cos\left(n\omega_0\left(t_a^{(s,n)}-t_b^{(s,n)}\right)\right)\right]\\ &-\mathbb{E}\left[\cos\left(n\omega_0\left(\hat{t}_a^{(s,n)}-t_a^{(s,n)}\right)\right)\right]\\ &-\mathbb{E}\left[\cos\left(n\omega_0\left(\hat{t}_b^{(s,n)}-t_b^{(s,n)}\right)\right)\right]\\ &-\mathbb{E}\left[\cos\left(n\omega_0\left(\hat{t}_b^{(s,n)}-t_a^{(s,n)}\right)\right)\right]\\ &-\mathbb{E}\left[\cos\left(n\omega_0\left(\hat{t}_a^{(s,n)}-t_b^{(s,n)}\right)\right)\right].\end{aligned} \tag{9}$$

Let the quantization interval l be small enough such that $E_z = t_z^{(s,n)} - \hat{t}_z^{(s,n)}$, $\forall z, s$, and n following a uniform distribution in the interval $[-l/2, l/2)$. In other words, we assume a uniformly distributed quantization error. Thus, we can calculate some terms of (9) as

$$\begin{aligned}\mathbb{E}\left[\cos\left(n\omega_0(\hat{t}_z^{(s,n)}-t_z^{(s,n)})\right)\right] &= \int_{-\frac{l}{2}}^{\frac{l}{2}}\frac{\cos(nE_z\omega_0)}{l}\,dE_z\\ &= \frac{2}{nl\omega_0}\sin\left(\frac{nl\omega_0}{2}\right),\ \forall z\in\{a,b\},\end{aligned} \tag{10}$$

$$\begin{aligned}\mathbb{E}\left[\cos\left(n\omega_0(\hat{t}_z^{(s,n)}-t_{z'}^{(s,n)})\right)\right] &= \frac{2}{nl\omega_0}\sin\left(\frac{nl\omega_0}{2}\right)\\ &\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos(n\omega_0(x-y))f_{T_a^{(s,n)},T_b^{(s,n)}}(x,y)\,dx\,dy,\\ &\forall z,z'\in\{a,b\},\ \text{and}\ z\neq z',\end{aligned} \tag{11}$$

and

$$\begin{aligned}\mathbb{E}\left[\cos\left(n\omega_0(\hat{t}_a^{(s,n)}-\hat{t}_b^{(s,n)})\right)\right] &= \frac{4\sin^2\left(\frac{nl\omega}{2}\right)}{l^2n^2\omega^2}\\ &\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\cos(n\omega_0(x-y))f_{T_a^{(s,n)},T_b^{(s,n)}}(x,y)\,dx\,dy,\end{aligned} \tag{12}$$

with $f_{T_a^{(s,n)},T_b^{(s,n)}}(x,y)$ being the joint probability density function (PDF) of $t_a^{(s,n)}$ and $t_b^{(s,n)}$. Finally, after substituting (10), (11), and (12) into (9), we obtain

$$\text{MSE}_{\text{RbCP}} = 2N - \sum_{n=1}^{N}\left[4\mathcal{Q}-(2\mathcal{Q}-1)^2\mathcal{I}\right], \tag{13}$$

where

$$\mathcal{Q} = \frac{1}{nl\omega_0}\sin\left(\frac{nl\omega_0}{2}\right) = \frac{M_{\text{RbCP}}}{2\pi}\sin\left(\frac{\pi}{M_{\text{RbCP}}}\right),$$

and

$$\mathcal{I} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{T_a^{(s,n)},T_b^{(s,n)}}(x,y)\cos(n\omega_0(x-y))dxdy.$$

It is evident that $-1 \leq \mathcal{I} \leq 1$. Therefore, MSE_{RbCP} can be lower and upper bounded by

$$1 - 4\mathcal{Q}^2 \leq \frac{\text{MSE}_{\text{RbCP}}}{N} \leq 4\left(\frac{1}{2} - \mathcal{Q}\right)\left(\frac{3}{2} - \mathcal{Q}\right), \quad (14)$$

Next, we investigate whether the signal representation by cosine phase method can perfectly reproduce the signal $x_s(t)$ (i.e., MSE $\to 0$) when $M_{\text{RbCP}} \to \infty$. Using the following approximation $\sin(x) \approx x$, we can write

$$\mathcal{Q} = \frac{nl\omega_0}{2nl\omega_0} = \frac{M_{\text{RbCP}}\pi}{2M_{\text{RbCP}}\pi} = \frac{1}{2}. \quad (15)$$

After substituting (15) into (13), it is easy to see that MSE_{RbCP} is equal to 0. Note that the equality is met for $\sin(x) = x$; however, such an approximation is more accurate as the argument x becomes smaller, i.e., $x \to 0$. Therefore, MSE_{RbCP} is asymptotically equal to 0.

IV. Transmission

A. Single transmitter

Here, we compare the MSE of the original signal $x_s(t)$ with its recovered version, $\hat{x}_s(t)$, under both the benchmark approach[3] and the proposed approach (i.e., signal representation by cosine phase approach). For the proposed approach, we assume that the signal $x_s(t)$ is represented according to (5); therefore, we calculate, quantize, and transmit the values of $t_a^{(s,n)}$ and $t_b^{(s,n)}$ every τ seconds. Moreover, we adopt an M_{RbCP}-bin quantization following (1) for the signal representation by cosine phase, similar to the benchmark approach.

By comparing (2) with (7), we can establish a relationship between the number of bins for the benchmark approach, i.e., M, and the number of bins for our proposed approach, i.e., M_{RbCP}, as

$$M_{\text{RbCP}} = M^{\frac{\tau W}{2N}}, \quad (16)$$

such that the bandwidth B and the transmission power P are the same for both.

B. Multiple transmitters

We consider two cases under a multiuser scenario. We first consider a case in which the transmission medium is shared without interference between users, i.e., an orthogonal sharing method is adopted. For example, each user has access to a portion of the available transmission band B, or equivalently, transmits for only a short period of time. In this case, the performance of each user can be obtained independently, similar to the case discussed previously in Section IV-A.

Next, we consider the case of nonorthogonal transmissions. Herein, the transmissions from all the sensor nodes are superimposed within the bandwidth B, and the receiver can apply an interference mitigation technique to recover the transmitted signals. In this instance, the nonorthogonal multiple access (NOMA) technique is adopted, where the signal-to-noise ratio (SNR) for a given sensor node is subject to degradation due to interference from other nodes. There are reported studies in the existing literature that have proposed some methods to maximize the bit rate of the sensor nodes for NOMA. For instance, Zhang *et al.* maximize the sum-weighted bit rate by jointly optimizing the three-dimensional placement of users and power allocation [12]. In this regard, the performance of a specific user can be evaluated through a peer-to-peer communication whose maximum communication rate is degraded by interference from other nodes. Therefore, we can assess the MSE of such a NOMA-based system as discussed in Section IV-A. For example, suppose that a given user i can transmit at rate R_i in a given configuration of a NOMA system. In this case, the following conditions must be met for the approach benchmarks and RbCP

$$W \log_2 M \leq R_i, \quad (17)$$

$$\frac{2N \log_2(M_{\text{RbCP}})}{\tau} \leq R_i, \quad (18)$$

respectively. By combining (17) and (18), we can write

$$M_{\text{RbCP}} = M^{\frac{\tau W}{2N}}. \quad (19)$$

Note that the result obtained for NOMA is the same as that presented in Section IV-A.

We assume that all S signals are τ-periodic with a bandwidth $W/2$. By noting that the benchmark approach and the proposed signal representation by cosine phase approach for the case of multiple sensor nodes require a transmission rate of $\log_2\left(M^{WS}\right)$ and $\log_2\left((M_{\text{RbCP}})^{2NS/\tau}\right)$, respectively, we can easily see that (16) is valid for the case of multiple sensor nodes. It turns out that the maximum number of bins for the proposed signal representation by cosine phase approach is given by

$$M_{\text{RbCP}} \leq \left(1 + \frac{P}{BN_0}\right)^{\frac{\tau B}{2NS}}. \quad (20)$$

V. Experiments and Discussions

In this section, we evaluate the MSE of the signal representation by cosine phase through Monte Carlo simulations. Recall that $\text{MSE}_{x,y}$ given in (3) provides the theoretical basis to compare against our proposed method. In short, such an MSE is achieved by transmitting at the channel capacity and applying sampling according to the Nyquist theorem. In addition, such a model adopts an M-bin quantization function $Q(\cdot)$ similar to (1). Hereafter, for ease of notation, we use the representation by cosine phase (RbCP) to denote a system that adopts the proposed approach of the signal representation, the M_{RbCP}-bin quantization function $Q(\cdot)$ as in (1), and transmits at the maximum rate.

[3]For simplicity, we use the term benchmark approach when considering a communication system that takes samples $x_s(t)$ at every $1/W$ seconds, performs M-bin quantization as in (1), and transmits at the Shannon channel capacity.

To elucidate the process of generating and filtering the signals used in our experiments[4], we first define R' as a uniform random variable in the interval $(-1, 1)$. Next, we can construct a vector $\mathbf{R} = \{r_1, r_2, ..., r_{|\mathbf{R}|}\}$ containing $|\mathbf{R}|$ independent realizations of R'. The wth term of the discrete Fourier transform of $\mathbf{R}$ can be calculated as

$$F(w) = T \sum_{k}^{|\mathbf{R}|} \mathbf{R}_k \exp(-i\omega_0 w t_k), \tag{21}$$

where $i^2 = -1$ is the imaginary unit, T and $t_k = -\tau/2 + Tk$ denote the time increment and the time given in seconds of the kth realization of R, respectively. Then, we can obtain the signal $x_s(t)$ by doing the inverse Fourier transform of $F(w)$ as follows

$$x_s(t_k) = A \sum_{w=1}^{N} F(w) \exp(i\omega_0 w t_k), \tag{22}$$

with $N = \lfloor \pi W/\omega_0 \rfloor$ and A being a constant to set the peak-peak value of $x_s(t_k)$.

In what follows, $\text{MSE}_{x,y}$ is calculated as in (3). Because all the signals $x_s(t)$ have the same value of W, the bandwidth B is divided equally among all S transmitters. Also, we assume a sampling rate R slightly higher than W. Rewriting (2) with the sampling rate R and setting M as largest possible integer value, we see that M is then given by

$$M = \left\lfloor \left(1 + \frac{P}{BN_0}\right)^{\frac{B}{RS}} \right\rfloor. \tag{23}$$

M_{RbCP} is given by the largest integer such that (20) is met. Thus, we have

$$M_{\text{RbCP}} = \left\lfloor \left(1 + \frac{P}{BN_0}\right)^{\frac{\tau B}{2NS}} \right\rfloor. \tag{24}$$

Furthermore, MSE_{RbCP} is theoretically estimated by the upper bound given by (14).

Fig. 1 compares the original signal with those recovered by RbCP and Benchmark Approach for $B = 15$ kHz, $S = 25$, $R = 16.8$, SNR $= -10$ dB, $W = 14$ Hz, $\tau = 1$ s, $N = 7$, $M = 30$, and $M_{\text{RbCP}} = 59$. Both methodologies effectively replicate the original signal. Although both solutions do not present a perfect match between the reproduced signals and the original, it is clear that there is a close similarity in the behavior.

In order to investigate the performance of both solutions in more detail, we evaluate the MSE against bandwidth B–providing an analysis centered on the network resource capacity of the system–and the peak-to-peak value of the original signal.

Fig. 2 shows the upper bound on the MSE_{RbCP}, MSE_{RbCP}, and $\text{MSE}_{x,y}$ versus B, where the curves (lines) are obtained by using (14) and (3) for the RbCP and Benchmark curves, respectively. Also, the markers are obtained through Monte Carlo simulations. It can be noted that the proposed upper bound becomes tight as B increases. The curves resemble a superposition of step functions and can be explained by rounding the expressions in (23) and (24). It is worth noting that as B increases, the RbCP method outperforms the Benchmark approach in terms of MSE. Although we cannot categorically verify such behavior by comparing (3) with (13), we can intuitively state the following: the larger the number of bins (M or M_{RbCP}), the smaller the MSE becomes. Furthermore, by realizing that M_{RbCP} grows faster than M when B increases for $\frac{\tau}{2N} > \frac{1}{R}$, we can expect that there is a value of B such that the RbCP approach outperforms the benchmark approach in terms of the MSE. This value is $B = 1.5$ kHz for the adopted parameters, as highlighted in Fig. 2.

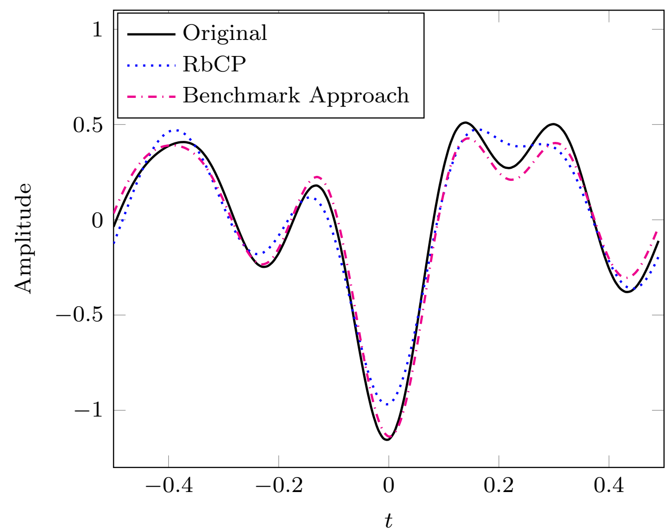

Fig. 1. Comparison between the reconstructed signal using the proposed model and the Benchmark Approach. The parameters are: $B = 15$ kHz, $S = 25$, SNR $= -10$ dB, $W = 14$ Hz, $\tau = 1$ s, $N = 7$, and the sampling rate for the Benchmark Approach is $R = 16.8$.

Fig. 3 examines the impact of the signal amplitude on the MSE. Fig. 3 plots the upper bound on the MSE_{RbCP}, MSE_{RbCP}, and $\text{MSE}_{x,y}$ versus the peak-to-peak value of the original signal, where the curves (lines) are obtained by using (14) and (3) for both the RbCP and Benchmark curves, respectively. Moreover, the markers are obtained through Monte Carlo simulations. $\text{MSE}_{x,y}$ increases with the square of the peak-to-peak value of the original signal, as described in (3), whereas MSE_{RbCP} is independent of the signal amplitude. In contrast, the RbCP approach requires that $t_a^{(s,n)}$ and $t_b^{(s,n)}$ have both real values; therefore, we may experience a deterioration in the performance of the RbCP approach because the solution of (6) could have an imaginary part for sufficiently high peak-peak values.

[4] All Python simulation files are available on GitHub at github.com/pedrogoria/Semantic-Functional-Communications

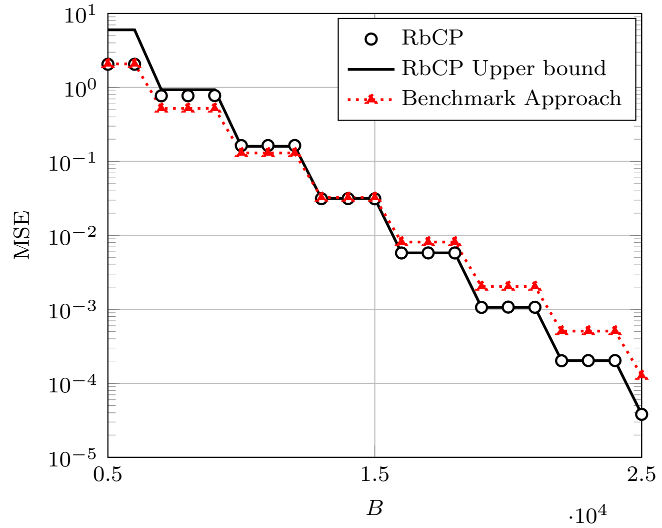

Fig. 2. MSE versus bandwidth B with $S = 25$, SNR $= -10$ dB, $W = 14$ Hz, $\tau = 1$ s, $N = 7$, and the sampling rate for the Benchmark Approach is 16.8.

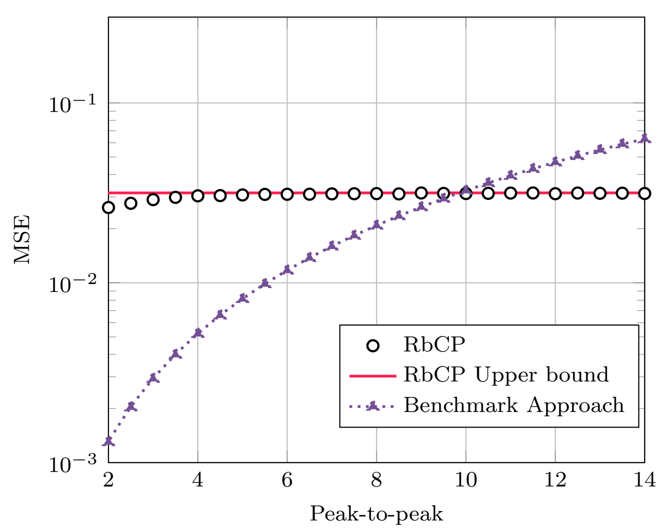

Fig. 3. MSE versus the peak-to-peak value of the signal with the parameters: $B = 15$ kHz, $S = 25$, SNR $= -10$ dB, $W = 14$ Hz, $\tau = 1$ s, $N = 7$, and the sampling rate for the Benchmark Approach is 16.8.

VI. Conclusion

This work has proposed a signal representation based on the phase shifts using a sum of cosines, which is tantamount to a sequence of impulses or events. To begin with, a closed formulation of the MSE for the proposed scheme has been derived. Furthermore, we have derived an upper and lower bound on the MSE for the proposed scheme, which are more tractable than the exact formula. For the scenarios discussed in the paper, the proposed signal representation can reduce the MSE between the reconstructed and original signals for bandwidth values B higher than a given threshold. This behavior has been observed mainly through Monte Carlo simulations, which constitutes an advancement to the state-of-the-art. A more comprehensive theoretical analysis would be beneficial to the research community and is worth being considered in future research investigations.

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References

[1] N. A. Pantazis, S. A. Nikolidakis, and D. D. Vergados, "Energy-efficient routing protocols in wireless sensor networks: A survey," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 2, pp. 551–591, Second Quarter 2013.

[2] N. A. Pantazis and D. D. Vergados, "A survey on power control issues in wireless sensor networks," *IEEE Commun. Surveys Tuts*, vol. 9, no. 4, pp. 86–107, Fourt Quarter 2007.

[3] M. de Castro Tomé, D. Gutierrez-Rojas, P. H. Nardelli, C. Kalalas, L. C. P. da Silva, and A. Pouttu, "Event-driven data acquisition for electricity metering: A tutorial," *IEEE Sensors Journal*, vol. 22, no. 6, pp. 5495–5503, 2022.

[4] X.-M. Zhang, Q.-L. Han, and B.-L. Zhang, "An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems," *IEEE Trans Ind. Informat.*, vol. 13, no. 1, pp. 4–16, Feb. 2017.

[5] M. de Castro Tomé, P. H. Nardelli, and H. Alves, "Long-range low-power wireless networks and sampling strategies in electricity metering," *IEEE Transactions on Industrial Electronics*, vol. 66, no. 2, pp. 1629–1637, 2018.

[6] K. A. Flanigan and J. P. Lynch, "Optimal event-based policy for remote parameter estimation in wireless sensing architectures under resource constraints," *IEEE Trans. Wireless Commun.*, vol. 21, no. 7, pp. 5293–5304, Jul. 2022.

[7] V. S. Varma, R. Postoyan, D. E. Quevedo, and I.-C. Morărescu, "Event-triggered transmission policies for nonlinear control systems over erasure channels," *IEEE Control Syst. Lett.*, vol. 7, pp. 2113–2118, 2023.

[8] P. E. Gória Silva, N. Marchetti, P. H. J. Nardelli, and R. A. A. de Souza, "Enabling semantic-functional communications for multiuser event transmissions via wireless power transfer," *Sensors*, vol. 23, no. 5, 2023.

[9] P. E. G. Silva, P. S. Dester, H. Siljak, N. Marchetti, P. H. Nardelli, and R. A. de Souza, "A novel semantic-functional approach for multiuser event-trigger communication," *Ad Hoc Networks*, vol. 159, p. 103496, 2024.
[10] P. E. G. Silva, P. H. J. Nardelli, A. S. de Sena, H. Siljak, N. Nevaranta, N. Marchetti, and R. A. A. de Souza, "Semantic-functional communications in cyber-physical systems," *IEEE Network*, vol. 38, no. 4, pp. 241–249, Jul. 2024.
[11] R. Gray and D. Neuhoff, "Quantization," *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2325–2383, 1998.
[12] R. Zhang, R. Tang, Y. Xu, and X. Shen, "Resource allocation for UAV-assisted NOMA systems with dual connectivity," *IEEE Wireless Commun. Lett.*, vol. 12, no. 2, pp. 341–345, Feb. 2023.